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Role and Properties of Wilson Lines in Transverse-Momentum-Dependent Parton Distribution Functions

N. G. Stefanis[*†]
Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany
E-mail: stefanis@tp2.ruhr-uni-bochum.de

I. O. Cherednikov[‡]
INFN Cosenza, Università della Calabria, I-87036 Rende (CS), Italy and
Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Russia
E-mail: igor.cherednikov@jinr.ru

A. I. Karanikas
University of Athens, Department of Physics, Nuclear and Particle Physics Section,
Panepistimiopolis, GR-15771 Athens, Greece
E-mail: akaran@phys.uoa.gr

We summarize the renormalization-group properties of transverse-momentum dependent (TMD) parton distribution functions (PDF)s arguing that in the light-cone gauge the overlapping ultraviolet and rapidity divergences cannot be solely controlled by (dimensional) regularization, but necessitate their renormalization. In doing so, we show that at the one-loop order this additional divergence entails an anomalous dimension which can be attributed to a cusp in the gauge contour at light-cone infinity. Then, we present a recent analysis of TMD PDFs which incorporates in the gauge links the Pauli term $\sim F^{\mu\nu}[\gamma_\mu, \gamma_\nu]$. This generalized treatment of gauge invariance is shown to be justified, in the sense that it does not modify the behavior of the leading-twist contribution, though it contributes to the anomalous dimension of that of twist-three. An important consequence of the inclusion of the spin-dependent Pauli term is the appearance of a constant phase—the same for the leading twist-two and subleading distribution functions—that ensues from the interaction of the Pauli term in the transverse gauge link with the gauge field accompanying the fermion. Remarkably, this phase has opposite sign for the Drell-Yan process as compared to the semi-inclusive DIS.

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[*]Speaker.
[†]Also at Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Russia.
[‡]Also at ITPM, Moscow State University, 119899 Moscow, Russia.

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1. Introduction and Theoretical Framework

One of the problems inherent in the definition of hadronic observables is how to ensure gauge invariance. This problem arises because correlators are nonlocal quantities which contain local operators that transform differently under gauge transformations, hence entailing a dependence on the gauge adopted. Clearly, physical quantities should not depend on the choice of the gauge we choose to work — this should merely be a matter of (calculational) convenience. To render *integrated* parton distribution functions (PDF)s gauge invariant, it is sufficient to insert into their definition a Wilson line — a gauge link — between the two Heisenberg quark operators that renders their product gauge invariant [1]. From the point of view of renormalizability, this operation introduces additional contributions to the anomalous dimension of the PDFs. These contributions stem from the local obstructions of the gauge contours: endpoints, cusps, and self-crossing points (see [2] for a technical exposition and references). It is to be emphasized that although the gauge link is nonlocal, no explicit path dependence is introduced, e.g., on the gauge-contour length. Actually, to ensure the gauge invariance of the PDFs it is even sufficient to use a straight lightlike line, because integrated PDFs are defined on the light cone and the only contribution from the gauge link to the anomalous dimension of the PDF comes from its endpoints (see, for instance, [3] and references cited therein). Hence, one has for the integrated PDF of a quark i in a quark a

$$f_{i/a}(x) = \frac{1}{2}\int \frac{d\xi^-}{2\pi} e^{-ik^+\xi^-} \langle P|\bar{\psi}_i(\xi^-,\mathbf{0}_\perp)\gamma^+[\xi^-,0^-|\mathscr{C}]\psi_i(0^-,\mathbf{0}_\perp)|P\rangle \ , \tag{1.1}$$

where

$$[\xi^-,0^-|\mathscr{C}] = \mathscr{P}\exp\left[-ig\int_{0^-[\mathscr{C}]}^{\xi^-} dz^\mu A_\mu^a(0,z^-,\mathbf{0}_\perp)t_a\right] \tag{1.2}$$

is a path-ordered gauge link (Wilson line) in the lightlike direction from 0 to ξ along the contour $\mathscr{C}$.

One may insert a complete set of states and split the gauge link $[\xi^-,0^-]$ into two gauge links connecting the points 0 and ξ through ∞. This is mathematically sound, provided the junction (hidden at infinity) of the two involved contours is smooth, i.e., entails only a trivial renormalization of the junction point so that the validity of the algebraic identity $[x_2,z\mid\mathscr{C}_1]\,[z,x_1\mid\mathscr{C}_2] = [x_2,x_1\mid\mathscr{C} = \mathscr{C}_1\cup\mathscr{C}_2]$ is ensured. This being the case, it is possible to associate each of the quark fields with its own gauge link because the attached contour has no bearing on the definition of $f_{i/a}(x)$. Then, the struck quark can be replaced by an "eikonalized quark"

