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A systematic hamiltonian analysis of exactly solvable models and their vacuum structure

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Simple two-dimensional models with massless and massive fermions are studied within the conventional (spacelike) and light front forms of the hamiltonian dynamics. The ultimate motivation is to understand deeper the relationship between these two formulations of quantum field theory. The models under study include versions of the derivative coupling model (Schroer, Rothe-Stamatescu), the Thirring and the Federbush model. The correct quantum Hamiltonians that incorporate knowledge of the operator solutions of the field equations, are derived. Some previous results are critically examined. While the derivative-coupling model is found to be equivalent in many respects to a free theory, the physical vacuum states of the massless Thirring and Federbush models can be obtained by means of a Bogoliubov transformation in the form of a coherent state quadratic in composite boson operators. The massive version of the Klaiber's current bosonization is constructed in the both space-like and light front forms. The massive Federbush model is shown to have the same interacting structure in the both schemes when the operator solutions are taken into account. We argue that the Federbush model is particularly suitable for a detailed nonperturbative comparison between the two forms of the relativistic dynamics.

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1. Introduction

Exactly solvable models are simple two-dimensional relativistic field theories in which operator solutions of the field equations can be found. They include versions of the model with derivative coupling [1, 2, 3], the massless Thirring model [4] and the massive Federbush model [5].

The exactly solvable models provide us with a suitable ground for analyzing the nonperturbative structure of the SL and LF versions of QFT as well as for a deeper understanding of the relationship between the two forms [6] of relativistic QFT. Looking at the main features of the two schemes, one notices striking mathematical as well as physical differences between them. Different physical structures include nature of field variables (dynamical versus dependent), different number of kinematical and dynamical Poincaré generators, status of the vacuum state, desription of "spinor" field in in two space-time dimensions. It is clear that physical predictions (amplitudes, correlation functions, etc.) should agree in both formalisms if these are applied in a mathematically correct manner. It is also important to develop the light front picture without merely rewriting mechanisms known in the conventional field theory to LF variables.

In the case of the exactly soluble models, SL and LF correlation functions can be calculated nonperturbatively and compared. In this way, the role of the vacuum state and of the operator part can be explicitly "visible", allowing one to understand the (dis)advantages of the two formalisms. For example, a potential weak point in the SL models is treatment of their vacuum states since one does not know the true lowest-energy eigenstate of the full Hamiltonian, just of its free part. We shall therefore critically examine the SL versions of the few exactly solvable models. In doing that, we shall express the SL Hamiltonians in terms of composite boson operators, obtained by bosonization of the fermion currents. In this way, interacting terms become bilinear and can be diagonalized by a Bogoliubov transformation: the true lowest-energy eigenstate will appear as a transformed Fock vacuum. The correlators should be calculated as its expectation values.

Another new element of our approach is a modification of the canonical procedure. The knowledge of the operator solutions (which tells us how the interacting field is composed from free fields) has to be taken into account in order to work with true field degrees of freedom. In an analogy to an elimination of nondynamical fields by using their constraints, this solution has to be inserted back to the original Lagrangian. This simple observation leads to remarkable changes in the form of the Lagrangian and Hamiltonian. This procedure is not the same as inserting Dirac equation to the Lagrangian (which would be an illegal step): the Dirac equation tells us what $\gamma^\mu \partial_\mu \Psi$ is while the operator solution implies knowledge directly of $\partial_\mu \Psi$. We shall find that the true Hamiltonian of the derivative-coupling model does not contain interaction, the interacting Hamiltonian of the Thirring model has opposite sign as expected naively and the SL and LF Hamiltonians of the Federbush model have analogous structure. In the usual treatment, their structure differs completely.

