PROCEEDINGS OF SCIENCE

# Light-Front Quantization of the Chern-Simons-Higgs Theory

**Usha Kulshreshtha***
*Department of Physics, Kirori Mal College*
*University of Delhi, Delhi-110007, India.*
*E-mail:* ushakulsh@gmail.com

Light-front quantization of Chern-Simons-Higgs theory is studied in the so-called symmetry phase using Hamiltonian, path integral and BRST formulations.

*Light Cone 2010 - LC2010*
*June 14-18, 2010*
*Valencia, Spain*

*Speaker.



PoS(LC2010)008

Gauge theories in two-space one-time dimensions involving Chern-Simons (CS) term coupled to matter fields describe excitations with fractional statistics [1]-[18]. Very recently we have studied the instant-form (IF) quantization (IFQ) [15] of this theory in the presence of the Higgs potential on the hyperplanes: $x^0 = t =$ constant, in the so-called symmetry phase (SP) of the Higgs potential [15]. The IFQ of this theory has also been studied by us [16] in the so-called broken (or frozen) symmetry phase of the Higgs potential [14, 15, 16, 17]. In this talk [18], I would consider the Hamiltonian [19, 20], path integral [21, 22] and BRST [23, 24, 25] formulations of this theory using the light-front (LF) dynamics [26, 27](on the hyperplanes defined by the LC time: $(\tau \equiv x^+ = \frac{(x^0+x^1)}{\sqrt{2}} = constant)$ [26, 27] in the symmetry phase of the Higgs potential. The study of a theory using the IFQ as well as the LFQ determines the constrained dynamics of the system completely [26, 27]. Also, different aspects of this theory have been studied by several authors in various contexts[1]-[18]. We first consider the LF [26, 27] Hamiltonian and path integral formulations of this theory (in the so-called symmetry phase of the Higgs potential) [16] under appropriate gauge-fixing. The Chern-Simons-Higgs theory in two-space one-time dimensions is defined by the action [1]-[18]:

$$S = \int \mathscr{L}_1(\Phi, \Phi^*, A^\mu) d^3x \tag{1a}$$

$$\mathscr{L} = \left[\frac{\kappa}{2}\varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda + (\tilde{D}_\mu \Phi^*)(D^\mu \Phi) - V(|\Phi|^2)\right] \tag{1b}$$

$$V(|\Phi|^2) = \alpha_0 + \alpha_2 |\Phi|^2 + \alpha_4 |\Phi|^4 = \lambda(|\Phi|^2 - \Phi_0^2)^2, \quad \Phi_0 \neq 0 \tag{1c}$$

$$D_\mu = (\partial_\mu + ieA_\mu), \quad \tilde{D}_\mu = (\partial_\mu - ieA_\mu), \quad \kappa = (\frac{\theta}{2\pi^2}) \tag{1d}$$

$$g^{\mu\nu} := \text{diag}(+1,-1,-1), \quad \mu,\nu = 0,1,2, \quad \varepsilon^{012} = \varepsilon_{012} = +1 \tag{1e}$$

Here $\theta$ is the Chern-Simons parameter. The Higgs potential is kept rather general, without making any specific choice fo r the parameters of the potential except that they are chosen such that the potential remains a double well potential with $\Phi_0 \neq 0$. The LF Lagrangian density of the theory reads:

$$\begin{aligned}\mathscr{L} := \Big[&\frac{\kappa}{2}[A^+\partial_+ A_2 - A^-\partial_- A_2 + A^-\partial_2 A^+ - A^+\partial_2 A^-] \\ &-\frac{\kappa}{2}A_2(\partial_+ A^+ - \partial_- A^-) + (\partial_+\Phi^*)\partial_-\Phi + (\partial_-\Phi^*)\partial_+\Phi \\ &+ieA^+\Phi\partial_+\Phi^* - ieA^+\Phi^*\partial_+\Phi - (\partial_2\Phi^*)\partial_2\Phi \\ &+ieA^-\Phi\partial_-\Phi^* - ieA^-\Phi^*\partial_-\Phi + 2e^2A^+A^-\Phi^*\Phi \\ &+ieA_2\Phi^*\partial_2\Phi - ieA_2\Phi\partial_2\Phi^* + e^2A_2^2\Phi^*\Phi - V(|\Phi|^2)\Big]\end{aligned} \tag{2}$$

