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B-meson physics from non-perturbative lattice heavy quark effective theory

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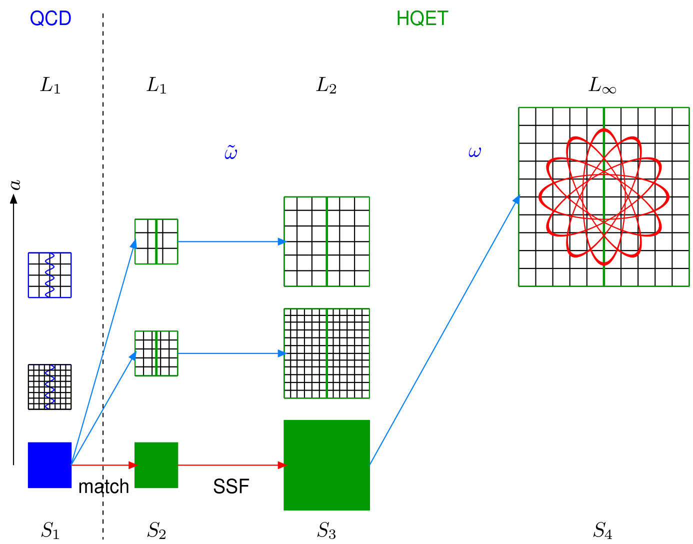
During the last years, the ALPHA collaboration has been developing and implementing a method based on Heavy Quark Effective Theory (HQET) to compute B-mesons observables through lattice simulations. Thanks to a non-perturbative matching to QCD, the theory is renormalizable at any order of the heavy quark mass expansion. In order to extract precisely the relevant matrix elements and masses, we use all-to-all propagators and solve an generalized eigenvalue problem (GEVP). We have shown in the quenched approximation that quantities like the b-quark mass $m_{\rm b}$, the heavy-light decay constant(s) or the B-meson spectrum can be computed precisely beyond the static approximation (including the first corrections in $1/m_{\rm b}$). More recently, we have started to include the sea quark effects, by working with $N_f = 2$ light flavors of dynamical fermions [1] . The computation of the matching parameters is almost finished, but concerning the extraction of the hadronic quantities for which we use some CLS ensembles [2], only one lattice spacing has been analyzed so far. In this proceeding we report on the status of this project and present some preliminary results. The strategy is sketched in the following figure:



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1. Introduction

The search for new physics is currently limited by the size of the theoretical uncertainties, mainly due to the errors affecting the hadronic quantities. This is specially true in the B physics area, where in many cases the errors are largely dominated by quantities like heavy-light decay constants or bag parameters, which are obtained by lattice simulation. If the recent progress of the lattice community in the light quark sector are very impressive, lattice simulations around the b-quark mass are still difficult. Various strategies have been developed (see e.g. [3–5]) based on effective theories (NRQCD, HQET), on relativistic formulation (Fermilab, RHQ) or a combination of both. As already mentioned in the abstract, the strategy followed by the ALPHA collaboration is based on non-perturbative HQET in the static approximation and at the $1/m_{\rm b}$ order (see [6] for a pedagogical introduction). This is a theoretically sound framework, in which the systematic errors are well under control. Based on our experience in the quenched approximation, we believe that the total uncertainty affecting the results can be made small enough to have an impact in the search for new physics, and maybe unveil BSM effects. This method has been tested in the quenched approximation to study the $B_{\rm s}$ meson spectrum [7], to determine the b-quark mass [8] and the decay constant $f_{B_{\rm s}}$ [9]. The computation is essentially done in two steps: as explained in section 2, the HQET parameters are obtained from a non-perturbative matching to QCD in a small volume, while the hadronic matrix element and masses are obtained from a subset of ensembles generated within the CLS effort (see section 3). We present our conclusions together with our preliminary results in section 4.