2. SL and LF derivative-coupling model

This is the simplest relativistic model [7] and turns out to be quite useful for illustration of the derivation of the correct Hamiltonians and vacuum states in the both quantization schemes as well as for comparison between them. The Lagrangian density of the derivative-coupling model is

$$\mathscr{L} = \frac{i}{2}\overline{\Psi}\gamma^\mu \overleftrightarrow{\partial_\mu} \Psi - m\overline{\Psi}\Psi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\mu^2\phi^2 - g\partial_\mu\phi J^\mu, \;\; J^\mu = \overline{\Psi}\gamma^\mu\Psi. \tag{2.1}$$

Both scalar and fermion field are massive. For $\mu = 0$ the theory is known as the Schroer's model [1], for interaction with the axial vector as the Rothe-Stamatescu model ($m = 0, \mu \neq 0$) [2].

The Euler-Lagrange equations following from the above Lagrangian are

$$i\gamma^\mu \partial_\mu \Psi = m\Psi + g\partial_\mu \phi \gamma^\mu \Psi, \quad \partial_\mu \partial^\mu \phi + \mu^2 \phi = g\partial_\mu J^\mu = 0. \tag{2.2}$$

We will use capital Greek letters for interacting Heisenberg fields and small letters for the free fields. The free scalar field $\phi(x)$ which enters into the operator solution

$$\Psi(x) =: e^{ig\phi(x)} : \psi(x), \;\; i\gamma^\mu \partial_\mu \psi(x) = m\psi(x) \tag{2.3}$$

is quantized as (our notation is $\hat{k}.x \equiv E(k^1)t - k^1 x^1, E(k^1) = \sqrt{k_1^2 + \mu^2}$)

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{+\infty} \frac{dk^1}{\sqrt{2E(k^1)}} \left[a(k^1)e^{-i\hat{k}.x} + a^\dagger(k^1)e^{i\hat{k}.x}\right]. \tag{2.4}$$

We will also need the free massive fermion field

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{+\infty} \frac{dp^1}{\sqrt{2E(p^1)}} \left[u(p^1)b(p^1)e^{-i\hat{p}.x} + v(p^1)d^\dagger(p^1)e^{i\hat{p}.x}\right],$$
$$u^\dagger(p^1) = (\sqrt{p^+}, \sqrt{p^-}), \;\; v(p^1)^\dagger = (\sqrt{p^+}, -\sqrt{p^-}), \;\; p^\pm = E(p^1) \pm p^1. \tag{2.5}$$

The conjugate momenta are calculated directly as $\Pi_\phi = \partial_0 \phi(x) - gJ^0$, $\Pi_\Psi = (i/2)\Psi^\dagger(x)$, $\Pi_{\Psi^\dagger} = \Pi_\Psi^\dagger(x)$. They lead to the Hamiltonian $H = H_{0B} + H^{'}$. H_{0B} is the free scalar-field part and

$$H^{'} = \int\limits_{-\infty}^{+\infty} \mathrm{d}x^1 \left[-i\Psi^\dagger \alpha^1 \partial_1 \Psi + m\Psi^\dagger \gamma^0 \Psi + g\partial_1 \phi J^1\right]. \tag{2.6}$$

Conventionally, one inserts the free field into the first (kinetic) term of $H^{'}$. This procedure yields $H_{0F} = \int\limits_{-\infty}^{+\infty} \mathrm{d}x^1 \left[-i\psi^\dagger \alpha^1 \partial_1 \psi + m\psi^\dagger \gamma^0 \psi\right]$. After inserting the current in the bosonized form,

$$j^\mu(x) = -\frac{i}{\sqrt{2\pi}} \int \frac{dk^1}{\sqrt{2k^0}} k^\mu \left\{c(k^1)e^{-i\hat{k}.x} - c^\dagger(k^1)e^{i\hat{k}.x}\right\}, \tag{2.7}$$

the interaction term in Eq.(2.6) becomes

$$H_{int} = \frac{g}{2\sqrt{\pi}} \int\limits_{-\infty}^{+\infty} \mathrm{d}k^1 \left[c^\dagger(k^1)a(k^1) + a^\dagger(k^1)c(k^1) + a^\dagger(k^1)c^\dagger(k^1) + a(k^1)c(k^1)\right]. \tag{2.8}$$

