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Hign-Energy Amplitudes in the Next-to-Leading Order

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High-energy scattering in the saturation region is described by the evolution of color dipoles. In the leading order this evolution is governed by the non-linear BK equation. To see if this equation is relevant for existing or future accelerators (like EIC or LHeC) one needs to know how big are the next-to-leading order (NLO) corrections. I review the calculation of the NLO corrections to high-energy amplitudes in QCD.

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In the framework of Wilson-line approach, the high-energy behavior of QCD amplitudes is determined by the rapidity evolution of Wilson lines. The typical example is the deep inelastic scattering (DIS) at small values of Bjorken x where the x_B dependence of structure functions is governed by the rapidity evolution of color dipoles. (A color dipole is a trace of a two-Wilson-line operator). At relatively high energies and for sufficiently small dipoles we can use the leading logarithmic approximation (LLA) where $\alpha_s \ll 1,\ \alpha_s \ln x_B \sim 1$ and get the non-linear BK evolution equation [1, 2] for the color dipoles $\hat{\mathcal{U}}^\eta(z_1,z_2)$:

$$\frac{d}{d\eta}\,\hat{\mathcal{U}}^\eta(z_1,z_2)$$
$$= \frac{\alpha_s N_c}{2\pi^2}\int d^2z_3\ \frac{z_{12}^2}{z_{13}^2 z_{23}^2}[\hat{\mathcal{U}}^\eta(z_1,z_3)+\hat{\mathcal{U}}^\eta(z_3,z_2)) - \hat{\mathcal{U}}^\eta(z_1,z_3) - \hat{\mathcal{U}}^\eta(z_1,z_3)\hat{\mathcal{U}}^\eta(z_3,z_2)] \quad (1)$$

where $\eta = \ln\frac{1}{x_B}$ and z_i are the transverse coordinates of the dipole. The first three terms correspond to the linear BFKL evolution [3] and describe the parton emission while the last term is responsible for the parton annihilation. For sufficiently low x_B the parton emission balances the parton annihilation so the partons reach the state of saturation with the characteristic transverse momentum Q_s growing with energy $1/x_B$ (for a review, see [4])

As usual, to get the region of application of the leading-order evolution equation one needs to find the next-to-leading order (NLO) corrections. In the case of the high-energy evolution equation (1) there is another reason why NLO corrections are important. Unlike the DGLAP evolution, the argument of the coupling constant in Eq. (1) is left undetermined in the LLA, and usually it is set by hand to be Q_s. The precise form of the argument of α_s should come from the solution of the BK equation with the running coupling constant, and the starting point of the analysis of the argument of α_s in Eq. (1) is the calculation of the NLO evolution.

We calculate the NLO corrections using the high-energy operator expansion of T-product of two vector currents in Wilson lines (see e.g the reviews [5]). Let us recall the general logic of an operator expansion. In order to find a certain asymptotical behavior of an amplitude by OPE one should

- Identify the relevant operators and factorize an amplitude into a product of coefficient functions and matrix elements of these operators
- Find the evolution equations of the operators with respect to the factorization scale
- Solve these evolution equations
- Convolute the solution with the initial conditions for the evolution and get the amplitude.

Since we are interested in the small-x asymptotics of DIS it is natural to factorize in rapidity: we introduce the rapidity divide η which separates the "fast" gluons from the "slow" ones.

As a first step, we integrate over gluons with rapidities $Y > \eta$ and leave the integration over $Y < \eta$ for later time. It is convenient to use the background field formalism: we integrate over gluons with $\alpha > \sigma = e^\eta$ and leave gluons with $\alpha < \sigma$ as a background field, to be integrated over later. Since the rapidities of the background gluons are very different from the rapidities of gluons in our Feynman diagrams, the background field can be taken in the form of a shock wave due to the