2. Computation of the relevant HQET parameters

At the $1/m_{\rm b}$ order, we write the HQET Lagrangian in the following way:

$$\mathscr{L}_{\rm HQET}(x) = \mathscr{L}_{\rm stat}(x) - \omega_{\rm kin}\mathscr{O}_{\rm kin}(x) - \omega_{\rm spin}\mathscr{O}_{\rm spin}(x)\,, \tag{2.1}$$

$$\mathscr{L}_{\rm stat}(x) = \overline{\psi}_{\rm h}(x)\,D_0\,\psi_{\rm h}(x)\,, \quad \mathscr{O}_{\rm kin}(x) = \overline{\psi}_{\rm h}(x)\mathbf{D}^2\psi_{\rm h}(x)\,, \quad \mathscr{O}_{\rm spin}(x) = \overline{\psi}_{\rm h}(x)\sigma\!\cdot\!\mathbf{B}\psi_{\rm h}(x)\,. \tag{2.2}$$

We are also interested in the time component of the heavy-light axial current A_0. Considering only the terms which contribute to zero spatial-momentum correlation functions, we write it as

$$A_0^{\rm HQET}(x) \;=\; Z_{\rm A}^{\rm HQET}\,[A_0^{\rm stat}(x) + c_{\rm A}^{(1)}A_0^{(1)}(x)]\,, \tag{2.3}$$

$$A_0^{(1)}(x) \;=\; \overline{\psi}_{\rm l}(x)\frac{1}{2}\gamma_5\gamma_i(\nabla_i^{\rm S} - \overleftarrow{\nabla}_i^{\rm S})\psi_{\rm h}(x)\,, \quad A_0^{\rm stat}(x) = \overline{\psi}_{\rm l}(x)\gamma_0\gamma_5\psi_{\rm h}(x)\,, \tag{2.4}$$

where $\nabla_i^{\rm S}$ denotes the symmetric derivative. The computation of $\omega_{\rm kin}, \omega_{\rm spin}, Z_{\rm A}^{\rm HQET}, c_{\rm A}^{(1)}$ and $m_{\rm bare}$ is done following the strategy presented in [1, 10]. In a volume of linear space extent $L_1 \sim 0.5$ fm, where the b-quark can be simulated with discretization effects under control, we compute a set of observables $\Phi_{i=1,\ldots,5}$ at four different lattice spacings and extrapolate them to the continuum [1]. In this set of simulations called S_1 in the figure, the light quark masses are set to 0, while for

[1] In the continuum and large volume limits, Φ_1 is proportional to the meson mass and Φ_2 to the logarithm of the decay constant, respectively. Φ_3 is used to determine the counter-term of the axial current, and $\Phi_{4,5}$ for the determination of the kinetic and magnetic term, respectively.

the (RGI) heavy quark mass M we have chosen nine different values covering a range between the charm to the bottom mass. In another set of simulations called S_2, where we use the same value of the physical volume, we compute the corresponding quantities in the effective theory (at the $1/m_\mathrm{b}$ order) and match them to their QCD counterpart. This matching can be written in the following way:

$$\Phi_i^{\mathrm{QCD}}(L_1,M,0) = \eta_i(L_1,a) + \sum_j \varphi_{ij}(L_1,a)\,\tilde{\omega}_j(M,a)\ , \tag{2.5}$$

where η and φ are computed by lattice simulations for different values of the lattice spacing a. In other words the matching equations determine the set of parameters $\tilde{\omega} = \varphi^{-1}\,[\Phi^{\mathrm{QCD}} - \eta]$. We then compute η and φ in a larger volume of space extent $L_2 = 2L_1$, and using the parameters $\tilde{\omega}(M,a)$ determined in the previous step we compute the observables $\Phi(L_2,M,0)$ according to a formula similar to eq. (2.5) but where L_1 is replaced by L_2 and $\tilde{\omega}$ by ω (note that in this step called S_3 the continuum limit can be taken because the divergences cancel out exactly). This procedure can then be re-iterated until the volume reached is large enough for finite size effects to be negligible, typically around $(2\,\mathrm{fm})^3$. In practice, it turns out that three different volumes are enough (L_1, L_2 and the large volume one). Thus, the HQET parameters that can be used in large volume simulations (denoted as S_4) are given by $\omega(M,a) = \varphi^{-1}(L_2,a)\,[\Phi(L_2,M,0) - \eta(L_2,a)]$.