The composite boson operators, satisfying $[c(k), c^\dagger(l)] = \delta(k-l)$, are given as

$$c(k^1) = \frac{i}{\sqrt{k^0}} \int dp^1 \left\{\theta\left(p^1 k^1\right)\right) \left[b^\dagger(p^1)b(p^1+k^1) - d^\dagger(p^1)d(p^1+k^1)\right] +$$
$$+ \varepsilon(p^1)\theta\left(p^1(p^1-k^1)\right)d(k^1-p^1)b(p^1)\right\}. \tag{2.9}$$

The Hamiltonian is non-diagonal. It can be diagonalized by a Bogoliubov transformation which for $m = 0$ is implemented by means of a unitary operator $U = \exp(iS)$ with [1]

$$S(\gamma) = -i \int_{-\infty}^{+\infty} \mathrm{d}k^1 \gamma(k) \big[c^\dagger(k^1) a^\dagger(-k^1) - c(k^1) a(-k^1)\big]. \tag{2.10}$$

The physical vacuum is then found in a complete disagreement with the LF results as

$$|\Omega\rangle = N \exp\big[\gamma(g) \int_{-\infty}^{+\infty} \mathrm{d}k^1 c^\dagger(-k^1) a^\dagger(k^1)\big] |0\rangle. \tag{2.11}$$

We shall sketch the LF analysis using the following notation: $x^\mu = (x^+, x^-)$, $p.x = \frac{1}{2}p^+x^- + \frac{1}{2}p^-x^+$, $p.p = m^2 \Rightarrow \hat{p}^- = m^2/p^+, \partial_+ = \partial/\partial x^+$, $\partial_- = \partial/\partial x^-$, $\psi^\dagger(x) = ({\psi_1}^\dagger, {\psi_2}^\dagger)$. The Lagrangian of the model expressed in terms of LF space-time and field variables has the form

$$\begin{aligned}\mathscr{L}_{lf} = i{\Psi_2}^\dagger \overleftrightarrow{\partial_+} \Psi_2 + i{\Psi_1}^\dagger \overleftrightarrow{\partial_-} \Psi_1 - m({\Psi_1}^\dagger \Psi_2 + {\Psi_2}^\dagger \Psi_1) + \\ + 2\partial_+\phi\partial_-\phi - \frac{1}{2}\mu^2\phi^2 - g\partial_+\phi j^+ - g\partial_-\phi j^-. \end{aligned} \tag{2.12}$$

The corresponding field equations read $2i\partial_+\Psi_2 = m\Psi_1 + 2g\partial_+\phi\Psi_2$, $2i\partial_-\Psi_1 = m\Psi_2 + 2g\partial_-\phi\Psi_1$. Inserting the second (constraint) equation into the Lagrangian leads to the free LF Hamiltonian:

$$P^- = \int_{-\infty}^{+\infty} \frac{\mathrm{d}x^-}{2} \Big[m\big({\psi_1}^\dagger\psi_2 + {\psi_2}^\dagger\psi_1\big) + \mu^2\phi^2\Big]. \tag{2.13}$$

The SL and LF forms lead seemingly to a completely different dynamics! The resolution of this contradiction is simple. The above considerations are wrong. The solution of the field equations have not been taken into account. Analogously to a treatment of a constraint, we have to insert the solution to the Lagrangian first, then calculate conjugate momenta and derive the Hamiltonian.