Lorentz contraction. To derive the expression of a quark (or gluon) propagator in this shock-wave background we represent the propagator as a path integral over various trajectories, each of them weighed with the gauge factor $\mathrm{Pexp}(ig\int dx_\mu A^\mu)$ ordered along the propagation path. Now, since the shock wave is very thin, quarks (or gluons) do not have time to deviate in transverse direction so their trajectory inside the shock wave can be approximated by a segment of the straight line. Moreover, since there is no external field outside the shock wave, the integral over the segment of straight line can be formally extended to $\pm\infty$ limits yielding the Wilson-line gauge factor

$$U^\eta_x \;=\; \mathrm{Pexp}\Big[ig\int_{-\infty}^{\infty} du\; p_1^\mu A^\sigma_\mu(up_1+x_\perp)\Big], \quad A^\eta_\mu(x) \;=\; \int d^4k\;\theta(e^\eta-|\alpha_k|)e^{ik\cdot x}A_\mu(k) \tag{2}$$

where the Sudakov variable α_k is defined as usual, $k=\alpha_k p_1+\beta_k p_2+k_\perp$. (We define the light-like vectors p_1 and p_2 such that $q=p_1-x_B p_2$ and $p_N=p_2+\frac{m_N^2}{s}p_1$ where p_N is the nucleon momentum). The structure of the propagator in a shock-wave background looks as follows:
[Free propagation from initial point x to the point of intersection with the shock wave z]
$\times$ [Interaction with the shock wave described by the Wilson-line operator U_z]
$\times$ [Free propagation from point of interaction z to the final point y].

The explicit form of quark propagator in a shock-wave background can be taken from Ref. [1]

$$\langle\hat\psi(x)\hat{\bar\psi}(y)\rangle \;\stackrel{x_*>0>y_*}{=}\; \int d^4z\;\delta(z_*)\frac{(\not x-\not z)}{2\pi^2(x-z)^4}\;\not p_2 U_z\frac{(\not z-\not y)}{2\pi^2(x-z)^4} \tag{3}$$

where we label operators by hats as usual. Hereafter use the notations $x_*=p_2^\mu x_\mu=\frac{\sqrt{s}}{2}x^+$, $x_\bullet=p_1^\mu x_\mu=\frac{\sqrt{s}}{2}x^-$ (and our metric is (1,-1,-1,-1)). Note that the Regge limit in the coordinate space corresponds to $x_*\to\infty, y_*\to-\infty$ while $x_\perp, y_\perp$ are fixed, see the discussion in Refs. [6, 7].

As we mentioned above, the result of the integration over the rapidities $Y>\eta$ gives the impact factor - the amplitude of the transition of virtual photon in two-Wilson-lines operator - "color dipole" $\mathscr{U}^\eta(z_1,z_2)=1-\frac{1}{N_c}\mathrm{tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}$. The LO impact factor is a product of two propagators (3)

$$(x-y)^4 T\{\hat{\bar\psi}(x)\gamma^\mu\hat\psi(x)\hat{\bar\psi}(y)\gamma^\nu\hat\psi(y)\} \;=\; \int\frac{d^2z_1d^2z_2}{z_{12}^4}\;I^{\mu\nu}_{\mathrm{LO}}(z_1,z_2)\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_2}\}\;+\;O(\alpha_s),$$
$$I^{\mathrm{LO}}_{\mu\nu}(z_1,z_2) \;=\; \frac{\mathscr{R}^2}{\pi^6(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\frac{\partial^2}{\partial x^\mu\partial y^\nu}\big[(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)-\frac{1}{2}\kappa^2(\zeta_1\cdot\zeta_2)\big]. \tag{4}$$

Here we introduced the conformal vectors [8, 9]

$$\kappa \;=\; \frac{\sqrt{s}}{2x_*}(\frac{p_1}{s}-x^2p_2+x_\perp)-\frac{\sqrt{s}}{2y_*}(\frac{p_1}{s}-y^2p_2+y_\perp), \quad \zeta_i \;=\; \big(\frac{p_1}{s}+z_{i\perp}^2p_2+z_{i\perp}\big), \tag{5}$$

and the notation $\mathscr{R}\;\equiv\;\frac{\kappa^2(\zeta_1\cdot\zeta_2)}{2(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}$. The above equation (4) is explicitly Möbius invariant.