$$\Psi(x^-|\Gamma) = \psi(x^-)[x^-,\infty^-|\Gamma] \equiv \psi(x^-)\mathscr{P}\exp\left[-ig\int_{\infty^-[\Gamma]}^{x^-} dz_\mu A_a^\mu(0^+,z^-,\mathbf{0}_\perp)t_a\right] \tag{1.3}$$

which is a contour-dependent Mandelstam fermion field [4] (with an analogous definition for the antifermion field). In this scheme, the gluon reconstitution in the gauge-invariant correlator for the integrated PDF involves gluons emanating either from the gauge links — giving rise to selfenergy-like diagrams — or contractions with the gluon self-fields of the Heisenberg operator for the struck quark which generate crosstalk-type diagrams. Note that for the sake of clarity and simplicity, we ignore bound states (spectators). As a result, one has

$$f_{i/a}^{\text{split}}(x) = \frac{1}{2}\sum_n\int \frac{d\xi^-}{2\pi} e^{-ik^+\xi^-} \langle P|\bar{\Psi}_i\left(\xi^-,\mathbf{0}_\perp|\mathscr{C}_1\right)|n\rangle\gamma^+\langle n|\Psi_i\left(0^-,\mathbf{0}_\perp|\mathscr{C}_2\right)|P\rangle \ . \tag{1.4}$$

This concept was carried over to *unintegrated* PDFs, i.e., to those PDFs which still depend on the transverse momenta — hence termed TMD PDFs. However, TMD PDFs with only longitudinal gauge links are not completely gauge invariant under different boundary conditions on the gluon propagator in the light-cone gauge $A^+ = 0$. The reason is that x^--independent gauge transformations are still possible under the same gauge condition. Hence, the naive collinear gauge-invariant TMD PDF definition as for the integrated case is inapplicable. Refurbishment is provided via the introduction of transverse gauge links which necessarily stretch out off the light cone to infinity [5, 6]. This generalizes Eq. (1.4) for a quark with $k_\mu = (k^+, k^-, \mathbf{k}_\perp)$ in a quark with $p_\mu = (p^+, p^-, \mathbf{0}_\perp)$ to the expression

$$f_{q/q}(x, \mathbf{k}_\perp) = \frac{1}{2}\int \frac{d\xi^-}{2\pi}\frac{d^2\boldsymbol{\xi}_\perp}{(2\pi)^2} \exp\left(-ik^+\xi^- + i\mathbf{k}_\perp\cdot\boldsymbol{\xi}_\perp\right) \Big\langle q(p)|\bar{\psi}(\xi^-, \boldsymbol{\xi}_\perp)[\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger \\ \times [\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]^\dagger \gamma^+ [\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \mathbf{0}_\perp][\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]\psi(0^-, \mathbf{0}_\perp)|q(p)\Big\rangle\Big|_{\xi^+=0} \tag{1.5}$$

in which

$$[\infty^-, \boldsymbol{\xi}_\perp; \xi^-, \boldsymbol{\xi}_\perp] \equiv \mathscr{P}\exp\left[ig\int_0^\infty d\tau\, n_\mu^- A_a^\mu t^a(\xi + n^-\tau)\right], \tag{1.6}$$

$$[\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \boldsymbol{\xi}_\perp] \equiv \mathscr{P}\exp\left[ig\int_0^\infty d\tau\, \mathbf{l}\cdot\mathbf{A}_a t^a(\boldsymbol{\xi}_\perp + \mathbf{l}\tau)\right] \tag{1.7}$$

are the lightlike and the transverse gauge link, respectively.

2. One-Loop Gluon Virtual Corrections in the $A^+ = 0$ Gauge

The pursuit of a proper definition of TMD PDFs is a long-standing problem that was not accomplished with the definition above. The reason is — frankly speaking — that nobody knows how the contour behaves at light-cone infinity when it ventures out in the transverse directions. This behavior has influence on the singularity structure of the gluon propagator in the light-cone gauge $A^+ = 0$, notably, $D_{\mu\nu}^{\mathrm{LC}}(q) = \frac{-i}{q^2-\lambda^2+i0}\left(g_{\mu\nu} - \frac{q_\mu n_\nu^- + q_\nu n_\mu^-}{[q^+]}\right)$, via the boundary conditions to go around its singularities. To estimate this influence, one has to calculate the one-loop virtual corrections in the $A^+ = 0$ gauge in conjunction with various boundary conditions (which absorb large-scale effects) and carry out the renormalization of the contour-dependent quark operators defined in Eq. (1.3). Two of us undertook this calculation, announced in [7, 2, 8], with a summary of the approach being given in [9]. The contributing diagrams are shown here in Fig. 1, while the corresponding algebraic expressions are given in Table 1 using the following symbolic abbreviations (the couplings g and g' below are labeled differently only in order to keep track of their origin; ultimately, they will be set equal):

(i) **Q**: struck quark $\psi_i(\xi) = e^{-ig[\int d\eta\, \bar{\psi}\hat{\mathscr{A}}\psi]}\ \psi_i^{\mathrm{free}}(\xi)$ — Heisenberg operator,
(ii) longitudinal gauge link: $[n^-]$,
(iii) transverse gauge link: $[\mathbf{l}_\perp]$,
(iv) g refers to the QCD Lagrangian — see item (i),
(v) coupling g' refers to the exponent of the gauge links, i.e., $g'\int_0^\infty d\tau \ldots$,
(vi) product $g'g'$ corresponds to path-ordered line integrals in the exponent of the gauge links, i.e., $g'g'\int_0^\infty d\tau \int_0^\tau d\sigma \ldots$.

