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TMD parton distributions and splitting functions

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We give a brief introduction to transverse momentum dependent (TMD) parton distribution functions, focusing on recent progress and open issues on operator matrix elements; factorization; lightcone singularities.

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While QCD factorization methods are well established in the case of scattering observables involving a single large mass scale [1], the treatment of processes with multiple mass scales is subtle. Transverse-momentum dependent (TMD) formulations come about in multiple-scale cases if one is to control potentially large contributions to higher orders of perturbation theory and to describe appropriately nonperturbative physics in the initial and final states of the collision.

This contribution gives a brief introduction to the topical area of TMD parton distributions, focusing on recent progress and open issues in the characterization of TMD pdfs in terms of gauge-invariant operator matrix elements. We discuss the physical picture leading to infrared subtraction factors; lightcone divergences in TMD splitting functions; issues on factorization.

We start in Sec. 1 by introducing basic concepts using the simpler case of the Sudakov form factor. Then in Sec. 2 we move to TMD parton distributions. In Sec. 3 we discuss the region of small x.

1. Infrared subtraction factors: the case of the Sudakov form factor

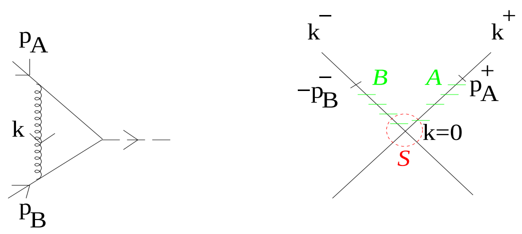
The case of the electroweak form factor of quarks (Fig. 1), although simpler than the treatment of general hard processes, serves to illustrate the role of gauge-invariant subtraction factors associated with infrared subgraphs in factorization formulas for physical cross sections. Suppose we look for a decomposition of the amplitude Γ in Fig. 1 as a sum of terms, one for each of the regions contributing to leading power in the hard scale,

$$\Gamma = \sum_{\text{regions } R} M_\Gamma(R) \; + \; \text{nonleading} \; , \tag{1.1}$$

subject to the requirements that i) the term for the hard region be integrable, and ii) the splitting between the terms be defined gauge-invariantly. This analysis is carried through in [2]. It corresponds to a factorization formula for a physical cross section $\sigma[\Gamma]$ of the schematic form

$$\sigma[\Gamma] = \int [dk] \; S \otimes C_A \otimes C_B \otimes H \; + \; \text{nonleading} \; , \tag{1.2}$$

where H is the hard term, S is the soft term, and C_A and C_B are the collinear terms.



(a) (b)

Figure 1: (a) Form factor graph; (b) collinear-to-A, collinear-to-B and soft regions.

It is shown in [2] that in order for the above requirements to be satisfied each term is to be supplemented with subtraction factors, which come from infrared subgraphs and are given by well-prescribed gauge-invariant operator matrix elements. This is pictured in Fig. 2 for the case of the soft term S. The top picture illustrates that in the soft region Γ is the one-loop contribution to the vacuum expectation value of two eikonal Wilson lines [3] taken along light-like directions $\widehat{p}_A$, $\widehat{p}_B$. This vacuum expectation value however has singularities from the collinear-to-A and collinear-to-B regions. The bottom picture depicts the method [2] to subtract these singularities by counterterms that are designed to be gauge-invariant and such that the collinear-to-A counterterm does not introduce spurious extra contributions in the collinear-to-B region, and viceversa. The term S resulting from these subtractions is thus still a good approximation to Γ in the soft region.

The infrared subtraction factors are identified in [2] for each of the terms in Eq. (1.2). Note that the need for infrared subtractions also emerges in recent analyses of the form factor [4] within the context of soft-collinear effective theory [5] (under the form, however, of counterterms that are not automatically gauge-invariant).

Methods similar to the ones just discussed for the form factor are applicable in the case of TMD formulations for general hard-scattering processes to treat endpoint singularities that affect the operator matrix elements defining TMD parton distributions. We move to this next.

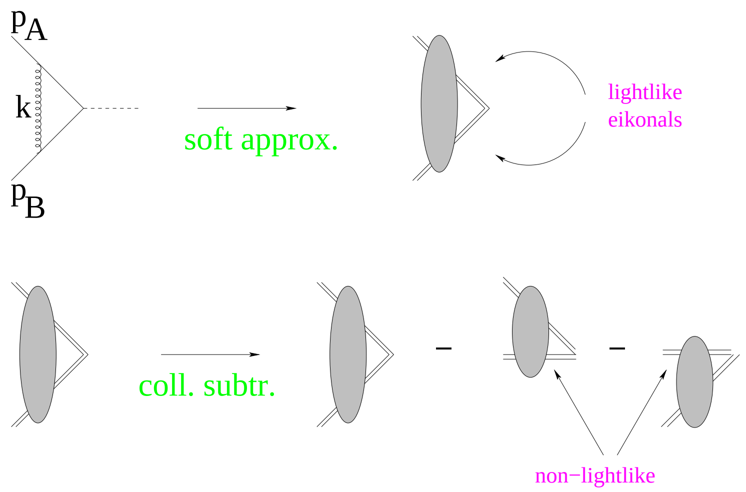


Figure 2: (top) Soft approximation by lightlike eikonals; (bottom) collinear subtractions to the soft term.

