Continuum limit results
from 2+1 flavor Domain Wall QCD

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Quantum Chromodynamics with two mass degenerate light quark flavors and an additional single heavier quark flavor has been simulated on the lattice using the domain wall fermion formulation. These simulations cover a range of dynamical pion masses between 290 and 420 MeV and were performed at two different values for the lattice spacing with $1/a = 1.73$ and 2.28 GeV and a linear lattice extent of 2.8 fm. This talk will focus on the determination of the kaon bag parameter B_K and the semi-leptonic kaon form factor K_{l3}. The various methods for extrapolating these quantities to the limit of physical light quark mass will be evaluated.

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The good chiral properties of the domain wall fermion formulation allow to extract many quantities relevant for phenomenological applications from lattice QCD simulations. In this report we will focus on the kaon bag parameter B_K and the semi-leptonic kaon form factor K_{l3} and also briefly comment on $K \to \pi\pi$ decays.

In continuing previous work [1] the RBC/UKQCD-Collaborations now simulated and ana-lyzed lattice QCD with 2 mass-degenerate light quark flavors and one heavy quark flavor, i.e. a 2+1 flavor simulation, at two different lattice scales using the domain wall fermion formulation. The light quark masses correspond to meson masses in the range of 290 to 420 MeV, whereas the heavier quark has been tuned to the mass of the strange quark as observed in Nature. Our coarser lattice spacing, where we simulated at two different values for the light quark masses, corresponds to a scale $1/a = 1.73(3)\,\mathrm{GeV}$, whereas on the finer lattice ($1/a = 2.28(3)\,\mathrm{GeV}$), three different light quark masses have been simulated. In the analysis also partially quenched quark masses were utilized, reaching down to meson masses of 225 MeV. The mass of the heavier strange quark was fixed separately on both sets of ensembles to reproduce its physical value in a first approxima-tion up to 10–15 per cent. Finally, we used the reweighting technique to fine tune the mass to its physical value. The physical, spatial volume in these simulations was held constant at $\approx (2.8\,\mathrm{fm})^3$.

For the extrapolation to the physical light quark masses as realized in Nature we used two different ansätze: one based on $\mathrm{SU}_L(2) \times \mathrm{SU}_R(2)$ chiral perturbation theory (chPT) and as an alternative a simple first order Taylor expansion in the quark masses, cf. [1, 2, 3]. The scales of our simulations and the two physical quark masses ($m_{\mathrm{ud}} = (m_{\mathrm{u}} + m_{\mathrm{d}})/2$ and m_s) were extracted using the masses of the pion, kaon, and the Ω^--baryon as input values.

1. Neutral kaon mixing

The mixing of of the neutral kaons due to indirect CP-violation is commonly described via the ε_K parameter in the unitarity triangle analysis. Using the operator product expansion, it is possible to separate the QCD from the electro-weak contributions. The former have to be treated non-perturbatively, e.g. by measuring the kaon bag parameter, representing the relevant hadronic matrix element, in lattice QCD simulations. Previously, the RBC/UKQCD collaborations pub-lished results obtained with a single lattice spacing [4, 1]. Here we present our updated preliminary value, now also including the continuum extrapolation, for the kaon bag parameter [2]

$$B_K^{\overline{\mathrm{MS}}}(\mu = 2\,\mathrm{GeV}) = 0.546(0.007)_{\mathrm{stat+spread}}(0.016)_{\mathrm{chiral\,extr}}(0.003)_{\mathrm{FV}}(0.014)_{\mathrm{ren}}.$$

The extrapolation to the physical point (determined as described above) has been performed us-ing either SU(2)-chPT (NLO) or an analytic ansatz based on a first order Taylor expansion, see Fig. 1. As our final value we take the average of these two ansätze and assign the difference as the systematic error due to the chiral extrapolation. The continuum limit is taken by including terms proportional to a^2 in LO in our extrapolation formulae and finally extrapolating to $a \to 0$. Finite volume (FV) corrections are included in the chPT fits. To quote our final result in the $\overline{\mathrm{MS}}$-scheme, the matrix element is first non-perturbatively renormalized in the RI/SMOM-scheme [5], which differs from the previously used RI/MOM scheme in that the momenta are defined symmet-rically with non-exceptional external momenta. We found this advantageous in further reducing the systematic error from the renormalization procedure. The final conversion to the NDR-scheme
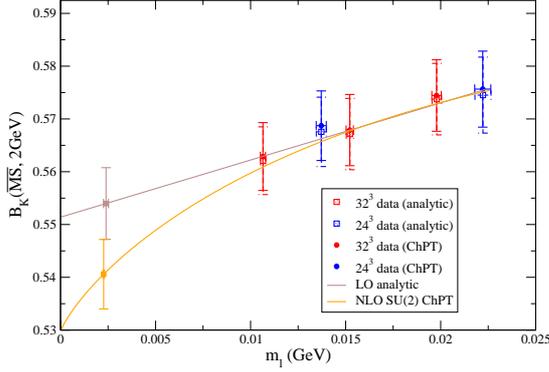
Figure 1: Extrapolation to the physical point for B_K using either NLO SU(2) chPT or an analytic ansatz. Plot courtesy of C. Kelly [2].

($\overline{\text{MS}}$) is performed using 1-loop perturbation theory. We indicate the systematic error from the renormalization procedure in our final result, and also enlarge the statistical error by the spread due to effects from $O(4)$-breaking seen in the extrapolation of the momenta in the non-perturbative renormalization. More details will be available in a forthcoming publication [6].

