PROCEEDINGS OF SCIENCE

The Pion Electromagnetic Form Factor in the Light-Front Approach

J. P. B. C. de Melo[a*†] **and T. Frederico**[b]
[a]*Laboratório de Física Teórica e Computação Científica*
Universidade Cruzeiro do Sul, 01506-000
[b]*Dep. de Física, Instituto Tecnológico de Aeronáutica, 12.228-900, São José dos Campos, Brazil*
E-mail: `joao.melo@cruzeirodosul.edu.br`

We study in the present work, the properties of pseudo-scalar light meson, in particular the elastic electromagnetic form factors and the decay constant for pseudoscalar mesons; also the transition form factor for the pion. In the case of the calculation for the elastic electromagnetic form factors and the decay constants, the model used is the covariant light front model (LFCM), utilized in a previus work in the heavy hadronic sector. For the transition form factor, the models of the wave functions employed are the gaussian and the hydrogen-atom wave function. The pion system is compose by constituents confined quark-antiquark pair bound state, wich in the present model is given by the Pauli-Villars regulators in order to regularize the Beth-Salpeter amplitudes.

LIGHT CONE 2010 Relativistic Hadronic and Particle Physics
June, 14-18, 2010, Valencia, Spain, France

*Speaker.
†www.cruzeirodosul.edu.br/lftc

PoS(LC2010)063

1. Introduction

In general, electromagnetic form factors are important font of informations about the internal structure of hadronic bound states. The few observables, for example the magnetic momentum and the total charge give many informations about the hadron in question. However, in the case of the light mesons, the relativistic behavior depend of the habronic substructure in terms of the fundamental constituents, quarks and gluons. The nonperturbative structure of hadrons bound states is a very hard task in terms of the quantum chromodynamics (QCD) degrees of freedom [1] and the phenomenological models [2, 3] wich parametrized the QCD properties of bound state mesons is able to give some answers in fit the experimental data [4, 5, 6].

The model utilized in the present work is the a relativistic constituent covariant quark model (Light-front covariant model,LFCM) [7], formulated with the light-front form of dynamics, proposed by Dirac [8] (see the reference [9] for details).

Here we study the electroweak properties of the light pseudo-scalar mesons, in particular, we address the electromagnetic form factors of the pion and kaon; also the transition form factor for the pion decay in two photons.

2. Bethe-Salpeter Amplitude Model and Wick Rotation

The vertex $meson - q\bar{q}$ model used to build the Bethe-Salpeter amplitude is writing with the function [2, 7]

$$\Lambda_M(k,p) = \frac{(k^2-m_1^2)\Gamma_M((p-k)^2-m_2^2)}{(k^2-\lambda^2+\imath\varepsilon)^n\;((p-k)^2-\lambda_M^2+\imath\varepsilon)^n}. \tag{2.1}$$

λ_M is a scale associated with the meson valence wave function and n is the power of the regulator; here, m_1 and m_2 are the quark and anti-quark masses within the hadronic bound state. The factors $(k^2-m_1^2)$ and $((p-k)^2-m_2^2)$ on the numerator of the vertex function, $\Lambda_M(k,p)$, avoids the cuts due the $q\bar{q}$ scattering if m_1+m_2 is smaller than the meson mass. The λ_M obeys the condition $2\;\lambda_M >$, in order to confine the meson bouns state.