In the NLO the coefficient function in front of the operators (1) is not Möbius invariant. As discussed in Refs. [5, 7, 10], formally the light-like Wilson lines are conformally (Möbius) invariant but the longitudinal cutoff $\alpha<\sigma=e^\eta$ in Eq. (1) violates this property. As was demonstrated in these papers, one can define a composite operator in the form

$$[\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_2}\}]^{\rm conf}_a = \mathrm{tr}\{\hat U^\sigma_{z_1}\hat U^{\dagger\sigma}_{z_2}\}$$
$$+\frac{\alpha_s}{2\pi^2}\int d^2z_3\,\frac{z_{12}^2}{z_{13}^2z_{23}^2}[\mathrm{tr}\{\hat U^\sigma_{z_1}\hat U^{\dagger\sigma}_{z_3}\}\,\mathrm{tr}\{\hat U^\sigma_{z_3}\hat U^{\dagger\sigma}_{z_2}\}-N_c\mathrm{tr}\{\hat U^\sigma_{z_1}\hat U^{\dagger\sigma}_{z_2}\}]\ln\frac{4az_{12}^2}{\sigma^2sz_{13}^2z_{23}^2}\;+\;O(\alpha_s^2) \tag{6}$$

where a is an arbitrary constant. It is convenient to choose the rapidity-dependent constant $a\to ae^{-2\eta}$ so that the $[\mathrm{tr}\{\hat U^\sigma_{z_1}\hat U^{\dagger\sigma}_{z_2}\}]^{\rm conf}_a$ does not depend on $\eta=\ln\sigma$ and all the rapidity dependence is encoded into a-dependence [5, 11]. In terms of composite dipoles (6) the operator expansion has the form:

$$T\{\hat{\bar\psi}(x)\gamma^\mu\hat\psi(x)\hat{\bar\psi}(y)\gamma^\nu\hat\psi(y)\} \;=\; \int\frac{d^2z_1d^2z_2}{z_{12}^4}\,\Big\{I^{\rm LO}_{\mu\nu}(z_1,z_2)\big[1 \tag{7}$$
$$+\frac{\alpha_s}{\pi}\big][\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_2}\}]_a+\int d^2z_3\;I^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3;a)[\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\}\mathrm{tr}\{\hat U_{z_3}\hat U^\dagger_{z_2}\}-N_c\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_2}\}]_a\Big\}$$

We need to choose the "new rapidity cutoff" a in such a way that all the energy dependence is included in the matrix element(s) of Wilson-line operators so the impact factor should not depend on energy. A suitable choice of a is given by $a_0=-\kappa^{-2}+i\epsilon=-\frac{4x_*y_*}{s(x-y)^2}+i\epsilon$ so we obtain

$$(x-y)^4T\{\hat{\bar\psi}(x)\gamma^\mu\hat\psi(x)\hat{\bar\psi}(y)\gamma^\nu\hat\psi(y)\} \tag{8}$$
$$=\int\frac{d^2z_1d^2z_2}{z_{12}^4}\,\Big\{I^{\mu\nu}_{\rm LO}(z_1,z_2)\big[1+\frac{\alpha_s}{\pi}\big][\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_2}\}]_{a_0}\;+\int d^2z_3\Big[\frac{\alpha_s}{4\pi^2}\frac{z_{12}^2}{z_{13}^2z_{23}^2}$$
$$\times\Big(\ln\frac{\kappa^2(\zeta_1\cdot\zeta_3)(\zeta_1\cdot\zeta_3)}{2(\kappa\cdot\zeta_3)^2(\zeta_1\cdot\zeta_2)}-2C\Big)I^{\mu\nu}_{\rm LO}+\;I^{\mu\nu}_2\Big][\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\}\mathrm{tr}\{\hat U_{z_3}\hat U^\dagger_{z_2}\}-N_c\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_2}\}]_{a_0}\Big\}$$

Here the composite dipole $[\mathrm{tr}\{\hat U^\sigma_{z_1}\hat U^{\dagger\sigma}_{z_2}\}]_{a_0}$ is given by Eq. (6) with $a_0=-\frac{4x_*y_*}{s(x-y)^2}+i\epsilon$ and $I^{\mu\nu}_2(z_1,z_2,z_3)$ is given by [12]