Inserting the solution of the Dirac eq. of the DCM in the form $\partial_\mu\Psi(x) = -ig\partial_\mu\phi(x)\Psi(x) + e^{-ig\phi(x)}\partial_\mu\psi(x)$ into $\mathscr{L}$ has a consequence that the interaction part cancels! The corresponding free-field conjugate momenta ($\Pi_\psi = i\psi^\dagger, \Pi_\phi = \partial_0\phi$) imply the Hamiltonian

$$H = \int_{-\infty}^{+\infty} \mathrm{d}x^1 \big[-i\psi^\dagger\alpha^1\partial_1\psi + m\psi^\dagger\gamma^0\psi + \frac{1}{2}\Pi_\phi^2 + \frac{1}{2}(\partial_1\phi)^2 + \frac{1}{2}\mu^2\phi^2\big], \tag{2.14}$$

which is just the sum of free Hamiltonians of the massive scalar and fermion fields. Correct Heisenberg equations are generated with this Hamiltonian:

$$i\partial_0\Psi(x) = -[H, \Psi(x)] = -i\alpha^1\partial_1\Psi + gj^0\Psi - gj^1\alpha^1\Psi. \tag{2.15}$$

Physical vacuum of the DCM coincides with the Fock vacuum. The only trace of the interacting theory is the non-canonical form of the anticommutation relation of the interacting fermion field

[1] Our approach here is a bit heuristic, operators have to be regularized. Mathematically correct treatment can be given by field operators considered as operator-valued distributions [8].

and the coupling-dependent correlation functions. The latter are expressed in terms of the free-field correlators, $D^{(+)}(x-y) = \langle 0|\phi(x)\phi(y)|0\rangle$, $S^{(+)}_{\alpha\beta}(x-y) = \langle 0|\psi_\alpha(x)\overline{\psi}_\beta(y)|0\rangle$, as

$$\langle vac|\Psi_\alpha(x)\overline{\Psi}_\beta(y)|vac\rangle = \langle 0| : e^{-ig\phi(x)} : \psi_\alpha(x)\overline{\psi}_\beta(y) : e^{ig\phi(y)} : |0\rangle = e^{g^2 D^{(+)}(x-y)} S^{(+)}_{\alpha\beta}(x-y). \quad (2.16)$$

We note that the DC model was used to illustrate the concept of an "infraparticle" [1]. The results obtained in the paper are valid for considered interacting Lagrangian, which however, as we have shown here, has nothing to do with the true Lagrangian of the original model.

In the LF case, the field equations are solved by $\Psi_2(x) = e^{-ig\phi(x)}\psi_2(x)$, $2i\partial_+\psi_2 = m\psi_1$, $\Psi_1(x) = e^{-ig\phi(x)}\psi_1(x)$, $2i\partial_-\psi_1 = m\psi_2$. Inserting these solutions into the LF Lagrangian yields

$$\mathscr{L}_{lf} = i{\psi_2}^\dagger \overset{\leftrightarrow}{\partial_+} \psi_2 + i{\psi_1}^\dagger \overset{\leftrightarrow}{\partial_-} \psi_1 - m({\psi_1}^\dagger\psi_2 + {\psi_2}^\dagger\psi_1) + 2\partial_+\phi\partial_-\phi - \frac{1}{2}\mu^2\phi^2. \quad (2.17)$$

The free Hamiltonian (2.13) follows. This result is the same as we obtained with the conventional treatment. The reason is that there is no kinetic (i.e., derivative) term in LF Hamiltonian.

The form of the correlation functions coincides with the space-like treatment. For example,

$$\langle 0|\Psi_1(x)\Psi_2^\dagger(y)|0\rangle = e^{g^2 D^{(+)}(x-y)} S^{(+)}_{12}(x-y),$$
$$S^{(+)}_{12}(x-y) = \langle 0|\psi_1(x)\psi_2^\dagger(y)|0\rangle = \int_0^\infty \frac{dp^+}{8\pi}\frac{m}{p^+} e^{-\frac{i}{2}p^+(x^- - y^- - i\varepsilon) - \frac{i}{2}\frac{m^2}{p^+}(x^+ - y^+ - i\varepsilon)},$$
$$S^{(+)}_{12}(z) = -\theta(z^2)\frac{m}{8}\Big[N_0\big(m\sqrt{z^2}\big) + i\,\mathrm{sgn}(z^+)J_0(m\sqrt{z^2})\Big] + \theta\big(-z^2\big)\frac{m}{4\pi}K_0\big(m\sqrt{-z^2}\big). \quad (2.18)$$