The theory is seen to possess five primary constraints:

$$\chi_1 = \Pi^+ \approx 0, \quad \chi_2 = (\Pi^- + \frac{\kappa}{2}A_2) \approx 0, \quad \chi_3 = (E - \frac{\kappa}{2}A^+) \approx 0 \tag{3a}$$

$$\chi_4 = [\Pi - \partial_-\Phi^* + ieA^+\Phi^*] \approx 0, \quad \chi_5 = [\Pi^* - \partial_-\Phi - ieA^+\Phi] \approx 0 \tag{3b}$$

Where $\Pi, \Pi^*, \Pi^+, \Pi^-$ and $E (:= \Pi^2)$ are the momenta canonically conjugate respectively to $\Phi, \Phi^*, A^-, A^+$ and $A_2$. After including these primary constraints in the canonical Hamiltonian density $\mathscr{H}_c$ with the help of the Lagrange multiplier field $s, u, v, w$ and $z$ the total Hamiltonian density $\mathscr{H}_T$ could be written as :

$$\begin{aligned}
\mathscr{H}_T = & \Big[(\Pi^+)s + (\Pi - \partial_-\Phi^* + ieA^+\Phi^*)w + (\Pi^* - \partial_-\Phi - ieA^+\Phi)z + (\Pi^- + \frac{\kappa}{2}A_2)u \\
& + (E - \frac{\kappa}{2}A^+)v + \frac{\kappa}{2}[A^-\partial_-A_2 - A^-\partial_2A^+ + A^+\partial_2A^- - A_2\partial_-A^-] \\
& - ieA^-\Phi\partial_-\Phi^* + ieA^-\Phi^*\partial_-\Phi - 2e^2A^+A^-\Phi^*\Phi + (\partial_2\Phi^*)\partial_2\Phi \\
& - ieA_2\Phi\partial_2\Phi^* + ieA_2\Phi^*\partial_2\Phi - e^2A_2^2\Phi^*\Phi + V(|\Phi|^2)\Big] \qquad (4)
\end{aligned}$$

Demanding that the primary constraint $\chi_1$ be preserved in the course of time, one obtains the secondary Gauss-law constraint: $\chi_6 = [ie(\Phi\partial_-\Phi^* - \Phi^*\partial_-\Phi) + 2e^2A^+\Phi^*\Phi] \approx 0$ The preservation of $\chi_2, \chi_3, \chi_4$ and $\chi_5$, as well as of $\chi_6$ for all times does not give rise to any further constraints. The theory is thus seen to possess only six constraints $\chi_i$. The constraints $\chi_4, \chi_5$ and $\chi_6$ could now be combined to give a new constraint: $\Omega = [ie(\Pi\Phi - \Pi^*\Phi^*)] \approx 0$ yielding the new set of constraints of the theory as: $\eta_1 = \chi_1, \quad \eta_2 = \chi_2, \quad \eta_3 = \chi_3, \quad \eta_4 = \Omega$. Further, the matrix of the Poisson brackets among the constraints $\eta_i$ , with $(i = 1, 2, 3, 4)$ is seen to be a singular matrix implying that the set of constraints $\eta_i$ is first-class and that the theory under consideration is gauge-invariant (GI). The divergence of the vector gauge current density of the theory could now be easily seen to vanish satisfying the continuity equation: $\partial_\mu j^\mu = 0$ , implying that the theory possesses at the classical level, a local vector-gauge symmetry. The action of the theory is indeed seen to be invariant under the local vector gauge transformations:

$$\delta\Phi = i\beta\Phi, \quad \delta\Phi^* = -i\beta\Phi^*, \quad \delta A^- = -\partial_+\beta, \quad \delta A_2 = -\partial_2\beta \qquad (5a)$$

$$\delta A^+ = -\partial_-\beta, \quad \delta\Pi^+ = o, \quad \delta E = \frac{-\kappa}{2}\partial_-\beta, \quad \delta\Pi^- = \frac{\kappa}{2}\partial_2\beta \qquad (5b)$$