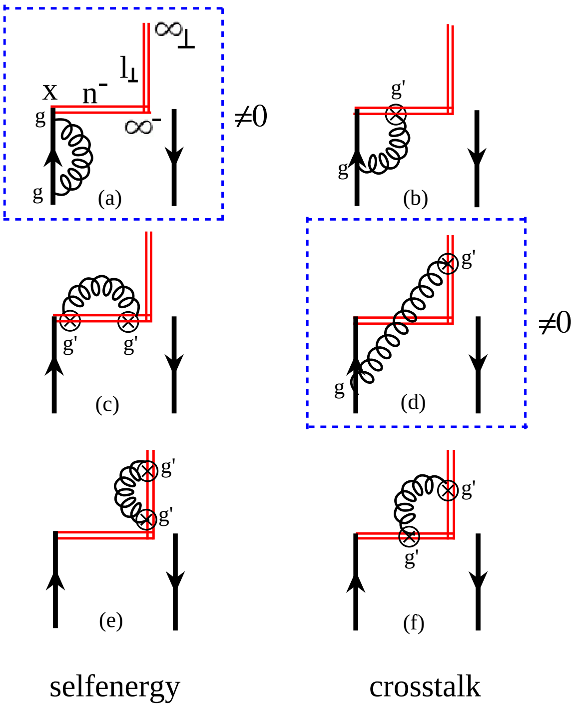

Figure 1: One-loop gluon virtual corrections to $f_{q/q}$ in the $A^+ = 0$ gauge. The double lines describe the gauge links attached to the fermions (heavy lines), while the curly lines represent gluons, and the symbol $\otimes$ denotes a line integral. The Hermitian-conjugate (mirror) diagrams are not shown.

Without going into too much detail, the results of this study show that the overlapping ultraviolet (UV) and rapidity divergences cannot be solely controlled by the dimensional (or any other) regularization. The ensuing divergence is of the type $(1/\varepsilon)\ln(\eta/p^+)$, which becomes infinite when $\eta \to 0$, and has, therefore, to be cured by an appropriate renormalization procedure. At this point it is important to mention that the terms on the diagonal in Table 1 represent selfenergy contributions, while all other terms are of the crosstalk type. In the gauge $A^+ = 0$ only the terms $\mathbf{QQ}$ and $\mathbf{Q}[\mathbf{l}_\perp]$ are non-vanishing. Moreover, the pole-prescription dependence in diagram (a) is canceled by its counterpart in (d) — see Fig. 1. Taking into account the mirror contributions to (a) and (d) (not shown in Fig. 1), one finds the following total contribution from virtual gluon corrections [2, 7]:

$$\Sigma_{\rm UV}^{\rm (a+d)}(\alpha_s,\varepsilon) = 2\frac{\alpha_s}{\pi}C_{\rm F}\left[\frac{1}{\varepsilon}\left(\frac{3}{4}+\ln\frac{\eta}{p^+}\right) - \gamma_E + \ln 4\pi\right] . \tag{2.1}$$

	struck quark	longitudinal gauge link	transverse gauge link
struck quark	$\mathbf{QQ} \Longleftrightarrow$ (a)	$\mathbf{Q}[n^-] \Longleftrightarrow$ (b)	$\mathbf{Q}[\mathbf{l}_\perp] \Longleftrightarrow$ (d)
longitudinal gauge link		$[n^-][n^-] \Longleftrightarrow$ (c)	$[n^-][\mathbf{l}_\perp] \Longleftrightarrow$ (f)=0
transverse gauge link			$[\mathbf{l}_\perp][\mathbf{l}_\perp] \Longleftrightarrow$ (e)

Table 1: Structure of the one-loop gluon virtual corrections to $f_{q/q}(x,\mathbf{k}_\perp)$ shown in Fig. 1.