J_0, N_0 and K_0 are the Bessel functions. The scalar-field function is $D^{(+)}(z) = m^{-1}S^{(+)}_{12}(z)$. The small imaginary (damping) factors guarantee existence of the corresponding integrals.

Note that the scalar-field correlation function diverges for $\mu = 0$ in both schemes. The LF calculation with the massless fermion field is inconsistent (yields vanishing $S^{(+)}_{11}(z)$.) The $m = 0$ limit of the LF fermion correlation function coincides however with the SL case.

The Lagrangian of the massive Rothe-Stamatescu model is identical with (2.1) except that the interaction term has j^μ_5 instead of j^μ. However, since the massive j^μ_5 current is not conserved, the model has a more complicated structure. Here, we will only make two remarks. First, due to nonconservation of the axial-vector current, the (pseudo)scalar field is no longer free. The Dirac equation seems naively to have an operator solution similar to the one from the DCM, $\Psi(x) = e^{-ig\gamma^5\phi(x)}\psi(x)$. However, this expression actually does not solve the equation due to $\{\gamma^\mu, \gamma^5\} = 0$. Thus, the massive RS model is not exactly solvable. On the other hand, the Dirac equation in the original RS model can be solved exactly but inserting the solution to the Lagrangian generates the free Hamiltonian. The overall picture is thus similar to the massive derivative-coupling model.

3. The Thirring model

The operator solution of the Thirring model was given by Klaiber [9] who also calculated n-point correlation functions. The model may seem obsolete and uninteresting today, but actually not all of its aspects have been adequately clarified. A systematic Hamiltonian study based on the model's solvability was not given so far.

The classical Lagrangian density of the Thirring model is

$$\mathscr{L} = \frac{i}{2}\overline{\Psi}\gamma^\mu \overleftrightarrow{\partial_\mu} \Psi - \frac{1}{2}gJ_\mu J^\mu, \;\; J^\mu = \overline{\Psi}\gamma^\mu\Psi. \tag{3.1}$$

The field equations are $i\gamma^\mu\partial_\mu\Psi(x) = gJ^\mu(x)\gamma_\mu\Psi(x)$ with $\partial_\mu J^\mu(x) = 0$. The simplest solution is

$$\Psi(x) = e^{-i(g/\sqrt{\pi})j(x)}\psi(x), \quad \gamma^\mu\partial_\mu\Psi(x) = 0, \quad j_\mu(x) = \frac{1}{\sqrt{\pi}}\partial_\mu j(x), \;\; J^\mu(x) = j^\mu(x). \tag{3.2}$$

It depends on the "integrated current $j(x)$. Free fields define the solution of the interacting model. Details of the canonical treatment of the model can be found in [10]. The main difference with respect to our previous analysis is that we insert the operator solution to the Lagrangian first:

$$\mathscr{L} = i\overline{\Psi}\gamma^\mu\big[-\frac{ig}{\sqrt{\pi}}\partial_\mu j\Psi + e^{-\frac{ig}{\sqrt{\pi}}j}\partial_\mu\psi\big] - \frac{g}{2}j_\mu j^\mu. \tag{3.3}$$

The first term in the bracket combines with the interaction term reversing its sign. We get

$$H = \int_{-\infty}^{+\infty} \mathrm{d}x^1\Big[-i\psi^\dagger\alpha^1\partial_1\psi - \frac{1}{2}g\big(j^0j^0 - j^1j^1\big)\Big] \equiv H_0 + H_{int}. \tag{3.4}$$

