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Update to the Bodek-Yang Unified Model for Electron- and Neutrino- Nucleon Scattering Cross Sections

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We construct a model for inelastic neutrino- and electron-nucleon scattering cross sections using effective leading order parton distribution functions with a new scaling variable ξ_w. Non-perturbative effects are well described using the ξ_w scaling variable, in combination with multiplicative K factors at low Q^2. Our model describes all inelastic charged lepton-nucleon scattering (including resonance) data (HERA/NMC/BCDMS/SLAC/JLab) ranging from very high Q^2 to very low Q^2 and down to the photo-production region. The model describes existing inelastic neutrino-nucleon scattering measurements, and is currently used in analyses of neutrino oscillation experiments in the few GeV region.

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Standard PDFs are extracted from global fits to various sets of deep inelastic (DIS) scattering data at high energies and high Q^2, where non-perturbative QCD effects are negligible. PDF fits are performed within the framework of QCD in either LO, NLO or NNLO.

In order to use the PDFs at low Q^2 we use a new scaling variable (ξ_w) to construct effective LO PDFs that account for the contributions from target mass corrections, non-perturbative QCD effects, and higher order QCD terms.

Our proposed scaling variable, ξ_w is derived as follows. Using energy momentum conservation, the fractional momentum, ξ carried by a quark in a proton target of mass M is

$$\xi = \frac{2xQ'^2}{Q^2(1+\sqrt{1+(2Mx)^2/Q^2})}$$
$$2Q'^2 = [Q^2+M_f{}^2-M_i{}^2]+\sqrt{(Q^2+M_f{}^2-M_i{}^2)^2+4Q^2(M_i{}^2+P_T^2)}$$

Here M_i is the initial quark mass with average initial transverse momentum P_T, and M_f is the mass of the final state quark. Assuming $M_i = 0, P_T = 0$ we construct following scaling variable

$$\xi_w = \frac{2x(Q^2+M_f{}^2+B)}{Q^2[1+\sqrt{1+(2Mx)^2/Q^2}]+2Ax},$$

where in general $M_f = 0$ (except for the case of charm-production in neutrino scattering for which M_f=1.32 GeV). The parameter A is used to account (on average) for the higher order QCD terms and dynamic higher twist in the form of an enhanced target mass term (the effects of the proton target mass is already taken into account in the denominator of ξ_w). The parameter B is used to account (on average) for the initial state quark transverse momentum, and also for the effective mass of the final state quark originating from multi-gluon emission. A non-zero B also allows us to describe data in the photoproduction limit (all the way down to Q^2=0).

In leading order QCD (e.g. GRV98 PDFs), $\mathscr{F}_{2,LO}$ for the scattering of electrons and muons on proton (or neutron) targets is given by the sum of quark and anti-quark distributions (each weighted the square of the quark charges):

$$\mathscr{F}^{e/\mu}_{2,LO}(x,Q^2) = \Sigma_i e_i^2 \left[xq_i(x,Q^2)+x\overline{q}_i(x,Q^2)\right].$$

Our proposed effective LO PDFs model includes the following:

1. The GRV98 LO Parton Distribution Functions (PDFs) are used to describe $\mathscr{F}^{e/\mu}_{2,LO}(x,Q^2)$. The minimum Q^2 value for these PDFs is 0.8 GeV^2.

2. The scaling variable x is replaced with the scaling variable ξ_w.

$$\mathscr{F}^{e/\mu}_{2,LO}(x,Q^2) = \Sigma_i e_i^2 \left[\xi_w q_i(\xi_w,Q^2)+\xi_w\overline{q}_i(\xi_w,Q^2)\right].$$

3. We multiply all PDFs by vector K factors such that they have the correct form in the low Q^2 photo-production limit. We use different forms for the sea and valence quarks.