With the the vertex function done above, the eletromagnetic form factor for the pseudoscalar mesons are given by the Mandelstam formula:

$$\left\langle p' \left| J_q^\mu\left(q^2\right) \right| p \right\rangle = \frac{N_c}{(2\pi)^4}\int d^4k\; Tr\left[\Lambda_{M'}\left(k,p'\right) S_F\left(k-p'\right) J_q^\mu S(k-p)\Lambda_M\left(k,p\right) S_F\;\left(k\right)\right]. \tag{2.2}$$

Where $S_F(p)$ is the Feynman propagator of the quark with the constituent mass m_q and Λ_M is the $meson - q\bar{q}$ vertex function presented in the last section. $N_c = 3$ is the number of quark colors and p^μ and p'^μ are the initial and final momenta of the system; q^μ is the momentum transfer. The electromagnetic form factor for pseudoscalr meson is calculated with the matrix of the electromagnetic current given by the equation:

$$\left\langle p' \left| J_q^\mu\left(q^2\right) \right| p \right\rangle = \left(p+p'\right) F_{PS}^{em}(q^2) \tag{2.3}$$

The weak decay constant of the pseudoscalar mesons is written as

$$\langle 0 \left| A^\mu(0) \right| p\rangle = \imath\sqrt{2} f_{ps} p^\mu, \tag{2.4}$$

where $A^{\mu} = \bar{q}(x)\gamma^{\mu}\gamma^{5}\frac{\tau}{2}q(x)$. The plus component of the electromgnetic current, $J^{+} = J^{0} + J^{3}$, is utilized to calculated the eletromagnetic form factor and the decay constant and the final expression for the pseudoscalar decay constant is

$$\imath p^2 f_{\pi} = \frac{m}{f_{\pi}} N_c \int \frac{dk^4}{(2\pi)^4} Tr\left[\not p \gamma^5 S(k)\gamma^5 S(k-p)\right] \Lambda_M(k,p). \quad (2.5)$$

In order to integrate the equation above, the Wick rotation [10] is formulated in the light-front approach [7]. With the instant form approach, the Wick rotation consist in change the time-momentum by the complex time momentum, $k_0 \longrightarrow \imath k_0$ [1]. The Wick rotation is utilized to solve the Beth-Salpeter (BS) equation in the Euclidian Space, because in the Minkowski space, the BS equation it difficult to solve because the singularities. Nevertheless, in the light-front case, the choice of the frame used to calculated the Feynnman amplitudes is important, because the poles position in k^{-} complex plane are frame dependent. The frame utilized for the calculation of the amplitudes with the light-front field theory are important, since is related with the breaking of the covariance of the eltromagnetic current by the non-valence contribuition to the matriz elements of the eletromagnetic current [11, 12, 13, 14, 15].

For the elastic electromagnetic form factors, the frame used in the literature in the past, is the Breit-frame with the condition $q^{+} = 0$, call in the literature as the Drell-Yan condition. This frame was accepted in the past as free of the zero modes or pair terms contribuition to the matriz elements of the electromagnetic current. But is not the case; not only the frame are important to calculated the electromagnetic processes, on the other hand, the component of the electromagnetic current utilized to extract the observables is also very important [13, 15]. In the reference [13] the pion meson electromagnetic elastic form factor was calculated with the plus and minus components of the electromagnetic current. In the case of the minus component of the current, besides the valence contribuition for the minus component of the electromagnetic current, we have contribuition of the non-valence components. The observables after the inclusion of the non-valence component are the same calculated with the equal time quantum field theory [1]. In the light-front approach, the Wick rotation is applied in order to avoid the singularities associated with the poles in the Bethe-Salpter amplitudes in the calculated matriz elements of the electromagnetic current [7].

The wick rotation in the light-front approach correspond to change the minus component of the quadri-momentum k^{-} by $k^{-}e^{\theta}$. Calculation with the Wick rotation formulated in the light-front envolve the θ angle of the rotation, which permit deform the integration contour to real for the complex plane.

3. Elastic Form Factors for Pion and Kaon

The observables for pseudoscalars mesons, pion and kaon, are calculated with the vertex (Eq. 2.1), present in the previus section. The calculation envolve the rotation Wick angle between $0 \leq \theta \leq 90^{0}$ and the equation below prove the consistence of the method:

$$\frac{F_{\pi}(q^2,\theta)}{F_{\pi}(q^2,\alpha)} = 1, \quad (3.1)$$

because the α and θ are two differents angles and given exact the same elastic electromagnetic form factor for the pion. The angles θ and α in the Eq. (3.1) are $30^0, 45^0, 60^0$ and 90^0, with $\theta \neq \alpha$.