From this expression one obtains for $f_{q/q}(x,\mathbf{k}_\perp)$ the anomalous dimension $\left(\gamma = \frac{\mu}{2}\frac{1}{Z}\frac{\partial\alpha_s}{\partial\mu}\frac{\partial Z}{\partial\alpha_s}\right)$

$$\gamma^{\rm LC}_{\rm one-loop} = \frac{\alpha_s}{\pi}C_{\rm F}\left(\frac{3}{4}+\ln\frac{\eta}{p^+}\right) = \gamma_{\rm smooth} - \delta\gamma\,, \tag{2.2}$$

where η is the rapidity parameter with $[\eta] = [\text{mass}]$ and $\delta\gamma$ represents the deviation from the anomalous dimension of the gauge-invariant quark propagator in a covariant gauge (see [3] and earlier references cited therein). As argued in [7, 2, 9], such an anomalous dimension can be associated with a cusp in the gauge contour at infinity and originates from the renormalization of the gluon interactions with this local contour obstruction. Therefore, one can claim that $\delta\gamma$ can be identified with the universal cusp anomalous dimension [10] at the one-loop order. But the choice of the gauge $A^+ = 0$ should not affect the renormalization properties of the TMD PDF. Thus, the definition of $f_{q/q}(x,\mathbf{k}_\perp)$ given by Eq. (1.5) has to be modified by a soft factor (counter term) [11]

$$R \equiv \Phi(p^+,n^-|0)\Phi^\dagger(p^+,n^-|\xi)\,, \tag{2.3}$$

where Φ and $\Phi^\dagger$ are appropriate eikonal factors to be evaluated along a jackknifed contour off the light cone (the explicit expressions and a graphic illustration can be found in [7, 2, 9]). We have shown there by explicit calculation that in the $A^+ = 0$ gauge with q^--independent pole prescriptions (advanced, retarded, principal value), the anomalous dimension associated with this quantity exactly cancels $\delta\gamma$, rendering the modified definition of the TMD PDF free from gauge artifacts. On the other hand, adopting instead a q^--dependent pole prescription (Mandelstam [12], Leibbrandt [13]), no anomalous-dimension anomaly appears and the soft factor reduces benignly to unity [8].

3. Inclusion of Pauli Spin Interactions

The conventional way to restore the gauge invariance of hadronic matrix elements is to use gauge links as those defined in Eqs. (1.6) and (1.7). However, this is only the minimal way to achieve this goal; it ignores the direct spin interactions because the gauge potential A_μ^a is spin-blind. To accommodate the direct interaction of spinning particles with the gauge field, one has to take into account the so-called Pauli term $\sim F^{\mu\nu}S_{\mu\nu}$, where $S_{\mu\nu} = \frac{1}{4}[\gamma_\mu,\gamma_\nu]$ is the spin operator. Following this generalized conception of gauge invariance, we promote the definition of the TMD PDF to [14]

$$\begin{aligned} f^{\Gamma}_{i/h}(x,\mathbf{k}_\perp) = \frac{1}{2}{\rm Tr}\int dk^-\int\frac{d^4\xi}{(2\pi)^4}e^{-ik\cdot\xi}\langle h|\bar\psi_i(\xi)[[\xi^-,\boldsymbol{\xi}_\perp;\infty^-,\boldsymbol{\xi}_\perp]]^\dagger[[\infty^-,\boldsymbol{\xi}_\perp;\infty^-,\boldsymbol{\infty}_\perp]]^\dagger \\ \times\Gamma[[\infty^-,\boldsymbol{\infty}_\perp;\infty^-,\mathbf{0}_\perp]][[\infty^-,\mathbf{0}_\perp;0^-,\mathbf{0}_\perp]]\psi_i(0)|h\rangle\cdot R\,, \end{aligned} \tag{3.1}$$

where Γ denotes one or more γ matrices in correspondence with the particular distribution in question, and the state $|h\rangle$ stands for the appropriate target. In the unpolarized case we have $|h\rangle = |h(P)\rangle$, with P being the momentum of the initial hadron, whereas for a (transversely) polarized target the state is $|h\rangle = |h(P),S_\perp\rangle$. The enhanced lightlike and transverse gauge links (denoted by double square brackets) contain the Pauli term and are given, respectively, by the following expressions:

$$[[\infty^-,\mathbf{0}_\perp;0^-,\mathbf{0}_\perp]] = \mathscr{P}\exp\left[-ig\int_0^\infty d\sigma\; u_\mu A_a^\mu(u\sigma)t^a - ig\int_0^\infty d\sigma\; S_{\mu\nu}F_a^{\mu\nu}(u\sigma)t^a\right]\,, \tag{3.2}$$

