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The asymmetry of the dimension two condensate

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I present recent analytical work concerning the electric-magnetic asymmetry in the dimension two condensate $\langle A_\mu^2 \rangle$ in pure Yang–Mills theory. We reproduce qualitatively the lattice results previously found by Chernodub and Ilgenfritz.

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1. Introduction

Recent years have witnessed a great deal of interest in the possible existence of mass dimension two condensates in gauge theories, see for example [1, 2, 3, 4, 6, 7, 5, 8, 9, 10, 11, 13, 14, 12] and references therein for approaches based on phenomenology, operator product expansion, lattice simulations, an effective potential and the string perspective. There is special interest in the operator

$$A^2_{\min} = \min_{U\in SU(N)} \mathscr{V}^{-1} \int d^4x (A^U_\mu)^2 \,, \tag{1.1}$$

which is gauge invariant due to the minimization along the gauge orbit. It should be mentioned that obtaining the global minimum is delicate due to the problem of gauge (Gribov) ambiguities [15]. As is well known, local gauge invariant dimension two operators do not exist in Yang-Mills gauge theories. The nonlocality of (1.1) is best seen when it is expressed as [16][1]

$$A^2_{\min} = \int d^4x \left[A^a_\mu \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) A^a_\nu - g f^{abc} \left(\frac{\partial_\nu}{\partial^2} \partial A^a \right) \left(\frac{1}{\partial^2} \partial A^b \right) A^c_\nu \right] + \mathscr{O}(A^4) \,. \tag{1.2}$$

The relevance of the condensate $\langle A^2_\mu \rangle_{\min}$ was discussed in [1, 2], where it was shown that it can serve as a measure for the monopole condensation in the case of compact QED.

All efforts so far have concentrated on the Landau gauge $\partial_\mu A_\mu = 0$. The preference for this particular gauge fixing is obvious since the nonlocal expression (1.2) reduces to the local operator $A^2_{\min} = A^2_\mu$. In the case of a local operator, the Operator Product Expansion (OPE) becomes applicable, and consequently a measurement of the soft (infrared) part $\langle A^2_\mu \rangle_{\mathrm{OPE}}$ becomes possible. Such an approach was followed in e.g. [8] by analyzing the appearance of $1/q^2$ power corrections in (gauge variant) quantities like the gluon propagator or strong coupling constant, defined in a particular way, from lattice simulations. Let us mention that already three decades ago attention was paid to the condensate $\langle A^2_\mu \rangle$ in the OPE context [17].

Recently, Chernodub and Ilgenfritz [12] have considered the asymmetry in the dimension two condensate. They performed lattice simulations, computing the expectation value of the electric-magnetic asymmetry in Landau gauge, which they defined as

$$\Delta_{A^2} = \langle g^2 A_0^2 \rangle - \frac{1}{d-1} \sum_{i=1}^{d-1} \langle g^2 A_i^2 \rangle \,. \tag{1.3}$$

At zero temperature, this quantity must, of course, be zero due to Lorentz invariance[2]. Necessarily it cannot diverge as divergences at finite T are the same as for $T = 0$, hence this asymmetry is, in principle, finite, and it can be computed without renormalization, for all temperatures. Their results are depicted in Figure 1. At high temperatures, general thermodynamic arguments predict a polynomial behavior $\propto T^2$, and this is also what the authors of [12] found[3]. For the low-temperature

[1] We will always work in Euclidean spacetime.

[2] We shall deliberately use the term Lorentz invariance, though we shall be working in Euclidean space throughout this paper.

[3] A perturbative computation gives a positive proportionality constant, in contrary to what is erronously [18] found in [12]. The lattice computations for $T < 6\, T_c$ find a negative proportionality constant, so one would expect the real high-temperature behavior to start yet later.

