Electron-Positron Radiative Annihilation : Timelike Virtual Compton Scattering

Asmita Mukherjee

Physics Department, IIT Bombay, Powai, Mumbai 400076 India
E-mail: mukherjee.asmita@gmail.com

We report on a recent work proposing measurements of the deeply virtual Compton amplitude (DVCS) $\gamma^* \rightarrow h \bar{h} \gamma$ in the timelike $t = (p_h + p_{\bar{h}})^2 > 0$ kinematic domain which is accessible at electron-positron colliders via the radiative annihilation process $e^+ e^- \rightarrow h \bar{h} \gamma$. 
1. Introduction

Deeply virtual compton scattering (DVCS) process, $ep \rightarrow e' \gamma p'$, where the intermediate photon virtuality $q^2 = (p' e - p e)^2 < 0$ and the momentum transfer to the target proton $t = (p' - p)^2 < 0$ are spacelike; provides access to the generalized parton distributions (GPDs) of the proton. They provide important information on the spin and spatial structure of the nucleon. The GPDs measure hadron structure at the amplitude level in contrast to the probabilistic properties of parton distribution functions. In the forward limit (zero momentum transfer) they reduce to ordinary parton distributions and $x$ moments of them are related to the form factors. The real part of the DVCS amplitude is measured through the interference with the Bethe-Heitler process and the imaginary part is measured through various spin asymmetries. In contrast, the process we consider is the radiative annihilation process $e^+ e^- \rightarrow h \bar{h} \gamma$ [2], which is accessible at electron-positron colliders and measures the timelike DVCS amplitude $\mathcal{M}(\gamma^* (q) \rightarrow \gamma h \bar{h})$ illustrated in Fig. 1(a). We will discuss possible measurements of the DVCS amplitude in the timelike or $t > 0$ kinematic domain, where $t = W^2 = (p_h + p_{\bar{h}})^2$ is the mass of the produced hadron pair. The hadronic matrix element is $C$-even since two photons attach to it. The same final state can also come from Bethe-Heitler processes, Fig. 1(b), where the hadronic part of the matrix element is $C$-odd. In addition to the process under consideration, doubly virtual Compton scattering, where one uses $e^+ e^- \rightarrow e^+ e^- h \bar{h}$ to measure the amplitude $\mathcal{M}(\gamma^* (q) \gamma^* (q') \rightarrow h \bar{h})$ with one or both initial photons highly spacelike, can also be measured at an electron-positron collider.

One can apply charge conjugation to the electron and positron in the initial state, thus relating two kinematic situations where the momentum and spin of the electron and positron are interchanged. The amplitudes change sign or not depending on the photon attachment to the initial electron line. The asymmetry obtained by interchanging the electron and positron is sensitive to the interference term between the $C$-even and $C$-odd amplitudes as [1]

$$A = \frac{\sigma - \sigma(e^+ \leftrightarrow e^-)}{\sigma + \sigma(e^+ \leftrightarrow e^-)} = \frac{2\text{Re}(\mathcal{M}(C = +) \times \mathcal{M}(C = -))}{|\mathcal{M}(C = +)|^2 + |\mathcal{M}(C = -)|^2},$$

(1.1)

which is sensitive to the relative phase of the $C$-even DVCS amplitude and the timelike form factors. The QED equivalents of these amplitudes, where hadrons are replaced by muons, usefully show that the magnitude of the $e^+ \leftrightarrow e^-$ asymmetry can be quite large.

We present a model calculation of the asymmetry for kinematic conditions of existing electron-positron colliders. Relevant kinematics is chosen for tau-charm factories, $s=14 \text{ GeV}^2$ (BEPCII) and B-factories, $s=112 \text{ GeV}^2$ (Babar at PEPII and Belle at KEKB).

Measurements of the radiative annihilation process can provide valuable new information on the analytic continuation of the DVCS amplitude, including the existence of a $J = 0$ fixed pole [3], which would be an amplitude that is constant in energy (and real in the spacelike case) though not constant in momentum transfer.

