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Power Suppressed Effects in $\bar{B} \to X_s\gamma$ at $O(\alpha_s)$

Soumitra Nandi[∗†]
Dip. Fisica Teorica, Univ. di Torino & INFN Torino, I-10125 Torino, Italy
E-mail: `nandi@to.infn.it`

In this talk I present $\alpha_s \frac{\Lambda^2_{QCD}}{m_b^2}$ corrections to the Wilson coefficients of the dimension five operators emerging from the Operator Product Expansion of inclusive radiative B decays. We discuss the impact of the resulting $O(\alpha_s\Lambda^2_{QCD}/m_b^2)$ corrections on the extraction of m_b and μ_π^2 from the moments of the photon spectrum.

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1. Motivation

Apart from the direct searches at colliders, low energy observables in flavor physics play an important role for an indirect search of NP; in this respect FCNC processes play an important role. The data from the decays of K, D and B mesons have so far been consistent with the Cabbibo-Kobayashi-Maskawa (CKM) paradigm of Standard Model (SM), however the flavor changing neutral current (FCNC) processes involving $b \to s$ transitions are expected to be sensitive to many sources of new physics (NP) since FCNC decays are rare (i. e. loop-suppressed) in the SM.

The inclusive radiative decays of the B meson is known to be a sensitive probe of the new physics. At the parton level, the decay process $B \to X_s\gamma$ is induced by a FCNC decay of the b, quark contained in the B meson, decays into a strange quark plus other partons, collectively indicated by the symbol X-partons, and a photon. The inclusive decay rate is given by,

$$\Gamma(b \to X_s\gamma)_{E_\gamma > E_0} = \frac{G_F^2\, m_b^5\, \alpha_{em}}{32\,\pi^4} |V_{ts}^* V_{tb}|^2 \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}(E_0, \mu_b), \tag{1.1}$$

where the Wilson Coefficients $C_i(\mu_b)$ are known at NNLO,

$$|C_{1,2}(\mu_b)| \sim 1,\ |C_{3,4,5,6}(\mu_b)| < 0.07,\ |C_7(\mu_b)| \sim -0.3,\ |C_8(\mu_b)| \sim -0.15. \tag{1.2}$$

$G_{ij}(E_0, \mu_b)$ determined by the matrix elements of the operators $O_1, \ldots.., O_8$, consists of perturbative and non-perturbative corrections. As per the perturbative corrections are concerned G_{ij} are fully known at Next-to-leading order (NLO); at the Next-to-NLO (NNLO) level G_{ij} $(i, j = 1, 2, 7, 8)$ have been considered so far, G_{77} and G_{78} are known in a complete manner, for complete list of references see [1]; in addition for complete NNLO calculations of G_{78} see [2].

The inclusive branching ratio in SM is given by [3],

$$\mathscr{B}(\bar{B} \to X_s\gamma)^{NNLO}_{E_\gamma > 1.6\,\mathrm{GeV}} = (3.15 \pm 0.23) \times 10^{-4}, \tag{1.3}$$

whereas the current experimental data tells us [4],

$$\mathscr{B}(\bar{B} \to X_s\gamma)^{exp}_{E_\gamma > 1.6\,\mathrm{GeV}} = (3.55 \pm 0.24 \pm 0.09) \times 10^{-4}. \tag{1.4}$$

Therefore SM prediction is consistent with the experiment, both have 7% error, useful to constrain many extensions of SM. As per the theoretical error is concerned it is dominated by the unknown non-perturbative corrections, and expected to be 5% from $O(\alpha_s \frac{\Lambda_{QCD}}{m_b})$ [1]. Experimental uncertainty is expected to reduce to 5% by the end of B factory era, it is desirable to reduce the theoretical uncertainty as much as possible both perturbative and non-perturbative.

While the total rate of $\bar{B} \to X_s\gamma$ is sensitive to new physics in flavor-changing transitions, Photon energy spectrum is largely insensitive to NP, it is almost completely determined by Standard Model physics. It is useful for precision studies of perturbative and non-perturbative strong interaction effects. The first moment, $\langle E_\gamma \rangle \sim m_b/2$, can be employed to extract information on the b quark mass; second moment is sensitive to average kinetic energy μ_π^2, see for instance [5, 6]. Measurements of m_b and μ_π^2 using $B \to X_s\gamma$ are complementary to the determinations using the inclusive moments of $B \to X_c\ell\bar{\nu}$; contribute in an important way to the global fits for extraction of V_{cb} and V_{ub}. During the last decade effort has been given to improve our understanding of photon spectrum, see for instance [7]; uncertainties of both perturbative and non-perturbative origin remain which need further investigations.