Symbols	Expressions	Figure 2	Value
$\mathscr{U}_1$	$\int_0^\infty d\tau\, \mathbf{l}\cdot\mathscr{A}(\mathbf{l}\tau)$	(a)	$\neq 0$
$\mathscr{U}_2$	$\int_0^\infty d\tau\, S\cdot\mathscr{F}(u\tau)$	(b)	$\neq 0$
$\mathscr{U}_3$	$\int_0^\infty d\tau\, S\cdot\mathscr{F}(\mathbf{l}\tau)$	—	0
$\mathscr{U}_4$	$\int_0^\infty d\tau \int_0^\tau d\sigma\, (\mathbf{l}\cdot\mathscr{A}(\mathbf{l}\tau))\, (\mathbf{l}\cdot\mathscr{A}(\mathbf{l}\sigma))$	—	0
$\mathscr{U}_5$	$\int_0^\infty d\tau \int_0^\tau d\sigma\, (\mathbf{l}\cdot\mathscr{A}(\mathbf{l}\tau))\, (S\cdot\mathscr{F}(\mathbf{l}\sigma))$	—	0
$\mathscr{U}_6$	$\int_0^\infty d\tau \int_0^\tau d\sigma\, (S\cdot\mathscr{F}(\mathbf{l}\tau))\, (\mathbf{l}\cdot\mathscr{A}(\mathbf{l}\sigma))$	—	0
$\mathscr{U}_7$	$\int_0^\infty d\tau \int_0^\tau d\sigma\, (S\cdot\mathscr{F}(u\tau))\, (S\cdot\mathscr{F}(u\sigma))$	(c)	0
$\mathscr{U}_8$	$\int_0^\infty d\tau \int_0^\tau d\sigma\, (S\cdot\mathscr{F}(\mathbf{l}\tau))\, (S\cdot\mathscr{F}(\mathbf{l}\sigma))$	—	0
$\mathscr{U}_9$	$\int_0^\infty d\tau \int_0^\infty d\sigma\, (\mathbf{l}\cdot\mathscr{A}(\mathbf{l}\tau))\, (S\cdot\mathscr{F}(u\sigma))$	(d)	$\neq 0$
$\mathscr{U}_{10}$	$\int_0^\infty d\tau \int_0^\infty d\sigma\, (S\cdot\mathscr{F}(\mathbf{l}\tau))\, (S\cdot\mathscr{F}(u\sigma))$	—	0

Table 2: Individual virtual-gluon contributions appearing in the evaluation of Eq. (3.4) up to $\mathscr{O}(g^2)$.

$$[[\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \mathbf{0}_\perp]] = \mathscr{P}\exp\left[-ig\int_0^\infty d\tau \mathbf{l}_\perp \cdot \mathbf{A}_\perp^a(\mathbf{l}\tau)t^a - ig\int_0^\infty d\tau S_{\mu\nu}F_a^{\mu\nu}(\mathbf{l}\tau)t^a\right] . \tag{3.3}$$

3.1 Gauge links with Pauli terms up to $\mathscr{O}(g^2)$

Adopting this reasoning, we have to calculate in the $A^+ = 0$ gauge the expression

$$[[\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \mathbf{0}_\perp]] \cdot [[\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]] = 1 - ig\left(\mathscr{U}_1 + \mathscr{U}_2 + \mathscr{U}_3\right) - g^2\left(\mathscr{U}_4 + \mathscr{U}_5 + \ldots \mathscr{U}_{10}\right) \tag{3.4}$$

with $F_a^{\mu\nu}(\infty^-, 0^+, \boldsymbol{\xi}_\perp) = 0, \quad \psi_i(\xi) = e^{[-ig\int d\eta\ \bar{\psi}\hat{\mathscr{A}}\psi]}\psi_i^{\text{free}}(\xi)$ and an analogous expansion for the transverse gauge links (see Table 2), whereas the contributing diagrams are displayed in Fig. 2. Let us quote here some important features of the presented theoretical framework referring for details to our recent work in Ref. [14]: (i) The Pauli term is not reparameterization invariant — unlike the usual Dirac term. Therefore, we have to use the dimensionful vectors $n_\mu^+ \to u_\mu^* = p^- n_\mu^+ , \quad n_\mu^- \to u_\mu = p^+ n_\mu^- , \quad \mathbf{l}_\perp \to p^+ \mathbf{l}_\perp$. (ii) The Pauli spin-interaction terms do not completely vanish along n^- in the $A^+ = 0$ gauge, whereas terms containing $\mathscr{F}(\mathbf{l}\tau)$ (or $\mathscr{F}(\mathbf{l}\sigma)$) cancel out in the product of the gauge links and $F_a^{\mu\nu}(\infty^-, 0^+, \boldsymbol{\xi}_\perp) = 0$. (iii) To the g^2-order level, the Pauli term reads

$$S\cdot\mathscr{F} \equiv S_{\mu\nu}\mathscr{F}^{\mu\nu} = 2S_{+-}\mathscr{F}^{+-} + 2S_{+i}\mathscr{F}^{+i} + 2S_{-i}\mathscr{F}^{-i} + S_{ij}\mathscr{F}^{ij} \tag{3.5}$$

and has the following non-zero components:

$$\mathscr{F}^{+-} = \partial^+\mathscr{A}^- , \ \mathscr{F}^{+i} = \partial^+\mathscr{A}^i , \tag{3.6}$$

$$\mathscr{F}^{-i} = \partial^-\mathscr{A}^i - \partial^i\mathscr{A}^- , \ \mathscr{F}^{ij} = \partial^i\mathscr{A}^j - \partial^j\mathscr{A}^i . \tag{3.7}$$

(iv) The diagrams (a)–(d) in Fig 2 represent virtual gluon corrections and contain UV and rapidity divergences that give rise to the anomalous dimension of the TMD PDF. In contrast, the diagrams (e)–(g), which describe real-gluon exchanges across the cut (vertical dashed line), contribute only finite terms.