$$\delta\Pi = [-i\beta\partial_-\Phi^* - e\beta A^+\Phi^* + i(e-1)\Phi^*\partial_-\beta], \quad \delta s = -\partial_+\partial_+\beta \qquad (5c)$$

$$\delta\Pi^* = [i\beta\partial_-\Phi - e\beta A^+\Phi - i(e-1)\Phi\partial_-\beta], \quad \delta u = -\partial_+\partial_-\beta \qquad (5d)$$

$$\delta w = (i\beta\partial_+\Phi + i\Phi\partial_+\beta), \quad \delta z = (-i\beta\partial_+\Phi^* - i\Phi^*\partial_+\beta) \qquad (5e)$$

$$\delta v = -\partial_+\partial_2\beta, \quad \delta\Pi_s = \delta\Pi_u = \delta\Pi_v = \delta\Pi_w = \delta\Pi_z = 0 \qquad (5f)$$

where $\beta \equiv \beta(x^+, x^-, x^2)$ is an arbitrary function of its arguments. Dirac quantization of the theory under the gauge: $\Psi_1 = \Phi \approx 0$ and $\Psi_2 = A^- \approx 0$ yields the non-vanishing equal light-cone-time commutators of the theory as:

$$[\Phi^*(x^+,x^-,x_2)\ ,\ \Pi(x^+,x^-,x_2)] = \frac{i\Phi^*}{\Phi}\delta(x^- - y^-)\delta(x_2 - y_2) \tag{6a}$$

$$[\Pi^*(x^+,x^-,x_2)\ ,\ \Pi(x^+,x^-,x_2)] = \frac{-i\Pi^*}{\Phi}\delta(x^- - y^-)\delta(x_2 - y_2) \tag{6b}$$

$$[\Phi^*(x^+,x^-,x_2)\ ,\ \Pi^*(x^+,x^-,x_2)] = i\delta(x^- - y^-)\delta(x_2 - y_2) \tag{6c}$$

$$[A^+(x^+,x^-,x_2)\ ,\ A_2(x^+,x^-,x_2)] = \frac{-i}{\kappa}\delta(x^- - y^-)\delta(x_2 - y_2) \tag{6d}$$

$$[A^+(x^+,x^-,x_2)\ ,\ \Pi^-(x^+,x^-,x_2)] = \frac{i}{2}\delta(x^- - y^-)\delta(x_2 - y_2) \tag{6e}$$

$$[A_2(x^+,x^-,x_2)\ ,\ \Pi(x^+,x^-,x_2)] = \frac{i}{2}\delta(x^- - y^-)\delta(x_2 - y_2) \tag{6f}$$

$$[\Pi^-(x^+,x^-,x_2)\ ,\ E(x^+,x^-,x_2)] = \frac{-i\kappa}{4}\delta(x^- - y^-)\delta(x_2 - y_2) \tag{6g}$$

Where $\Phi \approx 0$ represents the Coulomb gauge and $A^- \approx 0$ represents the light-cone coulomb gauge. In the path integral formulation, the transition to quantum theory is made by writing the vacuum to vacuum transition amplitude for the theory called the generating functional $Z[J_k]$ of the theory [16, 17, 18, 21, 22] under the gauge-fixing under consideration, in the presence of the external sources $J_k$ as:

$$\begin{aligned} Z[J_k] = \int [d\mu] \exp\Bigg[ i \int d^3x \Bigg[ & J_k\Phi^k + \Pi\partial_+\Phi + \Pi^*\partial_+\Phi^* + \Pi^+\partial_+A^- + \Pi^-\partial_+A^+ \\ & + E\partial_+A_2 + \Pi_s\partial_+s + \Pi_u\partial_+u + \Pi_v\partial_+v + \Pi_w\partial_+w + \Pi_z\partial_+z - \mathscr{H}_T \Bigg]\Bigg] \end{aligned} \tag{7}$$