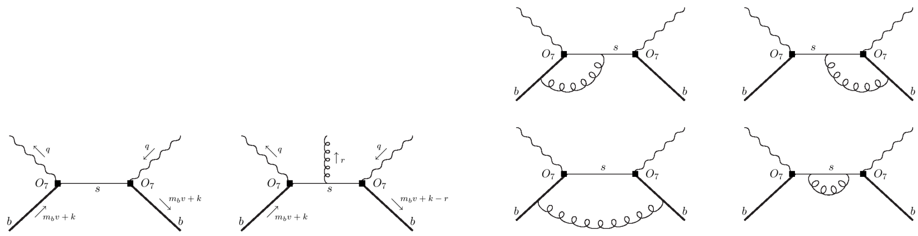

Figure 1: The imaginary parts of these tree-level (two figures starting from left) and one loop (four figures right-panel) diagrams contribute to the Wilson coefficients of the operators with dimension 3, 4 and 5.

2. $\alpha_s \frac{\Lambda_{QCD}^2}{m_b^2}$ corrections to $O_{7\gamma} - O_{7\gamma}$

Non-perturbative corrections induced by O_7 self interference is known through $\frac{1}{m_b^3}$ [8], we present the first calculation of the corrections to $\frac{\Lambda_{QCD}^2}{m_b^2}$ term in $B \to X_s\gamma$ decay rates and moments at order α_s. We computed the relevant Wilson coefficients at $O(\alpha_s)$ by expanding off-shell amputated Green functions around the b quark mass shell, and by matching them onto local operators in Heavy Quark Effective Theory (HQET). Our results allow for an improved analysis of the radiative moments. In particular, the inclusion of the $O(\alpha_s)$ perturbative corrections to the variance of the spectrum, permits the extraction of μ_π^2 at Next-to-Leading-Order (NLO).

Differential decay rate that is induced by 0_7 self-interference

$$d\Gamma_{77}(\bar{B} \to X_s\gamma) = \frac{G_F^2 \alpha_{\rm em} \overline{m}_b^2(\mu)}{16\pi^3 m_B} |V_{tb}V_{ts}^*|^2 |C_7^{\rm eff}(\mu)|^2 \frac{d^3q}{(2\pi)^3 2E_\gamma} W_{\mu\nu\alpha\beta} P^{\mu\nu\alpha\beta}. \tag{2.1}$$

Here, m_B is the mass of the B meson, q the momentum of the photon, $W_{\mu\nu\alpha\beta}$ and $P^{\mu\nu\alpha\beta}$ are the hadronic and photonic tensor respectively , for detail see [9]. The matching equation, helpful to determine the Wilson coefficients, is given by,

$$W_{\mu\nu\alpha\beta} P^{\mu\nu\alpha\beta} = -16\pi m_b \Big(c_{\dim 3} O_{\dim 3} + \frac{1}{m_b} c_{\dim 4} O_{\dim 4} + \frac{1}{m_b^2} c_{\dim 5} O_{\dim 5} + \dots \Big), \tag{2.2}$$

where $O_{\dim n}$ is an operator of dimension n that contains $n-3$ derivatives, and $c_{\dim n}$ is the corresponding Wilson coefficient that can be determined in perturbation theory.

In order to determine the tree-level Wilson coefficients we calculate the amputated Green functions corresponding to diagrams shown in Fig. 1, and with help of eq. 2.2 we are able to find out the relevant operators, given by

$$\begin{aligned} O_b^\mu &= \bar{b}\gamma^\mu b, \qquad\qquad O_2^{\mu\nu} = \bar{b}_v \frac{1}{2}\{iD^\mu, iD^\nu\} b_v \\ O_1^\mu &= \bar{b}_v iD^\mu b_v, \qquad O_3^{\mu\nu} = \bar{b}_v \frac{g_s}{2} G^{a\mu}{}_\alpha \sigma^{\alpha\nu} T^a b_v, \end{aligned} \tag{2.3}$$

and their corresponding Wilson coefficients [9]. The evaluation of the matrix elements of the operators involves the equation of motion of the effective theory, and leads to two additional matrix elements,

$$\lambda_1 = \frac{1}{2m_B} \langle \bar{B}(v) | \bar{b}_v (iD)^2 b_v | \bar{B}(v) \rangle, \qquad \lambda_2 = -\frac{1}{6m_B} \langle \bar{B}(v) | \bar{b}_v \frac{g_s}{2} G_{\mu\nu} \sigma^{\mu\nu} b_v | \bar{B}(v) \rangle. \tag{2.4}$$