From Fig. 2, we see that the gauge-link correlator contains contributions of two different types related to selfenergy- and crosstalk-type diagrams. To discuss the structure of the correlator in a compact way, it is useful to use the following symbolic abbreviations:

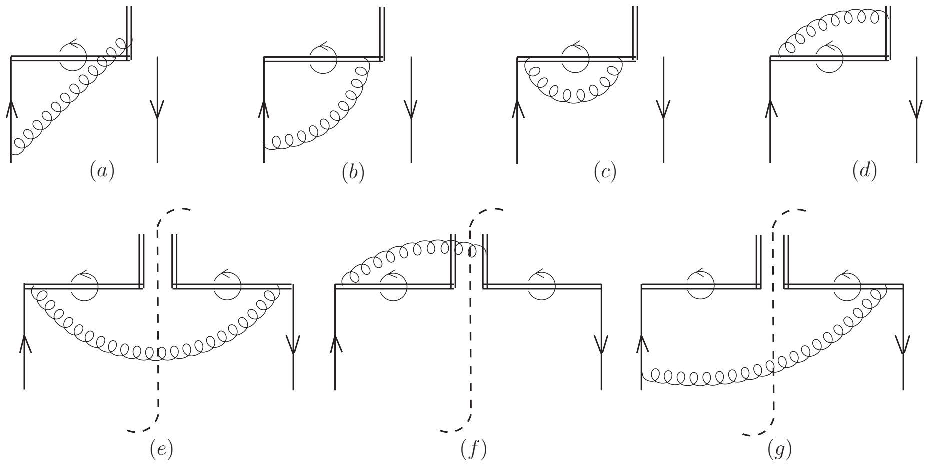

Figure 2: One-loop gluon virtual corrections to $f_{q/q}$ in the $A^+ = 0$ gauge. Graphs (a), (b), (c), and (d) describe virtual gluon corrections; graphs (e), (f), and (g) represent real-gluon exchanges across the cut (vertical dashed line). The double lines decorated with a ring represent enhanced gauge links containing the Pauli term.

$\mathbb{Q}$: Gauge self-field in the Heisenberg quark operator $\psi_i(\xi) = e^{-ig[\int d\eta\ \bar{\psi}\hat{\mathscr{A}}\psi]}\ \psi_i^{\text{free}}(\xi)$
$\mathbf{l}\cdot\mathscr{A}(\mathbf{l}\tau) \equiv \mathbb{A}^{\perp}$: Standard transverse gauge potential

$S\cdot\mathscr{F}(\mathbf{l}\tau) \equiv \mathbb{F}$: Tensor (Pauli) term

Then we obtain at $\mathscr{O}(g^2)$ the following results (consult Fig. 2 in conjunction with Table 2):
Selfenergy-type contributions

- $\mathbb{A}^{\perp}\mathbb{A}^{\perp}$: $\langle\mathscr{U}_4\rangle = 0$ not shown
- $\mathbb{F}^{-}\mathbb{F}^{-}$: $\langle\mathscr{U}_7\rangle = 0$ diagram (c) in Fig. 2
- $\mathbb{F}^{\perp}\mathbb{F}^{\perp}$: $\langle\mathscr{U}_8\rangle = 0$ not shown

Crosstalk-type contributions

- $\mathbb{Q}\mathbb{A}^{\perp}$: $\langle\mathscr{U}_1\rangle^{\text{UV}} = -\alpha_s C_{\text{F}}\frac{1}{\varepsilon}iC_\infty$ with $C_\infty = \{0(\text{adv}); -1(\text{ret}); -\frac{1}{2}(\text{PV})\}$ — diagram (a). This term cancels the pole-prescription-dependent term in the UV-divergent part of the fermion selfenergy $\mathbb{Q}\mathbb{Q}$.
- $\mathbb{Q}\mathbb{F}^{-}$: $\langle\mathscr{U}_2\rangle$ with $(\mathbb{Q}\mathbb{F}^{-})^{-} = \langle\mathscr{U}_2^{-}\rangle$ and $(\mathbb{Q}\mathbb{F}^{-})^{\perp} = \langle\mathscr{U}_2^{\perp}\rangle$ — diagram (b). Accordingly, for the leading twist-two TMD PDF, we find for the semi-inclusive DIS (SIDIS)

$$\Gamma_{\text{tw}-2}\langle\mathscr{U}_2^{-}\rangle + \langle\mathscr{U}_2^{-}\rangle^{\dagger}\Gamma_{\text{tw}-2} = \frac{i}{2}C_{\text{F}}\Gamma_{\text{tw}-2}\,, \tag{3.8}$$

$$\Gamma_{\text{tw}-2}\langle\mathscr{U}_2^{\perp}\rangle + \langle\mathscr{U}_2^{\perp}\rangle^{\dagger}\Gamma_{\text{tw}-2} = -\frac{i}{4}C_{\text{F}}\,\Gamma_{\text{tw}-2}\,. \tag{3.9}$$

