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A QCD sum rule study of two scalar mesons $f_0(980)$ and $a_0(980)$

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By using the QCD sum rules, we discuss masses of the two scalar mesons $f_0(980)$ and $a_0(980)$ which are observed to have the same mass in the scalar nonets of mass less than 1GeV. We assume the two scalar mesons to be the tetraquark states consisting of the scalar diquark–antidiquark state and the pseudoscalar diquark–antidiquark state. With the QCD sum rules up to the operators of energy dimension 10 with instanton contributions, we discuss the structures of the two scalar mesons and their masses.

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1. Introduction

There is a lot of controversy in understanding the scalar nonets of mass less than 1 GeV [1]. Although they are expected to have $q\bar{q}$ contents because of the similar flavor structures to the pseudoscalar meson nonets, but it is difficult to understand their mass spectrum and decay modes within such quark contents. These have raised other interpretations of them as various tetraquarks. In [3], in order to fix structure of the lightest scalar meson $\sigma(600)$ (or $f_0(600)$) as a tetraquark state, we used the instanton induced interaction between the quarks for $SU(2)_F$ [2] because $\sigma(600)$ has the same quantum number as the QCD vacuum. Applying the QCD sum rules to this approach, it was shown that $\sigma(600)$ could be understood as the tetrquark state consisting of the scalar diqaurk–antidiquark and pseudoscalar diquark–antidiquark with the equal weight and the opposite phase.

In here we extend such approach to the two scalar mesons $f_0(980)$ and $a_0(980)$ which belong to the octet in the scalar meson nonets with the same mass. We assume the two scalar mesons to be mixtures of the scalar diquark–antidiquark state and the pseudoscalar diquark–antidiquark state. We discuss their structures and masses with the QCD sum rules.

2. QCD sum rules for the two scalar mesons

The interpolating currents for the two scalar mesons $f_0(980)$ and $a_0(980)$ corresponding to the tetraquark states consisting of the scalar diquark–antidiquark and the pseudoscalar diquark–antidiquark can be written as

$$J_{f_0,a_0} = \alpha J^S_{f_0,a_0} + \beta J^{PS}_{f_0,a_0}\ . \tag{2.1}$$

Here $J^S_{f_0,a_0}$ ($J^{PS}_{f_0,a_0}$) means the interpolating current for the tetraquark state of the scalar diquark–antidiquark (the pseudoscalar diquark–antidiquark). Explicitly they are given by

$$\begin{aligned} J^S_{f_0,a_0} &= \frac{1}{\sqrt{2}}\varepsilon_{abc}\varepsilon_{ade}[(u^T_b C\gamma_5 s_c)(\bar{u}_d C\gamma_5 \bar{s}^T_e) \pm (d^T_b C\gamma_5 s_c)(\bar{d}_d C\gamma_5 \bar{s}^T_e)]\ , \\ J^{PS}_{f_0,a_0} &= \frac{1}{\sqrt{2}}\varepsilon_{abc}\varepsilon_{ade}[(u^T_b C s_c)(\bar{u}_d C \bar{s}^T_e) \pm (d^T_b C s_c)(\bar{d}_d C\gamma_5 \bar{s}^T_e)]\ , \end{aligned} \tag{2.2}$$

where the indices $a, b, c, \cdots$ denote color and the upper (lower) sign corresponds to $f_0(980)$ ($a_0(980)$).

In order to construct the QCD sum rules for the two scalar mesons, we need to calculate the correlators defined by

$$\Pi_{f_0,a_0} = i\int d^4x\, e^{iq\cdot x}\langle 0|J_{f_0,a_0}(x)J^\dagger_{f_0,a_0}(0)|0\rangle\ . \tag{2.3}$$

In Fig. 1, the diagrammatic representation of the correlator is shown. The diagrams in (a) are generated when the first term in $J^{S,PS}_{f_0,a_0}(x)$ is contracted with the first term in $J^{S,PS\dagger}_{f_0,a_0}(0)$. (When the second term in $J^{S,PS}_{f_0,a_0}(x)$ is contracted with the second term in $J^{S,PS\dagger}_{f_0,a_0}(0)$, the diagrams with replacing u by d in (a) of Fig. 1 are generated.) The last diagram in (a) can only contribute to the OPE (Operator Product Expansion) since it is fully connected. And the contributions from this diagram to the OPE are independent on the two mesons. As we discussed in [3], because of the chirality structure of the currents, this diagram yields the terms proportional to $\alpha^2+\beta^2$ and $\alpha^2-\beta^2$ in the QCD sum rules up the operators of energy dimension 10. The diagrams in (b) correspond to the contraction of the first term in $J^{S,PS}_{f_0,a_0}(x)$ with the second term in $J^{S,PS\dagger}_{f_0,a_0}(0)$ and vice versa. Since they