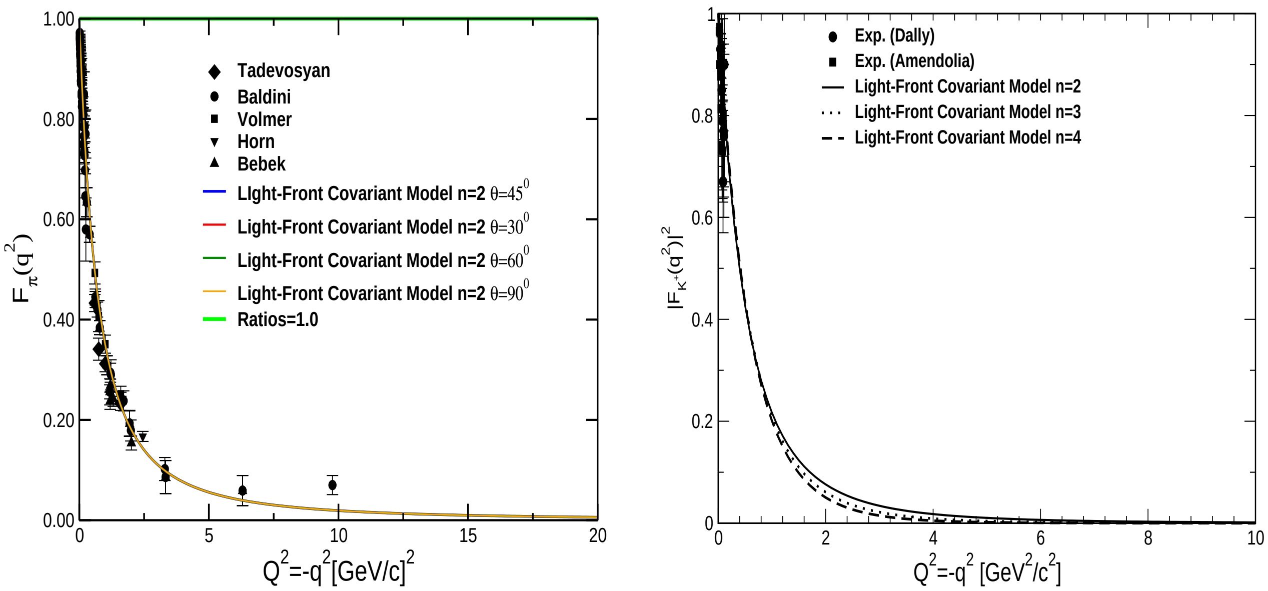

Figure 1: The pion and kaon elastic form factor calculated with the light-front covariant model (LFCM). The pion is calculated with the power vertex $n = 2$ and some differents θ Wick rotation angles, compared with the experimental word data for the elastic electromagnetic form factor [4, 5]. Also the calculation with Eq. (3.1) are showed and prove the consistence of the Wick rotation with the light-front field theory. In the right, the kaon elastic form factor calculated with the $n = 2, 3, 4$ in the LFCM is compared with the experimental data [4].

	n	r_{PS} (fm)	f_{PS}(MeV)	λ_M (MeV)
Pion (139 MeV)	2	0.576	92.4	542
$m_u = 220$ MeV	3	0.494	92.4	926
$m_d = 220$ MeV	4	0.456	92.4	1255
Exp.(Pion)		0.672	92.42	
Kaon (494 MeV)	2	0.474	113	648
$m_s = 0.508$ MeV	3	0.453	113	933
$m_u = 0.220$ MeV	4	0.450	113	1156
Exp.(Kaon)		0.560	113	