The interacting Hamiltonian has the simplest form in terms of composite operators $c(k^1), c^\dagger(k^1)$:

$$H_{int} = \frac{g}{\pi}\int_{-\infty}^{+\infty} dk^1|k^1|\Big[c^\dagger(k^1)c^\dagger(-k^1) + c(k^1)c(-k^1)\Big]. \tag{3.5}$$

Obviously $|0\rangle$ is not an eigenstate of $H = H_0 + H_{int}$. The true lowest-energy eigenstate of H can be found by its diagonalization using a suitable unitary operator $U(\gamma)$ [10, 11], $U(\gamma)HU^{-1}(\gamma)|0\rangle = 0$. It follows that $U^{-1}(\gamma)|0\rangle$ will be the physical vacuum state. Explicitly, one finds ($\gamma_d = \frac{1}{2}\mathrm{artanh}g/\pi$)

$$|\Omega\rangle = \exp\Big[-\frac{1}{2}\gamma_d\int_{-\infty}^{+\infty}\mathrm{d}p^1\big[c^\dagger(p)c^\dagger(-p) - c(p)c(-p)\big]\Big]|0\rangle. \tag{3.6}$$

It corresponds to a coherent state of pairs of composite bosons with zero total momentum, $P^1|\Omega\rangle = 0$. The vacuum $|\Omega\rangle$ also carries vanishing charge and axial charge and corresponds to the symmetric phase. The correlation functions can be calculated from the normal-ordered operator solution (3.2) using an infrared cutoff and the new vacuum state $|\Omega\rangle$. Calculations of the particle spectrum are also possible using the discrete plane-wave basis. They will be nontrivial since $[c(k^1), b(p^1)] \neq 0$.

4. The Federbush model

The Federbush model (FM) is the only known *massive* solvable model. It permits us to generalize the Klaiber's bosonization to the massive case and to search for the true physical ground state generalizing the SL treatment applied to the massless Thirring model. The Lagrangian of the FM describes two species of the fermion field interacting via specific current– current coupling,

$$\mathscr{L} = \frac{i}{2}\overline{\Psi}\gamma^\mu \overleftrightarrow{\partial_\mu} \Psi - m\overline{\Psi}\Psi + \frac{i}{2}\overline{\Phi}\gamma^\mu \overleftrightarrow{\partial_\mu} \Phi - \mu\overline{\Phi}\Phi - g\varepsilon_{\mu\nu}J^\mu H^\nu. \tag{4.1}$$

Here $J^\mu = \overline{\Psi}\gamma^\mu\Psi$, $H^\mu = \overline{\Phi}\gamma^\mu\Phi$. The field equations are

$$i\gamma^\mu\partial_\mu\Psi(x) = m\Psi(x) + g\varepsilon_{\mu\nu}\gamma^\mu H^\nu(x)\Psi(x), \quad i\gamma^\mu\partial_\mu\Phi(x) = \mu\Phi(x) - g\varepsilon_{\mu\nu}\gamma^\mu J^\nu(x)\Phi(x). \tag{4.2}$$

The relations $J^\mu(x) = \varepsilon^{\mu\nu}\partial_\nu j(x)/\sqrt{\pi}$, $H^\mu(x) = \varepsilon^{\mu\nu}\partial_\nu h(x)/\sqrt{\pi}$ define the "integrated currents" $j(x)$ and $h(x)$. They enter into the solutions in an "off-diagonal" way:

$$\Psi(x) = e^{-i\frac{g}{\sqrt{\pi}}h(x)}\psi(x),\ i\gamma^\mu\partial_\mu\psi(x) = m\psi(x), \quad \Phi(x) = e^{i\frac{g}{\sqrt{\pi}}j(x)}\phi(x),\ i\gamma^\mu\partial_\mu\phi(x) = \mu\phi(x). \tag{4.3}$$

The exponentials of the composite fields are more singular than in the massless case and have to be defined using the "triple-dot ordering" [12, 13] which generalizes the normal ordering defined through order by order subtractions of VEVs. We avoid this by bosonization of the massive current.

