

Light and heavy mesons in a soft-wall holographic model

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We study the spectrum and decay constants of light and heavy mesons in a soft-wall holographic approach, using the correspondence of string theory in Anti-de Sitter space and conformal field theory in physical space-time.

*Light Cone 2010: Relativistic Hadronic and Particle Physics
June 14-18, 2010
Valencia, Spain*

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1. Introduction

In a series of papers [1] Brodsky and de Téramond developed a semiclassical approximation to QCD - light-front holography (LFH)-, an approach based on the correspondence of string theory in Anti-de Sitter (AdS) space and conformal field theory (CFT) in physical space-time [2]. LFH [1] is one of the exciting features of the AdS/CFT correspondence. The LFH approach is a covariant and analytic model for hadron structure with confinement at large distances and conformal behavior at short distances. It is analogous to the Schrödinger theory for atomic physics. It provides a precise mapping of string modes $\Phi(z)$ in the AdS fifth dimension z to the hadron light-front wave functions (LFWF) in physical space-time in terms of the light-front impact variable ξ , which measures the separation of the quark and gluonic constituents inside a hadron. Therefore, different values of the holographic variable z correspond to different scales at which the hadron is examined. The mapping was obtained by matching certain matrix elements in the two approaches - string theory in AdS and light-front theory in Minkowski space-time. Meson and baryon physics was successfully described in the LFH approach [1]: the mass spectrum of meson and baryons (reproducing the Regge trajectories), pion leptonic constant, electromagnetic form factors of pion and nucleons, etc.

In order to break conformal invariance and incorporate confinement in the infrared (IR) region, two alternative AdS/QCD backgrounds have been suggested in the literature: the “hard-wall” approach [3] based on introducing a infrared (IR) brane cutoff in the fifth dimension, and the “soft-wall” approach [4]-[9], based on using a soft cutoff by introducing a background dilaton field in the AdS space or using a warp factor in the metric. Both approaches have certain advantages. One of the problems of the “hard-wall” scenario is a linear dependence of hadron masses $M \propto L$ at high values of the orbital momentum L instead of the quadratic behavior $M^2 \propto L$ (known as Regge trajectory). In fact, the “soft-model” was initiated in order to solve the problem of the hadronic mass spectrum. The “soft-model” has been applied to different aspects of hadron properties, including the hadron and glueball mass spectrum, the heavy quark potential, form factors, deep inelastic scattering, etc. [4]-[9]. Notice that in the LFH approach [1] both scenarios for AdS/QCD backgrounds (“hard-wall” and “soft-wall”) are used in order to map the string modes to the LFWF restricting to the massless quarks and then extending to the case of massive quarks. In Refs. [6] an alternative soft-wall holographic model has been developed, which provided an extension to hadrons with an arbitrary number of constituents. In Ref. [7, 8] meson wave functions in case of massive quarks have been derived using approaches developed in [1] and [6].

Notice that the LFH approach developed in Ref. [1] uses the so-called “negative” dilaton field profile ($e^{-\phi(z)}$ with $\phi(z) = -\kappa^2 z^2$), which was necessary to reproduce a massless pion and the behavior of the gravitational potential. In this paper we use the soft-wall approach with a “positive” dilaton field profile $\phi(z) = \kappa^2 z^2$, as suggested originally in Ref. [4]. In the context of the original Soft-Wall model [4] the positive sign in the dilaton profile is important to reproduce the correct behavior of Regge trajectories for higher spin states. In [8] we showed that the use of the positive dilaton in the LFH approach [1] is possible when the mass term of the AdS_{d+1} action is modified.

The proposed approach is applied to the study of the mass spectrum and decay properties of light and heavy mesons. The paper is structured as follows. First, in Section 2, we briefly discuss the basic notions of the approach. In Section 3 we consider the mass spectrum and decay properties of light and heavy mesons and summarize our results.