2. TMD parton distributions

Suppose we generalize the operator matrix elements that define ordinary parton distribution functions (pdfs) [6] to the case of operators at non-lightcone distances. For instance, for the quark distribution one has (Fig. 3)

$$\widetilde{f}(y) = \langle P|\overline{\psi}(y)V_y^\dagger(n)\gamma^+V_0(n)\psi(0)|P\rangle \quad . \tag{2.1}$$

Here ψ are the quark fields evaluated at distance $y = (0, y^-, y_\perp)$, where $y_\perp$ is in general nonzero, and V are eikonal-line operators. The TMD distribution is given by the double Fourier transform in y^- and $y_\perp$ of $\widetilde{f}$.

While Eq. (2.1) works at tree level [7] (including an extra gauge link at infinity in the case of physical gauge [8]), going beyond tree level requires treating lightcone singularities [9, 10, 11], associated with the $x \to 1$ endpoint, which are present even in dimensional regularization with an

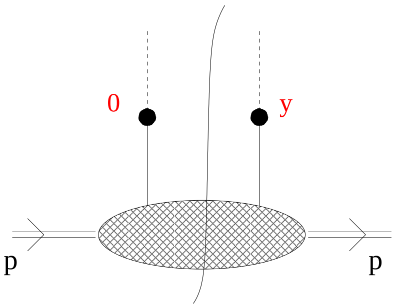


Figure 3: Correlator of two quark fields at distance y.

infrared cut-off. The explicit evaluation at one loop of these singularities in coordinate space is performed in [11], and gives

$$\widetilde{f}_1(y) = \frac{\alpha_s C_F}{\pi}\, p^+ \int_0^1 dv\, \frac{v}{1-v} \left\{ \left[e^{ip\cdot yv} - e^{ip\cdot y} \right] \Gamma(2-\frac{d}{2})\, (\frac{4\pi\mu^2}{\rho^2})^{2-d/2} \right.$$
$$\left. + \, e^{ip\cdot yv}\, \pi^{2-d/2}\, \Gamma(\frac{d}{2}-2)\, (-y^2\mu^2)^{2-d/2} + \cdots \right\} , \tag{2.2}$$

where μ is the dimensional-regularization scale and ρ is the infrared mass regulator. The first term in the right hand side of Eq. (2.2) corresponds to the case of ordinary pdfs. The lightcone singularity $v \to 1$, corresponding to the exclusive boundary $x = 1$, cancels in this term, but it is present, even at $d \neq 4$ and finite ρ, in subsequent terms.

These endpoint singularities come from gluon emission at large rapidity. They imply that, using the matrix element (2.1), the $1/(1-x)$ factors from real emission probabilities do not in general combine with virtual corrections to give $1/(1-x)_+$ distributions, but leave uncancelled divergences at fixed $k_\perp$. The endpoint singularities can be dealt with by the subtractive method [2, 12] discussed in the previous section in the case of the Sudakov form factor. This leads to well-prescribed counterterms [11] for the transverse momentum dependent splitting probabilities, which can be viewed as generalizing the plus-distribution regularization for $k_\perp \neq 0$. The role of infrared subtractions analogous to the ones above is discussed in [13] in the case of initial-state beam functions defined within the soft-collinear effective theory (see also [14]) to describe the incoming jet.

The subtractive treatment of lightcone singularities provides an alternative method, potentially more systematic, to the cut-off method [9, 15], in which the eikonal n in Eq. (2.1) is moved away from the lightcone. The subtractive approach has been used to study the relationship of the endpoint behavior at fixed $k_\perp$ with the cusp anomalous dimension [16]. We observe that the use of subtractive techniques may also be helpful to analyze issues of factorization and non-universality [17, 18] at TMD level. In the hadroproduction of nearly back-to-back hadrons, factorization is broken [17] by soft gluons exchanged between subgraphs in different collinear directions (see also the analyses [19, 20] for the Drell-Yan case). So the issue of factorization depends on developing a systematic treatment, as yet lacking, capable of handling overlapping divergences in infrared regions for complex observables that involve color charges in both initial and final states. The techniques above may prove to be useful for this.

More details and references can be found in the review articles [21].

3. TMD distributions at small x

As noted above, the case of back-to-back di-hadron or di-jet hadroproduction [17, 18] illustrates that, as a result of soft and collinear gluon correlations between initial and final states, a general TMD factorization formula is still lacking. In the case of small x, however, a TMD factorization result holds [22] thanks to the dominance of single gluon helicity at high energy. In this case, a TMD gluon distribution can be defined gauge-invariantly from the high-energy pole in physical cross sections.

The main reason why such a definition for TMD pdfs can be constructed in the high-energy limit is that [22] one can relate directly (up to perturbative corrections) the cross section for a *physical* process, say, photoproduction of a heavy-quark pair, to an unintegrated, transverse momentum dependent gluon distribution, much as, in the conventional parton picture, one does for DIS in terms of ordinary (integrated) parton distributions. On the other hand, the difficulties in defining a TMD distribution in the general case, over the whole phase space, can largely be associated with the fact that it is not obvious how to determine one such relation for general kinematics.

Applications of this observation to di-jet azimuthal correlations for $x \ll 1$ are investigated in [23]. Further phenomenological applications of the small-x TMD splitting functions [22] are in progress [24].

On the other hand, extensions of the above framework are needed if one is to take into account the nonlinear effects that can be expected to arise in the small x region from the high parton density. Work in this direction for dense targets and nuclei may be found in [25].

The techniques [26] have been proposed to incorporate the treatment of multiple-gluon rescattering graphs at small x starting from the operator matrix elements [6, 9, 10] for parton distributions. They may thus be helpful for extensions to the high density region that are aimed to retain accuracy also in the treatment of contributions from high p_T processes.

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