Here, the phase space variables of the theory are: $\Phi^k \equiv (\Phi, \Phi^*, A^-, A^+, A_2, s, u, v, w, z)$ wi th the corresponding respective canonical conjugate momenta: $\Pi_k \equiv (\Pi, \Pi^*, \Pi^+, \Pi^-, E,$ $\Pi_s, \Pi_u,\ \Pi_v, \Pi_w, \Pi_z)$. The functional measure $[d\mu]$ of the generating functional $Z[J_k]$ under the above gauge-fixing is obtained as :

$$\begin{aligned} [d\mu] = {} & [(ie\kappa\Phi)\delta^3(x^- - y^-)\delta^3(x_2 - y_2)][d\Phi][d\Phi^*][dA^+][dA^-][dA_2][ds][du][dv] \\ & [dw][dz][d\Pi][d\Pi^*][d\Pi^-][d\Pi^+][dE][d\Pi_s][d\Pi_u][d\Pi_v][d\Pi_w][d\Pi_z] \\ & \delta[\Pi^+ \approx 0]\delta[(\Pi^- + \frac{\kappa}{2}A_2) \approx 0]\delta[(E - \frac{\kappa}{2}A^+) \approx 0] \\ & \delta[(ie(\Pi\Phi - \Pi^*\Phi^*) \approx 0]\delta[\Phi \approx 0]\delta[A^- \approx 0] \end{aligned} \tag{8}$$

For the BRST formulation of the model, we rewrite the theory as a quantum system that possesses the generalized gauge invariance called BRST symmetry. For this, we first enlarge the Hilbert space of our gauge-invariant theory and replace the notion of gauge-transformation, which shifts operators by c-number functions, by a BRST transformation, which mixes operators with Bose and Fermi statistics. We then introduce new anti-commuting variable c and $\bar{c}$ (Grassman numbers on the classical level and operators in the quantized theory) and a commuting variable $b$ such that[23, 24, 25]:

$$\hat{\delta}\Phi = ic\Phi, \quad \hat{\delta}\Phi^* = -i\bar{c}\Phi^*, \quad \hat{\delta}A^- = -\partial_+ c, \quad \hat{\delta}A_2 = -\partial_2 c \tag{9a}$$

$$\hat{\delta}A^+ = -\partial_- c, \quad \hat{\delta}\Pi^+ = o, \quad \hat{\delta}E = -\frac{\kappa}{2}\partial_- c, \quad \hat{\delta}\Pi^- = \frac{\kappa}{2}\partial_2 c \tag{9b}$$

$$\hat{\delta}\Pi = [-ic\partial_-\Phi^* - ecA^+\Phi^* + i(e-1)\Phi^*\partial_- c], \quad \hat{\delta}v = -\partial_+\partial_2 c \tag{9c}$$

$$\hat{\delta}\Pi^* = [i\bar{c}\partial_-\Phi - e\bar{c}A^+\Phi - i(e-1)\Phi\partial_-\bar{c}], \quad \hat{\delta}s = -\partial_+\partial_+ c \tag{9d}$$

$$\delta w = (ic\partial_+\Phi + i\Phi\partial_+ c), \quad \delta z = (-ic\partial_+\Phi^* - i\Phi^*\partial_+ c) \tag{9e}$$

$$\hat{\delta}u = -\partial_+\partial_- c, \quad \hat{\delta}\Pi_u = \hat{\delta}\Pi_v = \hat{\delta}\Pi_w = \hat{\delta}\Pi_z = \hat{\delta}\Pi_s = 0 \tag{9f}$$

$$\hat{\delta}c = 0, \quad \hat{\delta}\bar{c} = b, \quad \hat{\delta}b = 0 \tag{9g}$$

with the property $\hat{\delta}^2 = 0$. We now define a BRST-invariant function of the dynamical variables to be a function $f$ such that $\hat{\delta} f = 0$. Now the BRST gauge-fixed quantum Lagrangian density $\mathscr{L}_{BRST}$ for the theory could be written as:

$$\begin{aligned}
\mathscr{L}_{BRST} := \Big[ & \frac{\kappa}{2}[-A^-\partial_- A_2 + A^-\partial_2 A^+ - A^+\partial_2 A^- + A_2\partial_- A^-] \\
& + ieA^-\Phi\partial_-\Phi^* - ieA^-\Phi^*\partial_-\Phi + 2e^2A^+A^-\Phi^*\Phi - (\partial_2\Phi^*)\partial_2\Phi \\
& + ieA_2\Phi\partial_2\Phi^* - ieA_2\Phi^*\partial_2\Phi + e^2A_2^2\Phi^*\Phi + \partial_-\Phi^*\partial_+\Phi \\
& + \partial_+\Phi^*\partial_-\Phi - ieA^+\Phi^*\partial_+\Phi + ieA^+\Phi\partial_+\Phi^* - \frac{\kappa}{2}A_2\partial_+ A^+ \\
& + \frac{\kappa}{2}A^+\partial_+ A_2 - V(|\Phi|^2) - \hat{\delta}[\bar{c}(\partial_+ A^- - \frac{1}{2}b)]\Big]
\end{aligned} \tag{10}$$

The last term in the above equation is the extra BRST-invariant gauge-fixing term. Proceeding classically, the Euler-Lagrange equation for $b$ reads: $b = (\partial_+ A^-)$ and the requirement $\hat{\delta}b = 0$ then implies $\hat{\delta}b = [\hat{\delta}(\partial_+ A^-)]$ leading finally to $\partial_+\partial_+ c = 0$. In introducing momenta one has to be careful in defining those for the fermionic variables. We thus define the bosonic momenta in the usual manner so that:

$$\Pi^+ := \frac{\partial}{\partial(\partial_+ A^-)}\mathscr{L}_{BRST} = -b \tag{11}$$

but for the fermionic momenta with directional derivatives we set

$$\Pi_c := \mathscr{L}_{BRST}\frac{\overleftarrow{\partial}}{\partial(\partial_+ c)} = \partial_+\bar{c}, \quad \Pi_{\bar{c}} := \frac{\overrightarrow{\partial}}{\partial(\partial_+\bar{c})}\mathscr{L}_{BRST} = \partial_+ c \tag{12}$$

implying that the variable canonically conjugate to $c$ is ( $\partial_+\bar{c}$) and the variable conjugate to $\bar{c}$ is ($\partial_+ c$). The quantum BRST-Hamiltonian density of the theory is :

$$\begin{aligned}
\mathscr{H}_{BRST} = \Big[ & \Pi_s\partial_+ s + \Pi_u\partial_+ u + \Pi_v\partial_+ v + \Pi_w\partial_+ w + \Pi_z\partial_+ z - e^2A_2^2\Phi^*\Phi \\
& + \frac{\kappa}{2}[A^-\partial_- A_2 - A^-\partial_2 A^+ + A^+\partial_2 A^- - A_2\partial_- A^-] \\
& - ieA^-\Phi\partial_-\Phi^* + ieA^-\Phi^*\partial_-\Phi - 2e^2A^+A^-\Phi^*\Phi + (\partial_2\Phi^*)(\partial_2\Phi) \\
& - ieA_2\Phi\partial_2\Phi^* + ieA_2\Phi^*\partial_2\Phi + V(|\Phi|^2) - \frac{1}{2}(\Pi^+)^2 + + \Pi_c\Pi_{\bar{c}}\Big]
\end{aligned} \tag{13}$$

In general, $c$ and $\bar{c}$ are independent canonical variables and one assumes that

$$\{\Pi_c, \Pi_{\bar{c}}\} = \{\bar{c}, c\} = \partial_+\{\bar{c}, c\} = 0, \quad \{\partial_+\bar{c}, c\} = (-1)\{\partial_+ c, \bar{c}\} \tag{14}$$

Here { , } means an anti-commutator. We thus see that the anticommulators in the above equation are non-trivial and need to be fixed. In order to fix these, we demand that c satisfy the Heisenberg equation: $[c, \mathscr{H}_{BRST}] = i\partial_+ c$ and using the property $c^2 = \bar{c}^2 = 0$ one obtains $[c, \mathscr{H}_{BRST}] = \{\partial_+\bar{c}, c\}\partial_+ c$ leading eventually to:

$$\{\partial_+\bar{c}, c\} = (-1)\{\partial_+ c, \bar{c}\} = i \tag{15}$$

The minus sign in the above equation is nontrivial and implies the existence of states with negative norm in the space of state vectors of the theory. The BRST charge operator $Q$ is the generator of the BRST transformations. It is nilpotent and satisfies $Q^2 = 0$. It mixes operators which satisfy Bose and Fermi statistics. The BRST charge operator of the present theory can be written as:

$$Q = \int dx^- dx_2 \left[ -ec(\Pi\Phi - \Pi^*\Phi^*) - i\partial_+ c[(\Pi^+ + \Pi^- + E) + \frac{\kappa}{2}(A_2 - A^+)] \right] \tag{16}$$