$$K^{vector}_{sea}(Q^2) = \frac{Q^2}{Q^2+C_s}, \quad K^{vector}_{valence}(Q^2) = [1-G_D^2(Q^2)]\frac{Q^2+C_{v2}}{Q^2+C_{v1}}$$

where $G_D = 1/(1+Q^2/0.71)^2$ is the proton elastic form factor. This form for the K factor for valence quarks is motivated by closure arguments and the Adler sum rule. At low Q^2, $[1-G_D^2(Q^2)]$ is approximately $Q^2/(Q^2+0.178)$. These modifications are included in order to describe low Q^2 data in the photoproduction limit (Q^2=0), where $\mathscr{F}_2^{e/\mu}(x,Q^2)$ is related to the photoproduction cross section according to

$$\sigma(\gamma p) = \frac{4\pi^2\alpha_{\rm EM}}{Q^2}\mathscr{F}_2^{e/\mu}(x,Q^2) = \frac{0.112mb}{Q^2}\mathscr{F}_2^{e/\mu}(x,Q^2)$$

4. We freeze the evolution of the GRV98 PDFs at a value of $Q^2 = 0.80$ GeV^2. Below this Q^2, $\mathscr{F}_2$ is given by;

$$\mathscr{F}_2^{e/\mu}(x,Q^2<0.8) = K^{vector}(Q^2)\mathscr{F}_{2,LO}^{e/\mu}(\xi_w, Q^2=0.8)$$

5. Finally, we fit for the parameters of the modified effective GRV98 LO PDFs (e.g. ξ_w) to inelastic charged lepton scattering data on hydrogen and deuterium targets (SLAC, BCDMS, NMC, H1). We obtain an excellent fit with the following initial parameters: A=0.419, B=0.223, and C_{v1}=0.544, C_{v2}=0.431, and C_{sea}=0.380, with $\chi^2/DOF = 1235/1200$. Because of these additional K factors, we find that the GRV98 PDFs need to be scaled up by a normalization factor N=1.011.

We now describe the second iteration of the fit. Theoretically, the K_i factors are not required to be the same for the u and d valence quarks or the u, d and s sea quarks and antiquarks. In order to allow flexibility in our effective LO model, we treat the K_i factors for u and d valence and sea quarks separately. As the predictions of our model are in good agreement with photoproduction data, and for much of the resonance region, we now proceed to include photo-production data in the fit. In order to get additional constraints on the different K factors for up and down quarks separately, we use both hydrogen and deuterium data.

The second iteration includes the additional photo-production and resonance data in the fit,.

$$K^{LW} = \frac{\nu^2 + C^{L-Ehad}}{\nu^2},\quad K^{vector}_{sea-strange}(Q^2) = \frac{Q^2}{Q^2 + C^{vector}_{sea-strange}},\quad K^{vector}_{sea-up}(Q^2) = \frac{Q^2}{Q^2 + C^{vector}_{sea-up}},$$

$$K^{vector}_{sea-down}(Q^2) = \frac{Q^2}{Q^2 + C^{vector}_{sea-down}},\quad K^{vector}_{valence-up}(Q^2) = K^{LW}[1-G_D^2(Q^2)]\frac{Q^2 + C^{vector}_{v2u}}{Q^2 + C^{vector}_{v1u}},$$

$$K^{vector}_{valence-down}(Q^2) = K^{LW}[1-G_D^2(Q^2)]\frac{Q^2 + C^{vector}_{v2d}}{Q^2 + C^{vector}_{v1d}}$$

The best fit is given by $A = 0.621 \pm 0.009$, $B = 0.380 \pm 0.004$, $C^{vector}_{v1d} = 0.341 \pm 0.007$, $C^{vector}_{v1u} = 0.417 \pm 0.024$, $C^{vector}_{v2d} = 0.323 \pm 0.051$, $C^{vector}_{v2u} = 0.264 \pm 0.015$, and $C^{L-Ehad} = 0.217 \pm 0.015$. The sea factors are $C^{vector}_{sea-down}$=0.621, C^{vector}_{sea-up}=0.363, and $C^{vector}_{sea-strange}$ was set to be the same as $C^{vector}_{sea-down}$. Here, the parameters are in units of $(GeV/c)^2$. The fit χ^2/DOF =2357/1717, and $N = 1.026 \pm 0.003$. The resonance data add to the χ^2/ndf because the fit only provides a smooth average over the resonances. No neutrino data are included in the fit.