$$(I_2)_{\mu\nu}(z_1,z_2,z_3) \tag{9}$$
$$=\frac{\alpha_s}{16\pi^8}\frac{\mathscr{R}^2}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\Big\{\frac{(\kappa\cdot\zeta_2)}{(\kappa\cdot\zeta_3)}\frac{\partial^2}{\partial x^\mu\partial y^\nu}\Big[\frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}{(\zeta_2\cdot\zeta_3)}+\frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_3)(\zeta_1\cdot\zeta_2)}{(\zeta_1\cdot\zeta_3)(\zeta_2\cdot\zeta_3)}$$
$$-\frac{(\kappa\cdot\zeta_1)^2}{(\zeta_1\cdot\zeta_3)}-\frac{\kappa^2(\zeta_1\cdot\zeta_2)}{(\zeta_2\cdot\zeta_3)}\Big]+\frac{(\kappa\cdot\zeta_2)^2}{(\kappa\cdot\zeta_3)^2}\frac{\partial^2}{\partial x^\mu\partial y^\nu}\Big[\frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_3)}{(\zeta_2\cdot\zeta_3)}-\frac{\kappa^2(\zeta_1\cdot\zeta_3)}{2(\zeta_2\cdot\zeta_3)}\Big]+(\zeta_1\leftrightarrow\zeta_2)\Big\}$$

The second step of the OPE program is the evolution equation(s) of color dipole with respect to rapitity ($\equiv$ our parameter a). The calculation performed in Refs. [10, 11] yields

$$2a\frac{d}{da}\big[\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_2}\}\big]_a \tag{10}$$
$$=\frac{\alpha_s}{2\pi^2}\int d^2z_3\,\frac{z_{12}^2}{z_{13}^2z_{23}^2}\big[\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\}\mathrm{tr}\{\hat U_{z_3}\hat U^\dagger_{z_2}\}-N_c\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_2}\}\big]_a\,\Big\{1+\frac{\alpha_s}{4\pi}\Big[b(\ln\frac{z_{12}^2\mu^2}{4}+2C)$$
$$-b\frac{z_{13}^2-z_{23}^2}{z_{12}^2}\ln\frac{z_{13}^2}{z_{23}^2}+(\frac{67}{9}-\frac{\pi^2}{3})N_c-\frac{10}{9}n_f\Big]\Big\}+\frac{\alpha_s^2}{16\pi^4}\int\frac{d^2z_3d^2z_4}{z_{34}^4}\Big[\Big\{\Big(-2+2\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2}\ln\frac{z_{12}^2z_{34}^2}{z_{14}^2z_{23}^2}$$
$$+\Big[\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2}\big(1+\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2-z_{14}^2z_{23}^2}\big)+\frac{2z_{13}^2z_{24}^2-4z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2-z_{14}^2z_{23}^2}\Big]\ln\frac{z_{13}^2z_{24}^2}{z_{14}^2z_{23}^2}\Big)\;+\;(z_3\leftrightarrow z_4)\Big\}$$

$$\times\ \Big[\Big(\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_3}\}\mathrm{tr}\{\hat{U}_{z_3}\hat{U}^\dagger_{z_4}\}\mathrm{tr}\{\hat{U}_{z_4}\hat{U}^\dagger_{z_2}\} - \mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_3}\hat{U}_{z_4}\hat{U}^\dagger_{z_2}\hat{U}_{z_3}\hat{U}^\dagger_{z_4}\}\Big) - (z_4 \to z_3)\Big]_a + \frac{z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2}$$

$$\times\ \Big\{2\ln\frac{z_{12}^2 z_{34}^2}{z_{14}^2 z_{23}^2} + \Big[1 + \frac{z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2 - z_{14}^2 z_{23}^2}\Big]\ln\frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2}\Big\}\Big[\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_3}\}\mathrm{tr}\{\hat{U}_{z_3}\hat{U}^\dagger_{z_4}\}\mathrm{tr}\{\hat{U}_{z_4}\hat{U}^\dagger_{z_2}\} - z_3 \leftrightarrow z_4\Big]_a\Bigg]$$

$$+\ \frac{\alpha_s^2 n_f}{2\pi^4}\int\frac{d^2z_3 d^2z_4}{z_{34}^4}\Big\{2 - \frac{z_{13}^2 z_{24}^2 + z_{23}^2 z_{14}^2 - z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2 - z_{14}^2 z_{23}^2}\ln\frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2}\Big\}[\mathrm{tr}\{t^a\hat{U}_{z_1}t^b\hat{U}^\dagger_{z_2}\}\mathrm{tr}\{t^a\hat{U}_{z_3}t^b(\hat{U}^\dagger_{z_4} - \hat{U}_{z_3})\}]_a$$

This kernel is a sum of the conformal part and the running coupling part proportional to $b = \frac{11}{3}N_c - \frac{2}{3}n_f$. It is worth noting that the linearization of this equation agrees with NLO BFKL kernel [13].