2. Approach

Our starting point is the action in AdS_{d+1} for a spin- J field $\Phi_J = \Phi_{M_1 \dots M_J}(x, z)$ – a symmetric, traceless tensor used in the LFH approach [1], where we perform two modifications: 1) use a positive dilaton profile $\phi(z) = \kappa^2 z^2$; 2) include a nontrivial z -dependence of the mass term coefficient $\mu_J^2 \rightarrow \mu_J^2(z)$ due to the interaction of the dilaton field with the matter field:

$$S_\Phi = \frac{(-1)^J}{2} \int d^d x dz \sqrt{g} e^{-\phi(z)} \left(\partial_N \Phi_J \partial^N \Phi^J - \mu_J^2(z) \Phi_J \Phi^J \right), \quad (2.1)$$

where $\mu_J^2(z) = \mu_J^2 + g_J \phi(z)$ is the "dressed" mass due to the interaction of the dilaton with Φ_J . Note that a similar modification of the mass term of the string mode dual to the spinor field describing nucleons has been done in the context of the Soft-Wall model in Ref. [9]. The coupling constant g_J will be fixed later, in order to get massless pion. The metric is defined as $ds^2 = (R/z)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$, $\eta_{\mu\nu} = \text{diag}(1, -1, \dots, -1)$, where R is the AdS radius, $g = |\det g_{MN}| = (R/z)^{2(d+1)}$, and g_{MN} is the metric tensor of $d+1$ space.

Next we restrict to the axial gauge $\Phi_{z\dots}(x, z) = 0$ and consider the string modes dual to hadrons with total angular spin J and four-momentum P and propagating in AdS space along the Poincaré coordinates: $\Phi_{v_1 \dots v_J}(x, z) = \sum_n \varphi_{nJ}(z) \int \frac{d^d P_n}{(2\pi)^d} e^{-iP_n x} \varepsilon_{v_1 \dots v_J}(P_n)$, where $v_1 \dots v_J$ are the Poincaré indices, n is the radial quantum number and $\varepsilon_{v_1 \dots v_J}(P_n)$ is the polarization tensor. Then doing the substitution $\varphi_{nJ}(z) = e^{\phi(z)/2} (R/z)^{J-\frac{d-1}{2}} \Phi_{nJ}(z)$ one can derive the Schrödinger-type equation of motion (EOM) for $\Phi_{nJ}(z)$ with effective potential $U_J(z) = \kappa^4 z^2 + (4a_J^2 - 1)/(4z^2) + 2\kappa^2(b_J - 1)$:

$$\left[-\frac{d^2}{dz^2} + U_J(z) \right] \Phi_{nJ}(z) = M_{nJ}^2 \Phi_{nJ}(z) \quad (2.2)$$

where $a_J = \sqrt{(d-2J)^2 + 4(\mu_J R)^2}/2$ and $b_J = (g_J R^2 + d - 2J)/2$.

Analytical solutions of Eq. (2.2) – eigenvalues and mass spectrum are:

$$\Phi_{nJ}(z) = \sqrt{\frac{2n!}{(n+a_J)!}} \kappa^{1+a_J} z^{1/2+a_J} e^{-\kappa^2 z^2/2} L_n^{a_J}(\kappa^2 z^2), \quad M_{nJ}^2 = 4\kappa^2 \left(n + \frac{a_J + b_J}{2} \right). \quad (2.3)$$

Restricting to $d = 4$ with $(\mu_J R)^2 = L^2 - (2 - J)^2$ [1] we fix $g_J R^2 = 4(J - 1)$ in order to get a massless pion. Therefore, in the case $d = 4$ we get $a_J = L$ and $b_J = J$ and the solutions of the Schrödinger equation read as:

$$\Phi_{nJ}(z) = \sqrt{\frac{2n!}{(n+L)!}} \kappa^{1+L} z^{1/2+L} e^{-\kappa^2 z^2/2} L_n^L(\kappa^2 z^2), \quad M_{nJ}^2 = 4\kappa^2 \left(n + \frac{L+J}{2} \right). \quad (2.4)$$

Because of $J = L$ or $J = L \pm 1$ our Soft-Wall model generates linear Regge trajectories in both quantum numbers n and J (or L): $M_{nJ}^2 \sim n + J$. Note that the string modes dual to the pseudoscalar $J^{PC} = 0^{-+}$ and scalar $J^{PC} = 0^{++}$ mesons, and correspondingly the vector $J^{PC} = 1^{--}$ and axial $J^{PC} = 1^{++}$ mesons, are different from each other (mass spectrum and wave functions) via the mass parameter of the string mode $(\mu_J R)^2$ depending explicitly on the orbital momentum L . Inclusion of chiral symmetry breaking effects in the AdS action and their impact on the hadron properties will be analyzed in the future.