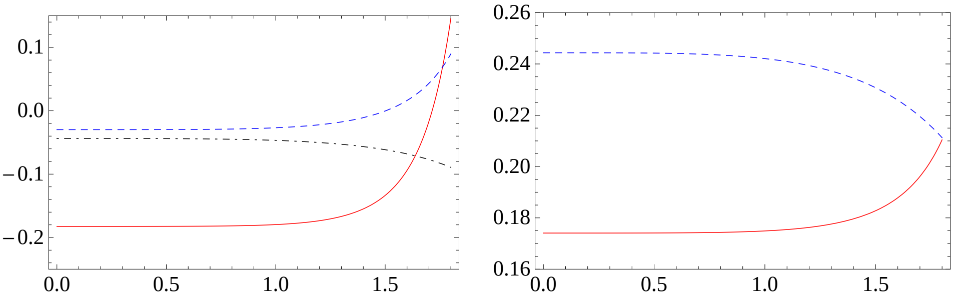

Figure 2: Ratio of NLO to leading order coefficients of λ_1 (left) and λ_2 (right) in the rate (red solid curves), the first moment (blue dashed curves) and the second moment (black dash-dotted curve) as a function of E_0.

In order to determine the Wilson coefficients at one loop level we calculate the amputated Green functions corresponding to the Feynman diagrams shown in Fig. 1; sixteen additional diagrams with a gluon radiated off an internal line will also contribute. it will give us the l.h.s of the matching equation (2.2),

$$f_0^{\mu}\left(z,\xi,\mu,\frac{1}{\varepsilon_{IR}}\right)v_\mu + f_{\lambda_1}\left(z,\xi,\mu,\frac{1}{\varepsilon_{IR}}\right)\frac{\lambda_1}{2m_b} + f_{\lambda_2}\left(z,\xi,\mu,\frac{1}{\varepsilon_{IR}}\right)\frac{\lambda_2}{2m_b}\,. \tag{2.5}$$

The r.h.s of the matching equation (2.2) will look like,

$$-16\pi m_b \sum_{n=3}^{\infty}\frac{1}{m_b^{n-3}}\left[c^{(0)}_{\dim n}\langle O_{\dim n}\rangle_{\text{1-loop}} + \left(\frac{\alpha_s}{4\pi}c^{(1)}_{\dim n} + \delta Z_{\dim n}c^{(0)}_{\dim n}\right)\langle O_{\dim n}\rangle_{\text{tree}}\right], \tag{2.6}$$

where $\langle O_{\dim n}\rangle_{\text{tree}}$ and $\langle O_{\dim n}\rangle_{\text{1-loop}}$ denote the tree-level and one-loop matrix elements of the operator $O_{\dim n}$ between B meson states, respectively, and $Z_{\dim n} = 1 + \delta Z_{\dim n}$ collects the Z-factors to render this expression ultraviolet finite. In the case at hand we only have to consider the one-loop matrix elements of the operators that have non-vanishing Wilson coefficients at the tree-level. The same holds for the tree-level matrix elements that are multiplied by Z-factors. For the renormalization constants, we use the on-shell scheme for the b spinors, and the $\overline{MS}$ scheme for the operator renormalization [9]. Requiring the equality of the eqs. 2.5 and 2.6 will determine the the infrared finite and gauge independent expressions for the Wilson coefficients [9] at one loop level.

In order to get a rough estimate of the size of the power-corrections at $O(\alpha_s)$ we set $\mu = m_b$ and use the numerical values $\alpha_s(m_b) = 0.22$, $m_b = 4.6\,\text{GeV}$, $\lambda_1 = -0.4\,\text{GeV}^2$ and $\lambda_2 = 0.12\,\text{GeV}^2$ to obtain $\Gamma_{77}|_{E_\gamma>1.8\,\text{GeV}}/\Gamma^{(0)}_{77} = 0.763 - 0.007 = 0.756$, a -0.9% effect. The effect of the new corrections on the rate varies with the cut, from -0.4% at $E_0 = 0$ to -0.9% at $E_0 = 1.8\,\text{GeV}$.

In Fig. 2 shows the ratios of NLO to leading order coefficients of $\lambda_{1,2}$ in the rate and in the first two moments as a function of the cut E_0, photon end point energy, using the same inputs. The NLO corrections to λ_2 are close to 20%. Note that in the right panel we have not shown a curve for the second central moment since λ_2 has a vanishing leading order coefficient. From second moment we expect to extract a higher value of λ_1, with our chosen input it is about 10%.

3. Conclusions

We present the first calculation of α_s corrections to $\frac{\Lambda_{QCD}^2}{m_b^2}$ in $B \to X_s\gamma$, the effect of NLO corrections on $B \to X_s\gamma$ rate is below 1% for E0 < 1:8 GeV. Our results allow for more precise evaluation of the moments of the photon distribution and will improve the determination of m_b and μ_π^2; Our method is applicable to inclusive semileptonic decay. $\alpha_s \frac{\mu_\pi^2}{m_b^2}$ corrections to the moments of $B \to X_c \ell \nu$ have been computed numerically, $\alpha_s \frac{\mu_G^2}{m_b^2}$ corrections are still unknown; we also believe analytical result might be easier to implement in the fitting codes.

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