I will not discuss here the steps 3 and 4 of our OPE program. The reason is that in DIS from nucleon or nucleus the evolution of color dipoles is non-linear and the analytic solution is not known at the present time. Even in the simpler case of forward $\gamma^*\gamma^*$ or onium-onium scattering where the dipole evolution is described by linear NLO BFKL equation, it is impossible to solve this equation since we do not know the argument of the coupling constant. Moreover, it is not known how to solve analytically this equation even if we take some simple model for the argument of coupling constant like the size of the parent dipole.

Still, the first step towards the solution would be to figure out the argument of coupling constant in the NLO BFKL equation. To get an argument of coupling constant we can use the renormalon-based approach and trace the quark part of the β-function proportional to n_f. In the leading log approximation $\alpha_s \ln\frac{p^2}{\mu^2} \sim 1,\ \alpha_s \ll 1$ the quark part of the β-function comes from the bubble chain of quark loops in the shock-wave background. Replacing the quark part of the β-function $-\frac{\alpha_s}{6\pi}n_f\ln\frac{p^2}{\mu^2}$ by the total contribution $\frac{\alpha_s}{4\pi}b\ln\frac{p^2}{\mu^2}$ we get [14, 15]

$$\begin{aligned}2a\frac{d}{da}\mathrm{Tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\} \;=\; & \frac{\alpha_s(z_{12}^2)}{2\pi^2}\int d^2z\ [\mathrm{Tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_3}\}\mathrm{Tr}\{\hat{U}_{z_3}\hat{U}^\dagger_{z_2}\} - N_c\mathrm{Tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}] \\ & \times\Big[\frac{z_{12}^2}{z_{13}^2 z_{23}^2} + \frac{1}{z_{13}^2}\Big(\frac{\alpha_s(z_{13}^2)}{\alpha_s(z_{23}^2)} - 1\Big) + \frac{1}{z_{23}^2}\Big(\frac{\alpha_s(z_{23}^2)}{\alpha_s(z_{13}^2)} - 1\Big)\Big] + \ldots\end{aligned} \tag{11}$$

where dots stand for the remaining α_s^2 terms irrelevant for the argument of α_s in the BK equation. When the sizes of the dipoles are very different the kernel of the above equation reduces to

$$\frac{\alpha_s(z_{12}^2)}{2\pi^2}\frac{z_{12}^2}{z_{13}^2 z_{23}^2}\,|z_{12}| \ll |z_{13}|,|z_{23}|, \qquad \frac{\alpha_s(z_{13})^2)}{2\pi^2 z_{13}^2}\,|z_{13} \ll |z_{12}|,|z_{23}|, \qquad \frac{\alpha_s(z_{23})^2)}{2\pi^2 z_{23}^2}\,|z_{23}| \ll |z_{12}|,|z_{13}| \tag{12}$$

so the argument of the coupling constant is the size of smallest dipole. The numerical approach to solution of the the NLO BK equation with this running coupling constant is presented in Ref. [16].

The main conclusion is that the rapidity factorization and high-energy operator expansion in color dipoles works at the NLO level. There are many examples of the factorization which are fine at the leading order but fail at the NLO level. I believe that the high-energy OPE has the same status as usual light-cone expansion in light-ray operators so one can calculate the high-energy amplitudes level by level in perturbation theory. There are many papers devoted to analysis of the high-energy amplitudes in QCD at the NLO level but all of them use traditional calculation of Feynman diagrams in momentum space. In our opinion, the high-energy OPE in color dipoles is technically more simple and gives us an opportunity to use an approximate tree-level conformal invariance in QCD. Moreover, the exact prescription for separating the coefficient functions (impact factors) and matrix elements is somewhat tricky in the traditional approach while it comes naturally in the framework of OPE logic.

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