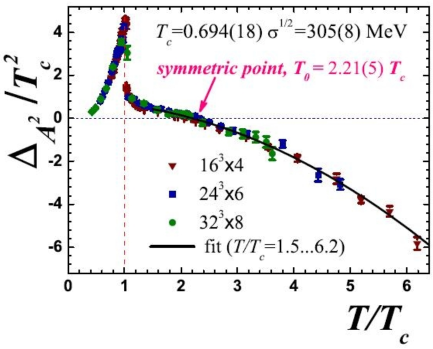

Figure 1: Lattice results for the electric-magnetic assymetry Δ_{A^2} as function of the temperature for SU(2) pure gauge theory, as found in [12].

behavior, however, one would expect an exponential fall-off with the lowest glueball mass in the exponent, $\Delta \sim e^{-m_{\mathrm{gl}}T}$. Instead, they found an exponential with a mass m significantly smaller than m_{gl}.

2. $\langle A_\mu^2 \rangle$ and Δ_{A^2} in the LCO formalism

In order to get more insight in the behavior of the asymmetry, we have investigated it using the formalism presented in [3]. A meaningful effective potential for the condensation of the *Local Composite Operator* (LCO) A_μ^2 was constructed by means of the LCO method. This is a nontrivial task due to the compositeness of the considered operator. We consider pure Euclidean SU(N) Yang–Mills theories with action

$$S_{\mathrm{YM+gf}} = \int d^4x \left(\frac{1}{4}(F_{\mu\nu}^a)^2 + S_{\mathrm{gf}} + b^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu \mathscr{D}_\mu^{ab} c^b \right) . \tag{2.1}$$

We couple the operator A_μ^2 to the Yang–Mills action by means of a source J:

$$S_J = S_{\mathrm{YM}} + \int d^4x \left(\frac{1}{2} J (A_\mu^a)^2 - \frac{1}{2}\zeta J^2 \right) . \tag{2.2}$$

The last term, quadratic in the source J, is necessary to kill the divergences in vacuum correlators like $\langle A^2(x) A^2(y) \rangle$ for $x \to y$, or equivalently in the generating functional $W[J]$, defined as

$$e^{-W[J]} = \int [\mathrm{fields}] e^{-S_J} . \tag{2.3}$$

The presence of the LCO parameter ζ ensures a homogenous renormalization group equation for $W[J]$. Its arbitrariness can be overcome by making it a function $\zeta(g^2)$ of the strong coupling constant g^2, allowing one to fix $\zeta(g^2)$ order by order in perturbation theory in accordance with the renormalization group equation.

In order to access the electric-magnetic asymmetry, a second source $K_{\mu\nu}$ is coupled to the traceless part of $A_\mu^a A_\nu^a$. This second operator will not mix with A_μ^2 itself, which allows control over the renormalization group of these two operators. Again a term quadratic in the new source must be added, introducing a second parameter $\omega(g^2)$ which can, again, be fixed order by order in accordance with the renormalization group equation. We have proven the all-order perturbative renormalizability of this extention of the formalism using the algebraic method based on the Ward identies [19].

In order to recover an energy interpretation, the terms $\propto J^2$ and k^2 can be removed by employing a Hubbard–Stratonovich transformation, amounting to inserting the following unities into the path integral:

$$1 = \int [d\sigma] e^{-\frac{1}{2\zeta}\int d^dx\left(\frac{\sigma}{g}+\frac{1}{2}A_\mu^2-\zeta J\right)^2} = \int [d\varphi_{\mu\nu}] e^{-\frac{1}{2\omega}\int d^dx\left(\frac{1}{g}\varphi+\frac{1}{2}A_\mu A_\nu-\omega\, k_{\mu\nu}\right)^2} , \tag{2.4}$$

with $\varphi_{\mu\nu}$ a traceless field, leading to the action

$$S = S_{\mathrm{YM}} + \int d^dx \left[\frac{1}{2\zeta}\frac{\sigma^2}{g^2} + \frac{1}{2\zeta g}\sigma A_\mu^2 + \frac{1}{8\zeta}(A_\mu^2)^2 + \frac{1}{2\omega}\frac{\varphi_{\mu\nu}^2}{g^2} + \frac{1}{2\omega g}\varphi_{\mu\nu}A_\mu A_\nu + \frac{1}{8\omega}(A_\mu^a A_\nu^a)^2\right] . \tag{2.5}$$