For high energy neutrino scattering on quarks and antiquarks, the vector and axial contributions are the same. At very high Q^2, where the quark parton model is valid, both the vector and axial K factors expected to be 1.0. Therefore neutrinos and antineutrino structure functions for an iso-scalar target are given by :

$$\mathscr{F}_2^{\nu}(x,Q^2) = \Sigma_i 2\left[\xi_w q_i(\xi_w,Q^2) + \xi_w \overline{q}_i(\xi_w,Q^2)\right].$$
$$x\mathscr{F}_3^{\nu}(x,Q^2) = \Sigma_i 2\left[\xi_w q_i(\xi_w,Q^2) - \xi_w \overline{q}_i(\xi_w,Q^2)\right].$$

There are two major differences between the case of electron/muon inelastic scattering and the case of neutrino and antineutrino scattering. In the neutrino case we have one additional structure function $\mathscr{F}_3^{\nu}(x,Q^2)$. In addition, at low Q^2 there should be a difference between the vector and axial K_i factors due a difference in the non-perturbative axial vector contributions. Unlike the vector $\mathscr{F}_2$ which must go to zero in the $Q^2 = 0$ limit, the axial part of $\mathscr{F}_2$ is non-zero in the $Q^2 = 0$ limit.

We account for kinematic and dynamic higher twist and higher order QCD effects in $\mathscr{F}_2$ by fitting the parameters of the scaling variable ξ_w and the K factors to low Q^2 data for $\mathscr{F}_2^{e/\mu}(x,Q^2)$. These should also be valid the vector part of $\mathscr{F}_2$ in neutrino scattering. However, the higher order QCD effects in $\mathscr{F}_2$ and $x\mathscr{F}_3$ are different. We account for the different scaling violations in $\mathscr{F}_2$ and $x\mathscr{F}_3$ by adding another correction $H(x,Q^2)$ to the expression for $x\mathscr{F}_3$

The differences between neutrinos and charged lepton scattering are accounted for in the following expressions:

$$\mathscr{F}_2^{\nu vector}(x,Q^2) = \Sigma_i K_i^{vector}(Q^2)\xi_w q_i(\xi_w,Q^2) + \Sigma_j K_j^{vector}(Q^2)\xi_w \overline{q}_j(\xi_w,Q^2)$$
$$\mathscr{F}_2^{\nu axial}(x,Q^2) = \Sigma_i K_i^{axial}(Q^2)\xi_w q_i(\xi_w,Q^2) + \Sigma_j K_j^{axial}(Q^2)\xi_w \overline{q}_j(\xi_w,Q^2)$$
$$x\mathscr{F}_3^{\nu}(x,Q^2) = 2H(x,Q^2)\left[\Sigma_i K_i^{xF3}\xi_w q_i(\xi_w,Q^2) - \Sigma_j K_j^{xF3}\xi_w \overline{q}_j(\xi_w,Q^2)\right]$$

Where i denotes $(valence - up)$, $(valence - down)$, $(sea - up)$, $(sea - down)$, and $(sea - strange)$. Detailed expressions are given in reference[1].

With the above assumptions we calculate the differential cross sections for neutrinos and antineutrino scattering. We also correct for nuclear effects in iron using the ratio of iron to deuterium structure functions as measured in muon and electron scattering experiments.

Our predictions are in good agreement with the CCFR CDHSW neutrino and antineutrino differential cross sections.

References

[1] A. Bodek and U.K. Yang, hep-ph/1011.6592.