The string mode $\Phi_{nJ}(z)$ can be directly mapped to the LFWF due to correspondence of AdS and light-front amplitudes. In particular, the holographic coordinate z is matched to the impact variable ζ in the LF formalism $z \rightarrow \zeta$, $\zeta^2 = \mathbf{b}_\perp^2 x(1-x)$, where \mathbf{b}_\perp is the impact separation and Fourier conjugate to the transverse momentum \mathbf{k}_\perp . In the case of two massless partons q_1 and \bar{q}_2 we obtain a relation between the AdS modes and meson LFWF $\tilde{\Psi}_{q_1\bar{q}_2}(x, \zeta)$ [7] given by

$$|\tilde{\Psi}_{q_1\bar{q}_2}(x, \zeta)|^2 = P_{q_1\bar{q}_2} x(1-x) f^2(x) \frac{|\Phi_{nJ}(\zeta)|^2}{2\pi\zeta}, \quad (2.5)$$

where $P_{q_1\bar{q}_2}$ is the probability of finding the valence Fock state $|q_1\bar{q}_2\rangle$ in the meson M . In the following we restrict to the case of $P_{q_1\bar{q}_2} = 1$ and only for the pion we consider $P_{q_1\bar{q}_2} < 1$ (see discussion in Ref. [7]). Here $f(x)$ is the longitudinal mode which is normalized as $\int_0^1 dx f^2(x) = 1$, where we have chosen $f(x) = 1$. Then the expressions for the meson LFWFs read:

$$\tilde{\Psi}_{q_1\bar{q}_2}(x, \mathbf{b}_\perp) = \sqrt{\frac{2n!}{(n+L)!}} \frac{\kappa^{1+L}}{\sqrt{\pi}} |\mathbf{b}_\perp|^L [x(1-x)]^{\frac{1+L}{2}} e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2} L_n^L(\kappa^2 \mathbf{b}_\perp^2 x(1-x)). \quad (2.6)$$

The meson LFWF (2.6) does not consider massive quarks. Inclusion of quark masses have been considered by us previously in [7, 8]. In particular, the meson LFWFs with massive quarks can be written down in the form of a product of transverse $\Phi(\zeta)$, longitudinal $f(x, m_1, m_2)$ and angular $e^{im\phi}$ modes [1]:

$$\tilde{\Psi}_{q_1\bar{q}_2}(x, \zeta, m_1, m_2) = \frac{\Phi_{nJ}(\zeta)}{\sqrt{2\pi\zeta}} f(x, m_1, m_2) e^{im\phi} \sqrt{x(1-x)}, \quad (2.7)$$

where m is the magnetic quantum number. For the longitudinal mode we will use the functional form

$$f(x, m_1, m_2) \equiv N f(x) \exp\left(-\frac{m_1^2/x + m_2^2/(1-x)}{2\lambda_{12}^2}\right), \quad (2.8)$$

containing quark masses and an additional scale parameter λ_{12} , where N is the normalization constant. Note, that the parameters κ and λ_{12} have different scaling. Later, in the analysis of the mass spectrum and decay constants of heavy-light meson, we will show that the dilaton parameter κ should scale as $\mathcal{O}(1)$ in the $1/m_Q$ expansion, where m_Q is the heavy quark mass, while the parameter λ_{12} should scale as $\mathcal{O}(m_Q^{1/2})$ and $\mathcal{O}(m_Q)$ in case of heavy-light mesons and heavy quarkonia, respectively. The meson mass spectrum in the case of massive quarks is given by [1]:

$$M_{nJ}^2 = \int_0^\infty d\zeta \Phi_{nJ}(\zeta) \left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(J-1) \right) \Phi_{nJ}(\zeta) + \int_0^1 dx \left(\frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right) f^2(x, m_1, m_2). \quad (2.9)$$