Starting from this, it is possible to compute the effective potential $V(\sigma,\varphi_{\mu\nu})$, given the correspondences

$$\langle\sigma\rangle = -\frac{g}{2}\langle A_\mu^2\rangle , \quad \langle\varphi_{\mu\nu}\rangle = -\frac{g}{2}\left\langle A_\mu A_\nu - \frac{\delta_{\mu\nu}}{d}A_\lambda^2\right\rangle . \tag{2.6}$$

Now we determine the values of ζ and ω from the renormalization group equations for the sources J and $K_{\mu\nu}$. For this, some anomalous dimensions and renormalization factors have to be computed up to one loop order higher than the intended loop order we are interested in. We have done this using the MINCER algorithm. The final result is up to one-loop order:

$$\zeta = \frac{N^2-1}{16\pi^2}\left[\frac{9}{13}\frac{16\pi^2}{g^2N} + \frac{161}{52}\right] , \qquad \omega = \frac{N^2-1}{16\pi^2}\left[\frac{1}{4}\frac{16\pi^2}{g^2N} + \frac{73}{1044}\right] . \tag{2.7}$$

3. Computation and minimalization of the action

The effective potential $V(\sigma,\varphi_{\mu\nu})$ can now be computed using standard techniques. We have taken the background fields σ and $\varphi_{\mu\nu}$ to have space-time independent vacuum expectation values and $\varphi_{\mu\nu}$ to be the traceless diagonal matrix $\mathrm{diag}(A, -\frac{1}{d-1}A, \ldots, -\frac{1}{d-1}A)$ [20].

3.1 Low temperatures

Computing the effective action up to one-loop order at zero temperature, yields only the minimum found in [3], which is what we expect. For finite but still not too high temperatures, the potential can be minimized numerically. The result is depicted in Figure 2. We see that the asymmetry rises at low temperatures, which agrees qualitatively with the findings of [12]. The low-temperature expansion of Δ_{A^2} reads

$$\Delta_{A^2} = (N^2-1)\frac{g^2\pi^2}{30}\left(1 - \frac{85}{1044}\frac{g^2N}{(4\pi)^2}\right)\frac{T^4}{m^2} , \tag{3.1}$$

and there is no correction to $\langle A_\mu^2\rangle$ at this order. Remark that we find a polynomial behavior $\propto T^4/m^2$ instead of an exponential. This does not agree with the lattice results, but in [12] the lowest

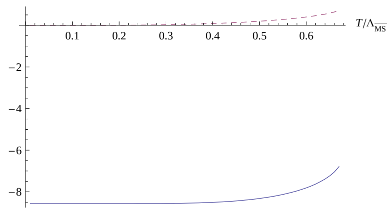

Figure 2: The $\langle g^2 A_\mu^2 \rangle$ condensate (full line) and the assymetry Δ_{A^2} (dashed line) as functions of the temperature, in units $\Lambda_{\overline{\mathrm{MS}}}$.

Figure 3: Left: The diagrams giving the Debye mass in HTL resummation. The ghost loop is not necessary [22]. **Right:** More diagrams that need to be resummed, coming from the σ and $\varphi_{\mu\nu}$ parts of the LCO Lagrangian. The dotted line is the σ or $\phi_{\mu\nu}$ propagator, and the labeled vertex depicts the additional vertices introduced by the formalism.

temperatures reached were $T = 0.4\ T_c$, where our expansion is not valid anymore. This also does not agree with the thermodynamic argument that, in a theory with a massgap, one would expect exponential scaling. However, the asymmetry does not get a gauge invariant meaning by going to the Landau gauge.

3.2 High temperatures

At temperatures higher than $0.67\ \Lambda_{\overline{\mathrm{MS}}}$, the minimum disappears. This signals a phase transition to the perturbative vacuum. In order to access this regime, it is possible to expand the effective potential for high temperatures, which yields

$$\langle g^2 A_\mu^2 \rangle = g^2(N^2-1)\frac{T^2}{4}\ , \qquad \Delta_{A^2} = g^2(N^2-1)\frac{T^2}{12}\ . \tag{3.2}$$

This coincides with the perturbative result.