The usual treatment yields contradictory dynamics: the SL Hamiltonian contains interaction,

$$H = \int_{-\infty}^{+\infty} dx^1 \Big[-\frac{i}{2}\psi^\dagger\alpha^1 \overleftrightarrow{\partial_1}\,\psi + m\psi^\dagger\gamma^0\psi - \frac{i}{2}\phi^\dagger\alpha^1\overleftrightarrow{\partial_1}\,\phi + \mu\phi^\dagger\gamma^0\phi - gj^0h^1 + gj^1h^0\Big], \tag{4.4}$$

while the LF Hamiltonian (obtained after inserting two fermion constraints), is free:

$$P^- = \int_{-\infty}^{+\infty}\frac{dx^-}{2}\Big[m\Big({\psi_1}^\dagger\psi_2 + {\psi_2}^\dagger\psi_1\Big) + \mu\Big({\phi_1}^\dagger\phi_2 + {\phi_2}^\dagger\phi_1\Big)\Big]. \tag{4.5}$$

Our approach leads to a different result. Inserting the solutions (4.3) into the Lagrangian, we get

$$\begin{aligned}\mathscr{L} &= \frac{i}{2}\psi^\dagger\gamma^0\gamma^\mu\overleftrightarrow{\partial_\mu}\,\psi - m\overline{\psi}\psi + \frac{i}{2}\phi^\dagger\gamma^0\gamma^\mu\overleftrightarrow{\partial_\mu}\,\phi - \mu\overline{\phi}\phi + g\varepsilon_{\mu\nu}j^\mu h^\nu,\\ H &= \int_{-\infty}^{+\infty}dx^1\big[-i\psi^\dagger\alpha^1\partial_1\psi + m\overline{\psi}\psi - i\phi^\dagger\alpha^1\partial_1\phi + \mu\overline{\phi}\phi + g\big(j^0h^1 - j^1h^0\big)\big].\end{aligned} \tag{4.6}$$

Both operators are expressed in terms of free fields and have an opposite sign (with respect to the conventional result) in the interaction piece. The interaction term is non-diagonal when expressed in terms of bosonized massive currents. A massive version of the Bogoliubov transformation is required. The massive analogues of Klaiber's operators $c(k^1)$ are surprisingly complicated [14].

The LF version of the Lagrangian (4.1) is:

$$\begin{aligned}\mathscr{L}_{lf} &= i\Psi_2^\dagger\overleftrightarrow{\partial_+}\,\Psi_2 + i\Psi_1^\dagger\overleftrightarrow{\partial_-}\,\Psi_1 - m\big(\Psi_2^\dagger\Psi_1 + \Psi_1^\dagger\Psi_2\big) + i\Phi_2^\dagger\overleftrightarrow{\partial_+}\,\Phi_2 + i\Phi_1^\dagger\overleftrightarrow{\partial_-}\,\Phi_1 - \\ &- \mu\big(\Phi_2^\dagger\Phi_1 + \Phi_1^\dagger\Phi_2\big) - \frac{g}{2}j^+h^- + \frac{g}{2}j^-h^+.\end{aligned} \tag{4.7}$$