In contrast with the pion, for the kaon not have new experimental data for observables, and it is difficult to compare new theoretical or phenomenological models in order to decide the best description to the kaon bound state [16, 17, 18]. However, phenomenological models are important, because these models given some insights about the possible answers for the strange sector of the hadronic mesons bound states. For the model here (LFCM), the elastic electromagnetic form factor for the kaon is calculated with the vertex function (Eq. 2.1) and for the present calcultion, have the angle for the Wick rotation fixed at 90^0 and the power of the vertex function running between

$n = 2,3,4$. The results for the elastic electromagnetic form factor at the Fig.2, in the kaon case, show at low energy a weak dependence with the power n of the vertex function. However between 1 and 2 GeV some difference for differents choices of the power n appear. At higher momentum transfer, up to 5.0 GeV^2, all n utilized here give the same electromagnetic form factor for the kaon. The parameters of the model are fixed by the pion and kaon decay constants [19].

4. Pion Decay

The pion decay process in two photons it a simple and important test to QCD. After the new experimental data from Babar [20], wich have studied the reaction $e^+e^- \to e^+e^-\pi^0$, the pion decay have a great attention [21, 22, 23, 24, 25]. The new data from Babar advocate the prevision made by perturbative QCD [26] for the pion decay is not satisfactory after 15 GeV^2. In the present work, the effective interaction lagrangian

$$\mathscr{L}^{int}_{\pi q} = -\imath \frac{m}{f_\pi} \vec{\pi} \cdot \vec{q} \gamma^5 \vec{\tau} q \tag{4.1}$$

is utilized, here m is the constituent quark mass, f_π is the weak decay constant and, $\vec{\pi}$ an $\vec{q}$ the pion and the quarks fields, the units are $\hbar = c = 1$. The amplitude for the transition of the pion in two photons is given by the triangle diagram. The tensor amplitude $T^{\mu\nu}$, have two components

$$T^{\mu\nu} = t_{\mu\nu}(k_1,k_2) + t_{\mu\nu}(k_2,k_1) \tag{4.2}$$

The tensor $t_{\mu\nu}(k_1,k_2)$ is obtained subsequent the traces in the spinor and flavour space are complete [1]:

$$t_{\mu\nu} = \frac{4}{3}\frac{M^2}{f_\pi} e_0^2 N_c \varepsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta I(k_1^2) \tag{4.3}$$

The integral $I(q^2)$ is

$$I(k_1^2) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{((k_2-k)^2 - M^2 + \imath\varepsilon)} \frac{1}{(k^2 - M^2 + \imath\varepsilon)((k_\pi - k)^2 - M^2 + \imath\varepsilon)} \,. \tag{4.4}$$

Were, N_c is the number of colors (=3), $k^\mu_\pi = k^\mu_1 + k^\mu_2$ is the π^0 momentum, $q^\mu = k^\mu_1$ is the space-like momentum transfer, and e_0 is the umit charge. The factor $1/3$, in Eq. 4.3, comes from the traces in the flavour space. The integration of the Eq.(4.4), are performade with the light-front coordinates, $k^+ = k^0 + k^3$ and $k^- = k^0 - k^3$, and after k^- integration, the pion aymptotic wave function appear [13, 27, 28]. The frame utilized have $q^+ = q^- = 0$ and the momentum transfer are transversal $q_\perp \neq 0$. The final neutral pion electromagnetic transition form factor is is **given** by the the central formula of the present work form-factor for a light-front of the quark bound-state wave-function:

$$F_{\pi^0}(-q^2) = \frac{\sqrt{N_c} M}{6\pi^{\frac{3}{2}}} \int \frac{dx d^2K_\perp}{(1-x)\sqrt{M_0}} \frac{\Phi_\pi(K^2)}{((\vec{K} x \vec{q})^2_\perp + M^2)} \,, \tag{4.5}$$

with the definitions

$$\vec{K}_\perp = (1-x)\vec{k}_\perp - x(\vec{k}_\pi - \vec{k})_\perp \,, \tag{4.6}$$