In order to compute higher-order corrections to this, it is necessary to perform a Hard Thermal Loop (HTL) resummation [21], as nonresummed perturbation theory leads to a tachyonic mass in our case[4]. In ordinary pure Yang–Mills theory at this order, HTL amounts to giving the timelike gluon a Debye mass $m_D^2 = \frac{N}{3} g^2 T^2$, which effectively resums the hard (high momentum) contributions of the diagrams left in Figure 3. In our formalism, however, there are four additional vertices giving rise to four extra diagrams that need to be resummed, shown at the right in Figure 3. Computing these additional diagrams, it turns out that they exactly cancel the lowest-order contribution from the condensate. Solving in the large-T limit yields

$$\langle g^2 A_\mu^2 \rangle = g^2(N^2-1)\left(\frac{T^2}{4} - \frac{m_D T}{4\pi} + \cdots\right)\ , \qquad \Delta_{A^2} = g^2(N^2-1)\left(\frac{T^2}{12} - \frac{m_D T}{36\pi} + \cdots\right)\ , \tag{3.3}$$

[4] The condensate and the mass generated by it differ by a negative multiplicative constant.

which is exactly what one would expect from perturbation theory.

4. Conclusions

The temperature dependence of the nonperturbative dimension two condensate and its electric-magnetic asymmetry have been studied analytically. At low temperatures, we find qualitative agreement with the lattice results. At high temperatures, the perturbative vacuum is recovered.

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References

[1] F. V. Gubarev, L. Stodolsky and V. I. Zakharov, Phys. Rev. Lett. **86** (2001) 2220.

[2] F. V. Gubarev and V. I. Zakharov, Phys. Lett. B **501** (2001) 28.

[3] H. Verschelde, K. Knecht, K. Van Acoleyen and M. Vanderkelen, Phys. Lett. B **516** (2001) 307.

[4] D. Dudal, H. Verschelde and S. P. Sorella, Phys. Lett. B **555** (2003) 126.

[5] D. Dudal, R. F. Sobreiro, S. P. Sorella and H. Verschelde, Phys. Rev. D **72** (2005) 014016.

[6] D. Dudal, H. Verschelde, R. E. Browne and J. A. Gracey, Phys. Lett. B **562** (2003) 87.

[7] D. Vercauteren and H. Verschelde, Phys. Lett. B **660** (2008) 432.

[8] Ph. Boucaud, A. Le Yaouanc, J. P. Leroy, J. Micheli, O. Pene and J. Rodriguez-Quintero, Phys. Rev. D **63** (2001) 114003.

[9] S. Furui and H. Nakajima, PoS **LAT2005** (2006) 291.

[10] F. V. Gubarev and S. M. Morozov, Phys. Rev. D **71** (2005) 114514.

[11] R. E. Browne and J. A. Gracey, JHEP **0311** (2003) 029.

[12] M. N. Chernodub and E. M. Ilgenfritz, Phys. Rev. D **78** (2008) 034036.

[13] O. Andreev, Phys. Rev. D **73** (2006) 107901.

[14] E. Ruiz Arriola and W. Broniowski, Phys. Rev. D **73** (2006) 097502.

[15] V. N. Gribov, Nucl. Phys. B **139** (1978) 1.

[16] M. Lavelle and D. McMullan, Phys. Rept. **279** (1997) 1.

[17] M. J. Lavelle and M. Schaden, Phys. Lett. B **208** (1988) 297.

[18] M. N. Chernodub, personal communication.

[19] D. Dudal, J. A. Gracey, N. Vandersickel, D. Vercauteren and H. Verschelde, Phys. Rev. D **80** (2009) 065017.

[20] D. Vercauteren and H. Verschelde, arXiv:1007.2789 [hep-th].

[21] J. O. Andersen and M. Strickland, Annals Phys. **317** (2005) 281.

[22] E. Braaten and R. D. Pisarski, Nucl. Phys. B **337** (1990) 569.