This equation implies that the set of states satisfying the constraints of the theory belong to the dynamically stable subspace of states $|\psi>$ satisfying $Q|\psi>= 0$, i.e., it belongs to the set of BRST-invariant states. Further the theory is seen to possess negative norm states in the fermionic sector and the existence of these negative norm states as free states of the fermionic part of $\mathscr{H}_{BRST}$ is irrelevant to the existence of physical states in the orthogonal subspace of the Hilbert space. The Hamiltonian is also invariant under the anti-BRST transformation given by:

$$\bar{\hat{\delta}}\Phi = -i\bar{c}\Phi, \quad \bar{\hat{\delta}}\Phi^* = ic\Phi^*, \quad \bar{\hat{\delta}}A^- = \partial_+\bar{c}, \quad \bar{\hat{\delta}}A_2 = \partial_2\bar{c} \tag{17a}$$

$$\bar{\hat{\delta}}A^+ = \partial_-\bar{c}, \quad \bar{\hat{\delta}}\Pi^+ = 0, \quad \bar{\hat{\delta}}\Pi^- = -\frac{\kappa}{2}\partial_2\bar{c}, \quad \bar{\hat{\delta}}E = \frac{\kappa}{2}\partial_-\bar{c} \tag{17b}$$

$$\bar{\hat{\delta}}\Pi = [i\bar{c}\partial_-\Phi^* + e\bar{c}A^+\Phi^* - i(e-1)\Phi^*\partial_-\bar{c}], \quad \bar{\hat{\delta}}s = \partial_+\partial_+\bar{c} \tag{17c}$$

$$\bar{\hat{\delta}}\Pi^* = [-ic\partial_-\Phi + ecA^+\Phi + i(e-1)\Phi\partial_- c], \quad \bar{\hat{\delta}}v = \partial_+\partial_2\bar{c} \tag{17d}$$

$$\bar{\hat{\delta}}w = (-i\bar{c}\partial_+\Phi - i\Phi\partial_+\bar{c}), \quad \bar{\hat{\delta}}z = (i\bar{c}\partial_+\Phi^* + i\Phi^*\partial_+\bar{c}) \tag{17e}$$

$$\bar{\hat{\delta}}u = \partial_+\partial_-\bar{c}, \quad \bar{\hat{\delta}}\Pi_s = \bar{\hat{\delta}}\Pi_u = \bar{\hat{\delta}}\Pi_v = \bar{\hat{\delta}}\Pi_w = \bar{\hat{\delta}}\Pi_z = 0 \tag{17f}$$

$$\bar{\hat{\delta}}\bar{c} = 0, \quad \bar{\hat{\delta}}c = -b, \quad \bar{\hat{\delta}}b = 0 \tag{17g}$$

with generator or anti-BRST charge

$$\bar{Q} = \int dx^- dx_2 \left[ e\bar{c}(\Pi\Phi - \Pi^*\Phi^*) + i\partial_+\bar{c}[(\Pi^+ + \Pi^- + E) + \frac{\kappa}{2}(A_2 - A^+)] \right] \tag{18}$$

We also have $\partial_+ Q = [Q, H_{BRST}] = 0$ and $\partial_+\bar{Q} = [\bar{Q}, H_{BRST}] = 0$ with $H_{BRST} = \int dx^- dx_2 \mathscr{H}_{BRST}$. We further impose the dual condition that both $Q$ and $\bar{Q}$ annihilate physical states, implying that: $Q|\psi>= 0 \quad and \quad \bar{Q}|\psi>= 0$. The states for which the constraints of the theory hold, satisfy both of these conditions and are in fact, the only states satisfying both of these conditions.