It means that for massive quarks the hadron masses are shifted due to the last term in the r.h.s. of Eq. (2.9). One should stress that the potential in Eqs. (2.9) is not complete. It includes confinement forces, but does not include in its full context effects of chiral symmetry breaking, which are

important for consistency with the infrared structure of QCD (see e.g. the discussion in Refs. [4, 5]). Moreover, it does not contain the one-gluon exchange term, which is sufficient for bottomia hadrons [10], and also hyperfine splitting terms. The master formula for meson masses including confinement, color Coulomb and hyperfine splitting effects reads [8]:

$$M_{nJ}^2 = 4\kappa^2 \left(n + \frac{L+J}{2} \right) + \int_0^1 dx \left(\frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right) f^2(x, m_1, m_2) - \frac{64\alpha_s^2 m_1 m_2}{9(n+L+1)^2} + \frac{32\pi\alpha_s \beta_S \nu}{9 \mu_{12}}, \quad (2.10)$$

where β_S is the spin projection factor, which is -3 for and 1 for $S = 0$ and $S = 1$ mesons, respectively; ν is a free parameter ν softening the original δ -functional form of the hyperfine-splitting potential; α_s is the QCD coupling and $\mu_{12} = 2m_1 m_2 / (m_1 + m_2)$.

3. Properties of light and heavy mesons

In the numerical analysis we restrict ourselves to the isospin limit $m_u = m_d = m$. We fix our free parameters (constituent quark masses, κ , λ_{12} , α_s and ν) from the description of the mass spectrum and decay constants of light and heavy mesons. Note that we use a unified value of the dilaton parameter κ for all meson states, as dictated by the AdS action.

The parameters are given in the following. For the constituent quark masses we have:

$$m = 420 \text{ MeV}, \quad m_s = 570 \text{ MeV}, \quad m_c = 1.6 \text{ GeV}, \quad m_b = 4.8 \text{ GeV}. \quad (3.1)$$

The unified value of the dilaton parameter for all mesons is fixed as $\kappa = 550 \text{ MeV}$. The hyperfine-splitting parameter is fixed as $\nu = 10^{-4} \text{ GeV}^3$. The strong coupling $\alpha_s \equiv \alpha_s(\mu_{12}^2)$ depends on quark flavor and is calculated consistently using the parametrization of α_s with “freezing” [11]:

$$\alpha_s(\mu^2) = \frac{12\pi/(33 - 2N_f)}{\ln((\mu^2 + M_B^2)/\Lambda^2)} \quad (3.2)$$

where N_f is the number of flavors, $\Lambda = 420 \text{ MeV}$ is the QCD scale parameter, $M_B = 854 \text{ MeV}$ is the background mass [8]. The dimensional parameters in the longitudinal wave functions are fitted as:

$$\begin{aligned} \lambda_{qq} &= 0.63 \text{ GeV}, \quad \lambda_{us} = 1.2 \text{ GeV}, \quad \lambda_{ss} = 1.68 \text{ GeV}, \quad \lambda_{qc} = 2.5 \text{ GeV}, \quad \lambda_{sc} = 3.0 \text{ GeV}, \\ \lambda_{qb} &= 3.89 \text{ GeV}, \quad \lambda_{sb} = 4.18 \text{ GeV}, \quad \lambda_{cc} = 4.04 \text{ GeV}, \quad \lambda_{cb} = 4.82 \text{ GeV}, \quad \lambda_{bb} = 6.77 \text{ GeV}. \end{aligned} \quad (3.3)$$

For the probabilities of the ground state pion and kaon we use the following values: $P_\pi = 0.6$ and $P_K = 0.8$, while for other mesons the probabilities are supposed to be equal to 1. The predictions of our approach for the light meson spectrum according to the $n^{2S+1}L_J$ classification are given in Table 1. For scalar mesons f_0 we present results for two limiting cases: for a nonstrange flavor content $f_0[\bar{n}n] = (\bar{u}u + \bar{d}d)/\sqrt{2}$ and for a strange one $f_0[\bar{s}s] = \bar{s}s$.