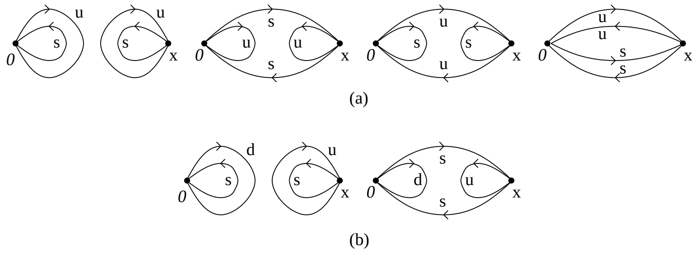

Figure 1: Diagrammatic representation of the correlator. Each quark line corresponds to the full quark propagator.

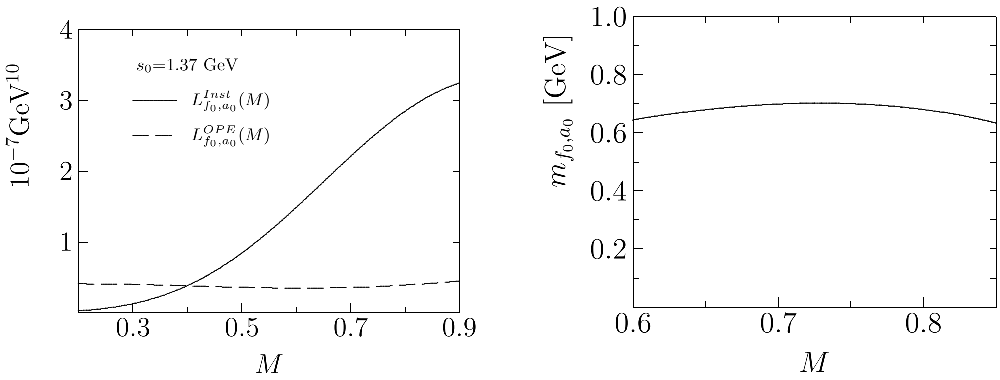

Figure 2: Left : Two contributions in the QCD sum rules Eq.(2.4). Right: Mass of $f_0(980)$ and $a_0(980)$. The threshold of the continuum state $s_0 = 1.37$GeV and $\alpha = \beta = 1$ are used.

are not fully connected, they cannot contribute to the QCD sum rules through the OPE. Therefore we get the same QCD sum rules for the two mesons from the OPE.

The last diagrams in (a) and (b) can contribute to the QCD sum rules through the direct instantons. The last diagram in (a) yields the same contributions from the direct instantons to the QCD sum rules for the two mesons. However, because of different sign between the two terms in the currents in Eq.(2.2), the last diagram in (b) yields the contributions from the direct instantons to the QCD sum rules for the two mesons proportional to $(\alpha-\beta)^2$ with the different sign. This inspection tells us that we should choose $\alpha = \beta$ in order to get the same mass for the two mesons.

Within the single narrow width resonance approximation and up to the operators of energy dimension 10, the QCD sum rule has the form [4]

$$L^{OPE}_{f_0,a_0}(M) + L^{Inst}_{f_0,a_0}(M) = 2 f^2_{f_0,a_0} m^8_{f_0,a_0} e^{-m^2_{f_0,a_0}/M^2} , \tag{2.4}$$

where f_{f_0,a_0}, m_{f_0,a_0} and M are the decay constant, mass of the two mesons and the Borel mass, respectively. The upper indices, *OPE* and *Inst* mean the contributions from the OPE and the direct instantons. We can see in the left panel of Fig. 2 that the contribution from the direct instantons is dominant in the QCD sum rules for $\alpha = \beta = 1$ and the threshold of the continuum state $s_0 = 1.37$GeV [5]. Moreover, the mass of the two mesons in the right panel of Fig. 2 is stable around 700MeV although it looks smaller than the experimental value.

3. Discussion

Based on the instanton induced interaction between light quarks for $SU(2)_F$, we assume the two scalar mesons $f_0(980)$ and $a_0(980)$ to be mixtures of the scalar diquark–antidiaquak state and pseudoscalar diquark–antidiquark state. From the QCD sum rules up to operators of energy dimension 10 with instanton contributions, it is shown that they could be understood as the tetraquark states consisting of the two types of diquarks with the equal weight and phase. However, this analysis could be incomplete since the mass of the two mesons from the QCD sum rules is smaller than the experimental value. In order to improve our analysis, therefore, more precise analysis for the instanton induced interaction in quarks for $SU(3)_F$ is needed since the two mesons have s quark. Furthermore, including the effects from large decay widths of the two mesons and from intermediate two meson states in the QCD sum rules is also needed.

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