We obtain a simple hadronic estimate by modeling the $C$-even $p \bar{p}$ timelike hadronic DVCS amplitude after an analysis of how it can be written in terms of several Lorentz structures multiplied by $C = +$ form factors. One of these is $R_V(\xi, W^2)$ and all of these can be related to the timelike
Electron-Positron Radiative Annihilation

Asmita Mukherjee

Figure 1: Processes contributing to $e^+e^- \rightarrow h^+h^-\gamma$: (a) the generic timelike DVCS process and (b) Bethe-Heitler processes.

generalized parton distributions, or generalized distribution amplitudes [4, 5]. We will keep only the $R_V$ term, which has the appearance of the QED amplitude multiplied by $R_V(\xi, W^2)$, and we will model this form factor in a simplified way where it depends only on $W^2 = t = (q - q')^2$ and is independent of $s$, or the overall $q^2$. One can say that this model simulates the C-even Compton amplitude as a $J = 0$ fixed pole amplitude with Regge behavior $s^0$ at fixed $t$. Of relevance here is an experimental result for the spacelike $C = +$ form factor $R_V(t)$ from real wide-angle Compton scattering. It is defined as the ratio of the measured real Compton amplitude $M(\gamma p \rightarrow \gamma' p')$ divided by the pointlike Klein-Nishina formula. $R_V(t)$ is measured to fall off as $1/t^2$ at large $t$ [6], consistent with PQCD and AdS/QCD counting rules, which in turn is consistent with what we do in the present context.

2. Cross section and Asymmetry

The process that we consider is,

$$e^+(p_{e^+}) + e^-(p_{e^-}) \rightarrow p(p_{h^+}) + \bar{p}(p_{h^-}) + \gamma(q')$$

and for comparison, we also consider the same process with $p$ and $\bar{p}$ replaced by $\mu^+$ and $\mu^-$, respectively.

The cross section for process (2.1) is [2]

$$d\sigma = \frac{\beta W (s - W^2)}{64(2\pi)^5 s^2} |\mathcal{M}|^2 dW d\Omega^* d\Omega,$$
electron-positron radiative annihilation

Asmita Mukherjee

Figure 2: Partonic diagrams for the case that the external photon is emitted from the hadrons.

where $|\mathcal{M}|^2$ is the matrix element summed over final and averaged over initial polarizations and we also use the notations

$$s = q^2 = Q^2; \quad s' = W^2; \quad \beta = \sqrt{1 - \frac{4m^2}{W^2}} \quad (2.3)$$

The solid angle $\Omega^*$ gives the direction of the outgoing proton or $\mu^+$ in the $p\bar{p}$ or $\mu^+\mu^-$ rest frame and $\Omega$ gives the direction of the incoming electron in the $e^+e^-$ rest frame.

Neglecting components that do not give large contributions in the Bjorken limit, the amplitude becomes

$$\mathcal{M}^{\mu\nu} = \frac{e^2}{2} (g^{\mu\nu} - p^\mu n^\nu - n^\mu p^\nu)$$

$$\times \int dx \left\{ \frac{1}{x + \xi + i\eta} + \frac{1}{x - \xi - i\eta} \right\}$$

$$\times \bar{u}(p_h) \left[ i \gamma^q + \frac{i}{2m} \sigma^{\alpha\beta} n_\alpha \Delta_\beta E^q \right] v(p_{\bar{h}})$$

$$- \frac{ie^2}{2} \epsilon^{\mu\nu\alpha\beta} p_{\alpha} n_{\beta} \int dx \left\{ \frac{1}{x + \xi + i\eta} - \frac{1}{x - \xi - i\eta} \right\}$$

$$\times \bar{u}(p_h) \left[ i \gamma^q \tilde{H}^q + \frac{n \cdot \Delta}{2m} \tilde{E}^q \right] v(p_{\bar{h}})$$

$$\equiv -e^2 \frac{\epsilon^{\mu\nu}}{2} \bar{u}(p_h) \left[ i R^q_{\nu} + \frac{i}{2m} \sigma^{\alpha\beta} n_\alpha \Delta_\beta R^q_T \right] v(p_{\bar{h}})$$

4
Asmita Mukherjee

Electron-Positron Radiative Annihilation

Figure 3: Asymmetries for $\gamma^* \rightarrow \gamma p \bar{p}$ and its muonic counterpart, plotted vs. the final fermion pair invariant mass, over a range beginning close to the $p \bar{p}$ threshold. The upper graph ($s = 112$ GeV$^2$) is for BELLE or Babar energies, and the lower graph ($s = 14.3$ GeV$^2$) is for BEPC II kinematics. The angles (in radians) and angular ranges are indicated on each plot.