One should stress that this approach correctly reproduces the mass spectrum of heavy-light mesons in the heavy quark limit when the heavy quark mass goes to infinity $m_Q \rightarrow \infty$ [12]

$$M_{qQ} = m_Q + \bar{\Lambda} + \mathcal{O}(1/m_Q), \quad (3.4)$$

Table 1: Masses of light mesons

Meson	n	L	S	Mass [MeV]			
π	0	0,1,2,3	0	140	1355	1777	2099
π	0,1,2,3	0	0		1355	1777	2099
K	0	0,1,2,3	0	495	1505	1901	2207
η	0,1,2,3	0	0	544	1552	1946	2248
$f_0[\bar{n}n]$	0,1,2,3	1	1	1114	1600	1952	2244
$f_0[\bar{s}s]$	0,1,2,3	1	1	1304	1762	2093	2372
$a_0(980)$	0,1,2,3	1	1	1114	1600	1952	2372
$\rho(770)$	0,1,2,3	0	1	804	1565	1942	2240
$\rho(770)$	0	0,1,2,3	1	804	1565	1942	2240
$\omega(782)$	0,1,2,3	0	1	804	1565	1942	2240
$\omega(782)$	0	0,1,2,3	1	804	1565	1942	2240
$\phi(1020)$	0,1,2,3	0	1	1019	1818	2170	2447
$a_1(1260)$	0,1,2,3	1	1	1358	1779	2101	2375

Table 2: Masses of heavy–light mesons

Meson	J^P	n	L	S	Mass [MeV]			
$D(1870)$	0^-	0	0,1,2,3	0	1857	2435	2696	2905
$D^*(2010)$	1^-	0	0,1,2,3	1	2015	2547	2797	3000
$D_s(1969)$	0^-	0	0,1,2,3	0	1963	2621	2883	3085
$D_s^*(2107)$	1^-	0	0,1,2,3	1	2113	2725	2977	3173
$B(5279)$	0^-	0	0,1,2,3	0	5279	5791	5964	6089
$B^*(5325)$	1^-	0	0,1,2,3	1	5336	5843	6015	6139
$B_s(5366)$	0^-	0	0,1,2,3	0	5360	5941	6124	6250
$B_s^*(5413)$	1^-	0	0,1,2,3	1	5416	5992	6173	6298

Table 3: Masses of heavy quarkonia $c\bar{c}$, $b\bar{b}$ and $c\bar{b}$

Meson	J^P	n	L	S	Mass [MeV]			
$\eta_c(2980)$	0^-	0,1,2,3	0	0	2997	3717	3962	4141
$\psi(3097)$	1^-	0,1,2,3	0	1	3097	3798	4038	4213
$\chi_{c0}(3415)$	0^+	0,1,2,3	1	1	3635	3885	4067	4226
$\chi_{c1}(3510)$	1^+	0,1,2,3	1	1	3718	3963	4141	4297
$\chi_{c2}(3555)$	2^+	0,1,2,3	1	1	3798	4038	4213	4367
$\eta_b(9390)$	0^-	0,1,2,3	0	0	9428	10190	10372	10473
$\Upsilon(9460)$	1^-	0,1,2,3	0	1	9460	10219	10401	10502
$\chi_{b0}(9860)$	0^+	0,1,2,3	1	1	10160	10343	10444	10521
$\chi_{b1}(9893)$	1^+	0,1,2,3	1	1	10190	10372	10473	10550
$\chi_{b2}(9912)$	2^+	0,1,2,3	1	1	10219	10401	10502	10579
$B_c(6276)$	0^-	0,1,2,3	0	0	6276	6911	7092	7209