These two results combine to produce a constant phase (unrelated to that found in [5])

$$\delta_{\text{tw}-2} = \alpha_s C_{\text{F}}\pi \tag{3.10}$$

which is also valid for the twist-three TMD PDF, i.e., $\delta_{\text{tw}-3} = \alpha_s C_F \pi$, but *flips sign* for the Drell-Yan (DY) process because it depends on the direction of the longitudinal gauge link. Hence, our analysis [14] predicts the important relation

$$\delta_{\text{SIDIS}} = -\delta_{\text{DY}} \,. \tag{3.11}$$

- $\mathbb{Q}\mathbb{F}^{\perp}$: $\langle \mathscr{U}_3 \rangle = 0$ not shown
- $\mathbb{A}^{\perp}\mathbb{F}^{\perp}$: $\langle \mathscr{U}_6 \rangle = -\langle \mathscr{U}_5 \rangle$, hence mutually canceling
- $\mathbb{A}^{\perp}\mathbb{F}^{-}$: $\langle \mathscr{U}_9 \rangle = \langle \mathscr{U}_9 \rangle^{\dagger}$ — diagram (d) in Fig. 2 — ("gluon mass" λ^2 drops out at the end):

$$\langle \mathscr{U}_9 \rangle = -\frac{1}{8\pi}\, C_F [\gamma^+, \gamma^-] \Gamma(\varepsilon) \left(4\pi \frac{\mu^2}{\lambda^2}\right)^{\varepsilon} \,. \tag{3.12}$$

This nontrivial Dirac structure entails

$$\Gamma_{\text{unpol.}} = \gamma^+ \quad : \Gamma_{\text{unpol.}}[\gamma^+, \gamma^-] = -[\gamma^+, \gamma^-]\Gamma_{\text{unpol.}} \,, \tag{3.13}$$

$$\Gamma_{\text{helic.}} = \gamma^+\gamma^5 \quad : \Gamma_{\text{helic.}}[\gamma^+, \gamma^-] = -[\gamma^+, \gamma^-]\Gamma_{\text{helic.}} \,, \tag{3.14}$$

$$\Gamma_{\text{trans.}} = i\sigma^{i+}\gamma^5 : \Gamma_{\text{trans.}}[\gamma^+, \gamma^-] = -[\gamma^+, \gamma^-]\Gamma_{\text{trans}} \,, \tag{3.15}$$

where obvious acronyms have been used. Taking into account the mirror diagrams (not shown in Fig. 2), the twist-two terms mutually cancel by virtue of the relation

$$[\gamma^+, \gamma^-]\Gamma_{\text{tw}-2} = -\Gamma_{\text{tw}-2}[\gamma^+, \gamma^-] = 2\Gamma_{\text{tw}-2} \,,$$

which permits a probabilistic interpretation of the twist-two TMD PDF as a density on account of $\mathbb{A}^{\perp}\mathbb{F}^{-} \to 0$. On the other hand, the twist-three TMD PDF gets a non-vanishing contribution to its anomalous dimension as one sees from

$$\Gamma_{\text{tw}-3}\langle \mathscr{U}_9 \rangle + \langle \mathscr{U}_9 \rangle^{\dagger}\Gamma_{\text{tw}-3} = -\frac{C_F}{4\pi}[\gamma^+, \gamma^-]\Gamma(\varepsilon)\left(4\pi \frac{\mu^2}{\lambda^2}\right)^{\varepsilon} \,.$$

- $\mathbb{F}^{\perp}\mathbb{F}^{-}$: $\langle \mathscr{U}_{10} \rangle = 0$ without assuming any particular form of the gauge field at light cone ∞.

3.2 Real-Gluon Contributions at $\mathscr{O}(g^2)$

Besides the virtual gluon corrections, there are also real gluon exchanges that contribute finite contributions to the TMD PDF. The main difference from the previously considered case is that now the discontinuity goes across the gluon propagator that has to be replaced by the cut one. Moreover, the Dirac structures, marked above by the symbol Γ, are sandwiched between Dirac matrices stemming from Pauli terms standing on different sides of the cut. The real-gluon contributions are specified in Table 3.