The LF current components are $j^+(x) = J^+(x) = 2:\psi_2^\dagger(x)\psi_2(x):$, $j^-(x) = J^-(x) = 2:\psi_1^\dagger(x)\psi_1(x):$, $h^+(x) = H^+(x) = 2:\phi_2^\dagger(x)\phi_2(x):$, $h^-(x) = H^-(x) = 2:\phi_1^\dagger(x)\phi_1(x)$. The coupled field equations

$$\begin{aligned}&2i\partial_+\Psi_2(x) = m\Psi_1 - gh^-\Psi_2,\ \ 2i\partial_-\Psi_1 = m\Psi_2 + gh^+\Psi_1,\\ &2i\partial_+\Phi_2(x) = \mu\Phi_1 + gj^-\Phi_2, \quad 2i\partial_-\Phi_1 = \mu\Phi_2 - gj^+\Phi_1.\end{aligned} \tag{4.8}$$

are solved in terms of the corresponding free fields (4.3) and the LF integrated currents,

$$j(x) = \frac{\sqrt{\pi}}{4}\int_{-\infty}^{+\infty}dz^-\,\varepsilon(x^- - z^-)j^+(x^+,z^-), \quad h(x) = \frac{\sqrt{\pi}}{4}\int_{-\infty}^{+\infty}dz^-\,\varepsilon(x^- - z^-)h^+(x^+,z^-). \tag{4.9}$$

The bosonized form of the LF Hamiltonian is quadratic in composite operators and diagonal:

$$P^-_g = \frac{g}{8\pi}\int\limits_{-\infty}^{+\infty} \mathrm{d}k^1 k^+ \Big[A^\dagger(k^+)D(k^+) + D^\dagger(k^+)A(k^+) - B^\dagger(k^+)C(k^+) - C^\dagger(k^+)B(k^+)\Big]. \qquad (4.10)$$

The operators $A(k^+),\ B(k^+),\ C(k^+)$ and $D(k^+)$ correspond to $j^+,\ h^+,\ j^-$ and h^-. Their form is as simple as the massless $c(k^1)$ in the SL case. For example,

$$A(k^+,x^+) = \frac{i}{\sqrt{k^+}}\int\limits_0^\infty dp^+\Big\{\big[b^\dagger(p^+)b(k^+ + p^+) - d^\dagger(p^+)d(p^+ + k^+)\big]e^{\frac{i}{2}\frac{m^2k^+x^+}{p^+(k^+p^+)}} + + d(p^+)b(k^+ - p^+)e^{-\frac{i}{2}\frac{m^2k^+x^+}{p^+(k^+-p^+)}}\Big\}. \qquad (4.11)$$

Complexities will enter in calculations of the correlation functions since the composite LF boson operators do not commute to the delta function at unequal LF times [14]. The SL Hamiltonian is not diagonal. It will be very interesting to see how the SL and LF schemes generate mutually consistent results for the correlators given the completely different vacuum structure in the two formulations of the relativistic dynamics of the Federbush model.

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References

[1] B. Schroer, Fort. Physik 1, 1 (1962).

[2] K. D. Rothe and O. I. Stamatescu, Ann. Phys. 95, 202 (1975).

[3] L. V. Belvedere and A. F. Rodrigues, J. Phys. A 40, 5193 (2007).

[4] W. E. Thirring, Ann. Phys. 3, 91 (1958).

[5] K. Federbush, Phys. Rev. 121, 1247 (1961).

[6] P. A. M. Dirac, Rev. Mod. Phys. 21, 392 (1949).

[7] F. Strocchi, *Selected topics on the general properties of QFT*, World Sci. Lect. Notes Phys. 51, 1993.

[8] P. Grangé and E. Werner, Nucl. Phys. Proc. Suppl. 161, 75 (2006).

[9] B. Klaiber, *The Thirring model*, in Lectures in Theoretical Physics, Vol. Xa, New York, 1968, p. 141.

[10] L. Martinovi**c**, Nucl. Phys. Proc. Suppl. 199, 147 (2010).

[11] L. Martinovi**c** and P. Grangé, submitted for publication.

[12] A. Wightman, in Cargése Lectures in Theoretical Physics, 1964, p.171.

[13] B. Schroer, T. T. Truong and P. Weisz, Ann. Phys. 102, 156 (1976).

[14] L. Martinovic, P. Grangé and B. Mutet, in preparation.