Now because $Q|\psi>= 0$, the set of states annihilated by $Q$ contains not only the set of states for which the constraints of the theory hold but also additional states for which the constraints of the theory do not hold in particular. This situation is, however, easily avoided by additionally imposing on the theory, the dual condition: $Q|\psi>=0$ and $\bar{Q}|\psi>=0$. Thus by imposing both of these conditions on the theory simultaneously, one finds that the states for which the constraints of the theory hold satisfy both of these conditions and, in fact, these are the only states satisfying both of these conditions because in view of the conditions on the fermionic variables $c$ and $\bar{c}$ one cannot have simultaneously $c$ , $\partial_+ c$ and $\bar{c}$, $\partial_+\bar{c}$, applied to $|\psi>$ to give zero. Thus the only states satisfying $Q|\psi>=0$ and $\bar{Q}|\psi>=0$ are those that satisfy the constraints of the theory and they belong to the set of BRST-invariant as well as to the set of anti-BRST-invariant states.

I thank Dayashankar Kulshreshtha and James Vary for the collaboration. I also thank James Vary for his kind hospitality at the Iowa State University which made this collaboration possible. I express my heart felt thanks to Professor Vicente Vento and Professor Joannis Papavassiliou and all other Organizers of the Conference for providing a very stimulating environment during the Conference. I have my very special thanks to the Gary McCartar Foundation for choosing me for the award of "Gary McCartar Award" for the year 2010 which helped me in participating in the LC2010 workshop at Valencia.

## References

[1] For a comprehensive recent review see e.g., "Aspects of Chern-Simons Theories", G.V. Dunne, hep-th/9902115, and references therein; A. J. Niemi and G.W. semenoff, Phys. Rev. Lett. **51**, 2077 (1983); A. N. Redilich, Phys. Rev. Lett. **52**, 18 (1984); Phys. Rev. **D29**, 2366 (1984); K. Ishikawa Phys. Rev. Lett. **53**, 1615 (1984); M. B. Paranjape, Phys. Rev. Lett. **55**, 2390 (1985); G. W. Semenoff and P. Sodano, Phys. Rev. lett. **57**, 1195 (1986).

[2] J. Leinaas and J. Myrheim, Nuovo Cimento **B37**, 1 (1977); F. Wilczek, Phys. Rev. Lett. **1982**, 957 (1982); R. B. Laughlin, Science, **242**, 525, (1988); A. Fetter, C. Hanna and R. B. Laughlin, Phys. Rev. **B39**, 9679 (1989); see also R. Prange and S. Girvin, "The quantum Hall Effect" (Springer Verlag, Berlin, 1990).

[3] S. Deser, R. Jackiw and S. Templeton, Phys. Rev. Lett. **48**, 975 (1982) ; Ann. Phys. (N.Y.), **140**, 372 (19 82); R. Jackiw and S. Templeton, Phys. Rev. **D23**, 2291 (1981).

[4] J. F. Schonfeld, Nucl. Phys. **B185**, 157 (1981).

[5] R. Mac Kenzie and F. Wilczek, Int. J. Mod. Phys. **A3**, 2827 (1988).

[6] Q. G. Lin, Guang-Jiong Ni, Class. Quant. Grav. **7**, 1261 (1990); Commun. Theor. Phys. **30**, 249 (1998); hep-th/9807177; Commun. Theor. Phys. **28**, 225 (1997); hep-th/9807176.

[7] F. Ferrari and I. Lazzizzera, Phys. Lett. **B395**, 250 (1997).

[8] M. Chaichian, W. F. Chen, and Z.Y. Zhu, Phys. Lett. **B 387**, 785 (1996).

[9] D. Bak, R. Jackiw, S. Y. Pi, Phys. Rev. **D49**, 6778 (1994); see also R. Jackiw, Annals Phys. (N.Y.) **201**, 83 (1990); R. Jackiw and S. Y. Pi, Phys. Rev. **D42**, 3500 (1990); *ibid* **D48**, 3929 (1990); Phys. Rev. Lett. **64**, 2969 (1990); *ibid* **66**, 2682 (1991); Nucl. Phys. B (Proc. Suppl.) **33C**, 104 (1993).