and

$$M_0^2 = \frac{K_\perp^2 + M^2}{x(1-x)} . \tag{4.7}$$

M_0 is the free mass operator; the pion wave-function, $\Phi(K^2)$, is normalized to one. The phenomenological light-front wave functions, $\Phi(k^2)$, utilized at this point are the Gaussian, $e^{((-4/3)\ R_{nr}^2 k^2)}$ and the Hydrogen-atom, $1/(R_{nr}^{-2} + k^2)^2$, which depende of two independent parameters, the quark mass and the non-relativistic charge radius R_{nr}. The parameters are fixed by the experimental pion decay constant $f_\pi = 93.4\ MeV$ [19].

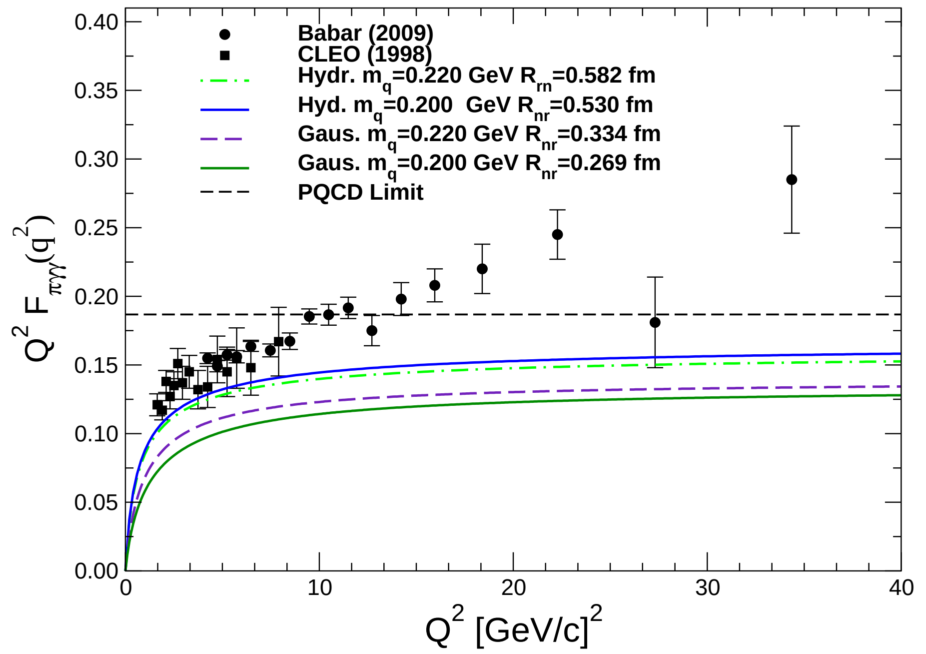

Figure 2: The pion transition form factor calculated with the Hidrogen and Gaussian models of light-front wave functions. The results are compared with the experimental data [20, 29].

5. Results and Conclusion

In the present work we have calculated the elastic eletromagnetic form factor for the pseudoscalar mesons, pion and kaon with the LFCM and the results are compared with the experimental data. The parameters utilized in the present work are given in the table in the text. The table show some results for the pion not present in the figure 1; but also, the powers $n = 3,4$, describe the pion elastic form factor very well. In the pion case, the new data existe, but is not the case of the kaon. Both mesons are presented in the figure 1. The LFCM, describe very well the electromagnetic pion elastic form factor for the power $n = 2$ up to 10 $(GeV/c)^2$ (see the figure).The kaon electromagnetic form factor presente here, is calculated with the powers $n = 2,3$ and 4 and the compared with the experimental data [4]. In the case of the pion transition form factor, the models of the wave functions utilized here is in agreement with the data from the reference [29] at low momentum transfer, however, in higher momentum, the models wave functions employed here is not agree with the Babar data [20]. The results exposed for the pion transition form factor need more explorations front the Babar new data [20].