The form factors $R^q_V, R^q_T, R^q_A,$ and $R^q_P$ are [7, 8]

$$
R^q_V(\xi, W^2) = \int \frac{x}{x^2 - \xi^2 - i\eta} H^q(x, \xi, W^2),
$$

$$
R^q_T(\xi, W^2) = \int \frac{x}{x^2 - \xi^2 - i\eta} E^q(x, \xi, W^2),
$$

$$
R^q_A(\xi, W^2) = \int \frac{\xi}{x^2 - \xi^2 - i\eta} R^q(x, \xi, W^2),
$$

$$
R^q_P(\xi, W^2) = \int \frac{\xi}{x^2 - \xi^2 - i\eta} E^q(x, \xi, W^2)
$$

The full amplitude will depend on

$$
R_V(\xi, t) = \sum e^2_q R^q_V(\xi, t),
$$

with similar equations for $V \rightarrow T, A, P$. The asymmetries, Eq. (1.1), arise from interference between the C-odd and C-even amplitudes.

In modeling the hadronic part of the C-even diagrams, we keep just the term $R_V(\xi, W^2)$. Further, recall that photon-hadron amplitudes can expect a $J = 0$ fixed pole, which would be a term with flat energy dependence and a form factor like dependence on the momentum transfer to the hadrons, $W^2$. We give $R_V$ the same $W^2$ dependence as an electromagnetic form factor and normalize by

$$
|R_V(\xi, W^2)| = \frac{4}{3} F_1(W^2).
$$

The multiplicative factor is estimated from the expressions for $R_V(\xi = 0, W^2)$ and $F_1(W^2)$, Eq. (2.5), and expecting domination by $u$-quarks and approximate mean momentum fraction $x \approx 1/2$. 

\[5\]
Support for this normalization and shape comes from data on spacelike wide angle Compton scattering. The numerically most important form factor here is \( R_V(t) \), and data shows that while \( R_V(t) \) does drop less rapidly with increasing \(|t|\) than \( F_1(t) \), it does not do so by a lot, and that \( R_V(t) = (4/3)F_1(t) \) is a decent representation of the data.

Further in our modeling, we note that the Lorentz structure that multiplies \( R_V(\xi,W^2) \) in the Bjorken limit is the same as one obtains from the QED amplitude in the Bjorken limit. If we are not deeply in the Bjorken region, we can argue that the Lorentz structure of the amplitude is better represented by the QED amplitude, including the final fermion mass and multiplied by \( R_V(\xi,W^2) \).

Fig. 3 shows two asymmetry plots, one at \( s = 112 \text{ GeV}^2 \) relevant for Belle or Babar energies and at \( s = 14.3 \text{ GeV}^2 \) relevant for BEPC II energies. The asymmetries are for cross sections integrated over a stated range of angles, and plotted versus final hadronic mass \( W \). Since the sign of the asymmetry changes with \( \phi \), one should not integrate over more than half the range of that angle; if desired, one can integrate over fairly broad ranges of \( \theta \) and \( \theta^* \). For comparison, and to indicate the mass sensitivity for the selected \( s \) and \( W \), the plots also include the asymmetries expected for the purely muonic case. The asymmetries can be large and measurable when the kinematics are well chosen.

3. Summary and Conclusions

We studied deeply virtual Compton scattering amplitude in the timelike domain in electron positron radiative annihilation process. We introduced a forward-backward asymmetry to isolate the \( C = + \) amplitude, where both photons couple to the hadrons, in an interference with the \( C = - \) amplitude where one one photon couple to the hadron. By choosing a simple model we have showed that the asymmetry can be quite large.

4. Acknowledgments

We thank the organizers of LC2010 for the invitation and International Light Cone Advisory Committee for Gary Mccartor Travel Award.

References