[10] R. Banerjee, Nucl. Phys. **B390**, 681, 1993; R. Banerjee, Annals Phys. **222**, 254 (1993).

[11] Todahiko Kimura, Prog. Theor. Phys. **81**, 1109 (1989); W. Ogura, Phys. Lett. **229**, 61 (1989).

[12] V. Ya. Fainberg, N.K. Pak, and M.S. Shikakhwa, J. Phys. **A30**, 3947 (1997).

[13] Y. Igarashi, H. Imai, S. Kitakado, J. Kubo, and H. So, Mod. Phys. Lett. **A5**, 1663 (1990); W.T. Kim and Y. J. Park, Phys. Lett. **B336**, 376 (1994).

[14] D. Boyanovsky, Phys. Rev.**D42**, 1179 (1990); see also D. Boyanovsky and R. Blankenbecler, Phys. Rev. **D31**, 3234 (1985).

[15] U. Kulshreshtha, D. S. Kulshreshtha, H.J.W. Mueller-Kirsten and J. P. Vary, Physica Scripta **79**, 045001 (2009).

[16] U. Kulshreshtha, Physica Scripta **75**, 795 (2007).

[17] U. Kulshreshtha and D. S. Kulshreshtha, Canadian J. Phys. **86**, (2008).

[18] For the details of this work see U. Kulshreshtha, D. S. Kulshreshtha and J.P. Vary, Physica Scripta **80**, (2010) - in press.

[19] P. A. M. Dirac, Can. J. Math 2, 129 (1950); M. Henneaux and C. Teitelboim, “Quantization of Gauge Systems”, Princeton University Press, Princeton, New Jersey, 1992; D. M. Gitman and I.V. Tyutin, "Quantization of Fields with Constraints" (Springer Verlag 1990).

[20] U. Kulshreshtha and D. S. Kulshreshtha, Canad. J. Phys. **82**, 569 (2004); Canad. J. Phys. **82**, 843 (2004); U. Kulshreshtha,Int. J. Theor. Phys. **41**,273 (2002).

[21] P. Sanjanovic, Annals Phys. (N.Y.) **100**, 227 (1976).

[22] U. Kulshreshtha and D. S. Kulshreshtha, Phys. Lett. **B555**, 255 (2003); U. Kulshreshtha and D.S. Kulshreshtha, Eur. Phys. J. **C29**, 453 (2003); Int. J. Theor. Phys. **43**, 2355 (2004); Int.J. Theor. Phys. **44**,587 (2005).

[23] C. Becchi, A. Rouet and A. Stora, Phys. Lett. **B52**, 344 (1974); V. Tyutin, Lebedev Report No. FIAN-39 (unpublished).

[24] D. Nemeschansky, C. Preitschopf and M. Weinstein, Ann. Phys.(N.Y.) **183**, 226 (1988).

[25] U. Kulshreshtha, D. S. Kulshreshtha and H. J. W. Mueller-Kirsten, Zeit. f. Phys. **C60**, 427 (1993); U. Kulshreshtha, D. S. Kulshreshtha and H. J. W. Mueller-Kirsten, Zeit.f. Phys. **C64**,169 (1994); U. Kulshreshtha, D. S. Kulshreshtha and H. J. W. Mueller-Kirsten, Phys. Rev. **D47**, 4634 (1993).

[26] P. A. M. Dirac, Rev. Mod. Phys.**21**, 392 (1949); for a recent review on Light-Cone Quantization, see e.g., S. J. Brodsky, H. C. Pauli and S. S. Pinsky, Phys. Rep. **301**, 299 (1998).

[27] U. Kulshreshtha, Int. J. Theor. Phys. **46** (2007); U. Kulshreshtha, Helv. Phys. Acta **71**, 353 (1998); Canad. J. Phys. **79**, 1085 (2001); U. Kulshreshtha and D. S. Kulshreshtha, Int. J. Theor. Phys. **37**, 2539 (1998); Canad. J. Phys. **80**, 791 (2002); Int. J. Theor. Phys. **41**, 2395 (2002).