References

[1] C. Itzykson and J. Zuber, Quantum Field Theory, McGraw-Hill Book Co., (1980).

[2] B. El-Bennich, J .P. B. C. de Melo, B. Loiseau, J.-P. Dedonder, T. Frederico. Braz.J.Phys.**38**(2008) 465.

[3] J. P. B. C. de Melo and T. Frederico, Phys.Proc.Suppl.**199** (2010) 276.

[4] S. R. Amendolia et al., Phys. Lett. **178** (1986) 116.
E. B. Dally et al., Phy. Rev. Lett. **45** (1980) 232.

[5] R. Baldini *et al.*, Eur. Phys. J. C **11**, (1999) 709; Nucl. Phys. A **666& 667**, (2000) 3.
The Jefferson Lab F(pi) Collaboration (J. Volmer *et al.*), Phys. Rev. Lett. **86**, (2001) 1713.
The Jefferson Lab F(pi) Collaboration (T. Horn *et al.*), Phys. Rev. Lett. **97**, (2006) 192001.
The Jefferson Lab F(pi) Collaboration (V. Tadevosyan *et al.*), Phys. Rev. **C 75**, (2007) 055205.

[6] S. R. Amendolia et al., Phys. Lett. **B277**, 168 (1986).

[7] J. P. B. C. de Melo, E. Gambim and T. Frederico, PoS LC2008, (2008) 003.

[8] P. Dirac, Rev. Mod. Phys. **21**, (1949) 392.

[9] Stanley J. Brodsky, Hans-Christian Pauli and Stephen S. Pinsky, Phys.Rept. **301** (1998)299. .

[10] G. C. Wick, Phys. Rev. **96**, (1954) 1124.

[11] J. P. B. C. de Melo and T. Frederico, Phy. Rev. **C55**, (1997) 2043.

[12] H. W. L. Naus, J. P. B. C. de Melo and T. Frederico, Few-Body Syst. **24**, (1998) 99.

[13] J. P. B. C. de Melo, and T. Frederico, Phys. Rev. **59**, (1999) 574.

[14] J. P. B. C. de Melo, T. Frederico, E. Pace and G. Salmé, Nucl. Phy. **A707**, (2002) 399.

[15] B. L. G. Baker, H. M. Choi, C. -R. Ji, Phys. Rev. **D63**, (2001) 074014.

[16] Fabiano P. Pereira, J. P. B. C. de Melo, T. Frederico and Lauro Tomio, Nucl.Phys. **A790**, (2007) 610.

[17] O. A. T. Dias, V. S. Filho and J. P. B. C. de Melo, Nucl.Phys.Proc.Suppl. **199** (2010) 281.

[18] Udit Raha and Andreas Aste, Phys. Rev. **D79**, 034015 (2009).

[19] Review of Particle Physics, PDG, Journal of Physics G (2010).

[20] B. Auber et al., Phys. Rev. **D80** , (2009) 05002.

[21] A. E. Dorokhov, hep-ph/1003.469v2.

[22] E. Ruiz Arriola and W. Broniowski, Phys. Rev. **D 81** (2010) 094021.

[23] Xing-Gang Wu and Tao Huang, hep-ph/1005.3359v2.

[24] H. L. L. Roberts, C. D. Roberts, A. Bashir, L. X. G. Guerrero and P. C. Tandy, nu-th/1009.0067v1.

[25] S. Noguera and V. Vento, Eur. Phys. J. **A** (2010) 11029.

[26] G. P. Lepage and S. J. Brodsky, Phy. Rev. **D22**, (1980) 2157.

[27] T. Frederico, G.A. Miller, Phys. Rev. **D45** (1992) 4207.

[28] J. P. B. C. de Melo, T. Frederico, E. Pace and G. Salme, Nucl.Phys. **A707** (2002) 399.

[29] H.-J. Behrend et al., Z. Phys. **C49** (1991) 401