3. Extraction of HQET hadronic matrix elements and preliminary results

In the large volume limit, at the first order of the $1/m_\mathrm{b}$ expansion our main observables are given by

$$m_\mathrm{B} = m_\mathrm{bare} + E^{\mathrm{stat}} + \omega_\mathrm{kin}\,E^{\mathrm{kin}} + \omega_\mathrm{spin}\,E^{\mathrm{spin}}\,, \tag{3.1}$$

$$m_\mathrm{B} - m_{\mathrm{B}^*} = \frac{4}{3}\omega_\mathrm{spin}\,E^{\mathrm{spin}}\ , \tag{3.2}$$

$$\begin{aligned}\log(a^{3/2} f_\mathrm{B}\sqrt{m_\mathrm{B}/2}) &= \log(Z_\mathrm{A}^{\mathrm{HQET}}) + \log(a^{3/2}p^{\mathrm{stat}}) + b_\mathrm{A}^{\mathrm{stat}} a m_\mathrm{q} \\ &\quad + \omega_\mathrm{kin} p^{\mathrm{kin}} + \omega_\mathrm{spin} p^{\mathrm{spin}} + c_\mathrm{A}^{(1)} p^{\mathrm{A}^{(1)}},\end{aligned} \tag{3.3}$$

where $b_\mathrm{A}^{\mathrm{stat}}$ is an improvement coefficient. The HQET energies $E^{\mathrm{stat}}, E^{\mathrm{kin}}, E^{\mathrm{spin}}$ and the matrix elements $p^{\mathrm{stat}}, p^{\mathrm{A}^{(1)}}, p^{\mathrm{kin}}, p^{\mathrm{spin}}$ have been measured on a subset of configuration ensembles produced within the CLS effort [2] with $\mathrm{N_f} = 2$ flavors of $O(a)$-improved Wilson-Clover fermions. We have employed a GEVP analysis in order to control the excited states contamination. The chiral extrapolations are performed using pion masses down to 250 MeV (and the coupling $g_{\mathrm{B}^*\mathrm{B}\pi}$ computed in [11]). In principle these quantities should be calculated at various values of the lattice spacing in order to take the continuum limit. This continuum extrapolation will be left for future work, instead here we just compute the observables at one (quite small) value of the lattice spacing $a = 0.07\,\mathrm{fm}$. From our experience in the quenched approximation, we do not expect the discretization effects to be visible within our present accuracy. More details of this computation can be found in [1].

4. Preliminary results and conclusion

For the b-quark mass, including the $1/m_\mathrm{b}$ terms, we find :

$$m_\mathrm{b}^{\overline{\mathrm{MS}}}(m_\mathrm{b}^{\overline{\mathrm{MS}}})\Big|_{\mathrm{N_f}=2}^{\mathrm{HQET}} = 4.276(25)_{r_0}(50)_{\mathrm{stat+renorm}}(?)_a\ \ \mathrm{GeV}\,. \tag{4.1}$$

where the first error comes from the uncertainty on r_0, while the second error includes the statistical error on $aE^{\rm stat}$, the uncertainty on the chiral extrapolation and the error on the quark mass renormalization constant. At the same order our result for the heavy-light decay constant $f_{\rm B}$ reads

$$f_{\rm B}{}^{\rm HQET}_{\rm N_f=2} = 178(16)(?)_a\,{\rm MeV}\,, \tag{4.2}$$

where the first error includes the statistical uncertainty on matrix elements, the systematics coming from chiral extrapolation and the uncertainty on the physical scale of r_0. In both cases the "$(?)_a$" indicates that a continuum limit is not yet performed, but we expect the discretization effects to be small. These first results are promising and once we have controlled cut-off effects by simulation at several lattice spacings, we plan to extent our project to the computation of other hadronic quantities like the $B-\bar{B}$ mixing or $B\to\pi$ semileptonic form factors as well as the spectrum of hadrons with a b-flavor.

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