Table 4: Decay constants f_P in MeV of pseudoscalar mesons

Meson	Data [13]	Our
π^-	$130.4 \pm 0.03 \pm 0.2$	131
K^-	$156.1 \pm 0.2 \pm 0.8$	155
D^+	206.7 ± 8.9	167
D_s^+	257.5 ± 6.1	170
B^-	193 ± 11	139
B_s^0	$253 \pm 8 \pm 7$	144
B_c	$489 \pm 5 \pm 3$ [14]	159

Table 5: Decay constants f_V in MeV of vector mesons

Meson	Data [13]	Our	Meson	Data [13]	Our
ρ^+	210.5 ± 0.6	170	ρ^0	154.7 ± 0.7	120
D^*	$245 \pm 20_{-2}^{+3}$ [15]	167	ω	45.8 ± 0.8	40
D_s^*	$272 \pm 16_{-20}^{+3}$ [16]	170	ϕ	76 ± 1.2	58
B^*	$196 \pm 24_{-2}^{+39}$ [15]	139	J/ψ	277.6 ± 4	116
B_s^*	$229 \pm 20_{-16}^{+41}$ [15]	144	$\Upsilon(1s)$	238.5 ± 5.5	56

where the scale parameter $\bar{\Lambda}$ is of order $\mathcal{O}(1)$, and the mass splitting of vector and pseudoscalar states $\Delta M_{qQ} = M_{qQ}^V - M_{qQ}^P$, which is of order $1/m_Q$:

$$\Delta M_{qQ} = \frac{2}{M_{qQ}^V + M_{qQ}^P} \left(\kappa^2 + \frac{64\pi\alpha_s}{9} \frac{\beta_S v}{m_q} \right) \sim \frac{1}{m_Q}. \quad (3.5)$$

where parameters κ and λ_{qQ} scale as $\kappa \sim \mathcal{O}(1)$ and $\lambda_{qQ} \sim \mathcal{O}(m_Q^{1/2})$. Note that this scaling is also consistent with the scaling of the leptonic decay constants of heavy-light mesons $f_P \sim f_V \sim 1/\sqrt{m_Q}$. We also correctly reproduce the expansion of the heavy quarkonia mass in the heavy quark limit: $M_{Q_1\bar{Q}_2} = m_{Q_1} + m_{Q_2} + E + \mathcal{O}(1/m_{Q_{1,2}})$, where E is binding energy.

We present results for mass spectrum and decay constants of light and heavy mesons in Tables 1-5. Note that with the universal value of the dilaton scale parameter $\kappa = 550$ MeV, we can well reproduce data for the coupling constants of light mesons. For heavy-light mesons we need a bit larger value of the parameter κ , because the leptonic decay constants are proportional to κ . For the description of leptonic decay constants of heavy quarkonia we need an even larger value of κ . In particular, it should be roughly 2, 3 and 4 times larger for $c\bar{c}$, $c\bar{b}$ and $b\bar{b}$ states, respectively, than the unified value 550 MeV.

In conclusion, we present a detailed analysis of the mass spectrum and decay properties of light, heavy-light mesons and heavy quarkonia in an holographic soft-wall model using conventional sign of the dilaton profile $\phi(z) = \kappa^2 z^2$. In our calculations we consider one-gluon exchange and hyperfine splitting corrections phenomenologically by modifying the potential. We showed that obtained results for heavy-light mesons are consistent with constraints imposed by HQET. In future work we plan to improve the description of the meson data and extend our formalism to baryons.

Acknowledgments

The authors thank Stan Brodsky, Guy de Téramond, Herry Kwee and Oleg Andreev for useful discussions. This work was supported by the DFG under Contract No. FA67/31-2 and No. GRK683. This research is also part of the European Community-Research Infrastructure Integrating Activity “Study of Strongly Interacting Matter” (HadronPhysics2, Grant Agreement No. 227431), Russian President grant “Scientific Schools” No. 3400.2010.2, Federal Targeted Program “Scientific and scientific-pedagogical personnel of innovative Russia” Contract No.02.740.11.0238. Work supported by Fondecyt (Chile) under Grant No. 1100287. A. V. acknowledges the financial support from Fondecyt grants 3100028 (Chile).

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