Using the same symbolic notation as in the previous subsection, we briefly remark that

- $\mathbb{F}^{-}\mathbb{F}^{-}$: $\langle \mathscr{U}_{11} \rangle \to 0$ (at least power-suppressed $\sim p^-$)
- $\mathbb{A}^{\perp}\mathbb{F}^{-}$: $\langle \mathscr{U}_{12} \rangle + \langle \mathscr{U}_{12} \rangle^{\dagger} \sim \Gamma[\gamma^+, \gamma^-] + [\gamma^+, \gamma^-]\Gamma = 0$
- $\mathbb{Q}\mathbb{F}^{-}$: $\langle \mathscr{U}_{13}^{-} \rangle + \langle \mathscr{U}_{13}^{-} \rangle^{\dagger}$ and $\langle \mathscr{U}_{13}^{\perp} \rangle + \langle \mathscr{U}_{13}^{\perp} \rangle^{\dagger}$ mutually cancel up to a power-suppressed term.

Symbols	Expressions	Figure 2
$\mathscr{U}_{11}$	$\int_0^\infty d\tau \int_0^\infty d\sigma\ (S\cdot\mathscr{F}(u\tau))\,\Gamma\,(S\cdot\mathscr{F}(u\sigma+\xi^-;\boldsymbol{\xi}_\perp))$	(e)
$\mathscr{U}_{12}$	$\int_0^\infty d\tau \int_0^\infty d\sigma\ (\mathbf{l}\cdot\mathscr{A}(\mathbf{l}\tau))\,\Gamma\,(S\cdot\mathscr{F}(u\sigma+\xi^-;\boldsymbol{\xi}_\perp))$	(f)
$\mathscr{U}_{13}$	$\int_0^\infty d\sigma\Gamma\,(S\cdot\mathscr{F}(u\sigma+\xi^-;\boldsymbol{\xi}_\perp))$	(g)

Table 3: Individual real-gluon contributions to $\mathscr{O}(g^2)$ corresponding to the diagrams $(e),(f),(g)$ in Fig. 2.

4. Highlights and Conclusions

We argued that the dimensional regularization of overlapping UV and rapidity divergences in TMD PDFs is not sufficient to render the TMD PDF finite — one needs renormalization [7, 2]. To remedy this deficiency, a soft factor [11] along a jackknifed contour off the light cone was introduced into the definition of the TMD PDF [7] whose anomalous dimension cancels in leading loop order the cusp anomalous dimension entailed by this overlapping divergence (with a full-fledged discussion being given in [2]). The modified TMD PDF reproduces the standard integrated PDF and is controlled by an evolution equation with the same anomalous dimension as one finds in covariant gauges with no dependence on the adopted pole prescription for the gluon propagator — this would be impossible without the soft renormalization factor (see [2] and for a more dedicated discussion [15]). In particular, using the $A^+ = 0$ gauge in conjunction with the Mandelstam-Leibbrandt pole prescription [12, 13], no anomalous-dimension defect appears and thus the soft factor becomes trivial. An important finding of this approach is that the anomalous dimension of the unpolarized TMD PDF for SIDIS and the DY process is the same, i.e., $\gamma_{f_{q/q}}^{\rm SIDIS} = \gamma_{f_{q/q}}^{\rm DY}$, albeit the sign of the ε term in the gluon propagator $\frac{1}{q^+ + i\varepsilon}$ is different for these two processes — irrespective of the boundary condition applied. Quite recently, Collins discussed alternative ways to redefine the TMD PDFs in such a way as to avoid rapidity divergences [16].

We also presented a new scheme for gauge-invariant TMD PDFs which includes the direct interaction of spinning particles with the gauge field by means of the Pauli term in the longitudinal and transverse gauge links. In some sense, the Pauli spin interaction is the abstract analogue of a Stern-Gerlach apparatus — sort of — and gives rise through the transverse gauge link to a constant phase $\delta = \alpha_s C_{\rm F}\pi$, which is the same for twist-two and twist-three TMD PDFs, but flips sign when the direction of the gauge link is reversed — thus breaking universality. As a result, one finds $\delta_{\rm DY} = -\delta_{\rm SIDIS}$. To facilitate calculations, we developed in Ref. [14] Feynman rules for enhanced gauge links — longitudinal and transverse — which supplement those derived before for the standard gauge links by Collins and Soper [1]. Because the Pauli term contributes to the anomalous dimension of the twist-three TMD PDF, the evolution of such quantities is more delicate and may require the modification of the renormalization factor to preserve its density interpretation.

Bottom line: Our results — most significant amongst them the appearance of a non-universal phase — may stimulate both theoretical and experimental activities. On the other hand, T-even and T-odd TMD PDFs may become "measurable" on the lattice, so that it seems possible that non-trivial Wilson lines, as those we discussed in this presentation, may be revealed in the future.

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