2. Kaon semi-leptonic form factor

The kaon semi-leptonic form factor $f_+^{K\pi}(0)$ in the K_{l3} mode of the $K \to \pi$ decay has been calculated on the Domain Wall QCD ensembles with the coarser lattice spacing [7, 8]. As is customary in lattice calculations of this quantity, the deviation $\Delta f = f_+^{K\pi}(0) - 1 - f_2(f_0, m_\pi, m_K)$ from unity in the SU(3)-flavor limit and the analytic contribution f_2 is determined. Using spatial periodic boundary conditions, the discrete momenta accessible on the lattice need to be interpolated to zero momentum, which was done using a pole ansatz for $f_+^{K\pi}(q^2)$ in the earlier publication [7]. This introduces some unwanted model dependence in the final result. To overcome this shortcoming, partially twisted boundary conditions were introduced recently [9] allowing to tune the lattice momenta close to zero. This method has been successfully applied in the K_{l3} form factor calculation [8]. In the left panel of Fig. 2 the q^2-interpolation of the previous result is compared to the new data at q^2 close to zero, basically confirming the validity of the pole ansatz used previously. The right panel of Fig. 2 shows the chiral extrapolation to the physical point using SU(3) chPT resulting in [8]

$$f_+^{K\pi} = 0.9599(0.0034)_{\text{stat}}(^{+0.0031}_{-0.0043})_{\text{chiral extr}}(0.0014)_{\text{discretization}}.$$

The systematic error for the chiral extrapolation was estimated by allowing the pion decay constant in the SU(3) chiral limit, f_0, to vary in the range of 100–131 MeV. The discretization error takes into account the missing continuum extrapolation. The latter will be addressed in future work, when also data from the finer lattice spacing will be taken into account. Combining the above result for the form factor with the experimental result $|V_{us} f_+(0)| = 0.2163(5)$ [10] gives $V_{us} = 0.2253(14)$.

3. $K \to \pi\pi$

Calculating the amplitudes in the $K \to \pi\pi$ matrix elements via SU(3)-chPT from those in $K \to \pi$ and $K \to$ vacuum resulted in large systematic uncertainties [11] since the SU(3)-chPT for the simulated pion masses was not converging well. Recent approaches therefore focus on directly accessing the matrix elements via two-pion states on the available domain wall ensembles. For details we refer to [12, 13].
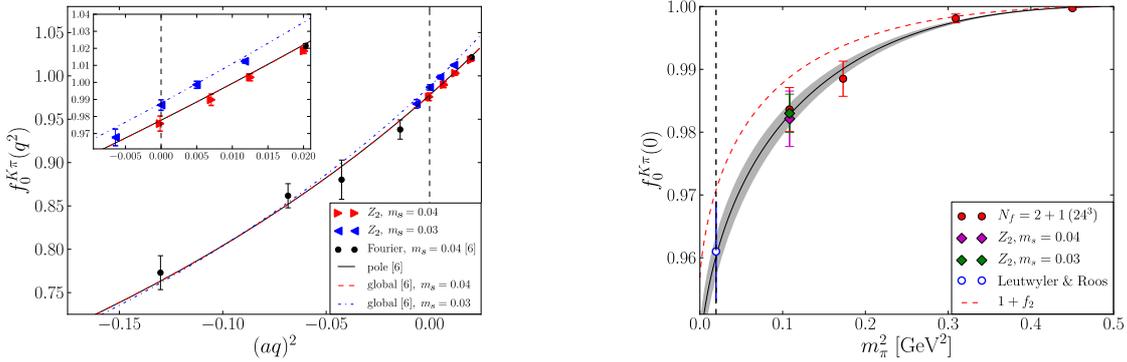
Figure 2: *Left panel:* interpolation in q^2 for K_{l3}, *right panel:* extrapolation to physical light quark masses. Plots courtesy of P. Boyle et al. [8].

4. Final remarks

Selected results from the Domain Wall QCD research programme conducted by the RBC/UKQCD-Collaborations were presented with a special focus on kaon physics. Future simulations will aim at further reducing the simulated pion masses to reduce the current dominant systematic error from the extrapolation to the physical point. This will have to be achieved on coarser lattice spacings in order to keep the finite volume error well under control in numerically affordable simulations. For first results and how to still limit the discretization error on coarser lattices we refer to [14].

References

[1] RBC-UKQCD, C. Allton *et al.*, Phys. Rev. **D78**, 114509 (2008), arXiv:0804.0473.

[2] RBC-UKQCD, C. Kelly, to be published in the proceedings of Lattice 2010 .

[3] RBC-UKQCD, Y. Aoki *et al.*, (2010), arXiv:1011.0892.

[4] RBC-UKQCD, D. J. Antonio *et al.*, Phys. Rev. Lett. **100**, 032001 (2008), arXiv:hep-ph/0702042.

[5] C. Sturm *et al.*, Phys. Rev. **D80**, 014501 (2009), arXiv:0901.2599.

[6] RBC-UKQCD, P. A. Boyle *et al.*, in preparation .

[7] P. A. Boyle *et al.*, Phys. Rev. Lett. **100**, 141601 (2008), arXiv:0710.5136.

[8] P. A. Boyle *et al.*, (2010), arXiv:1004.0886.

[9] P. A. Boyle, J. M. Flynn, A. Jüttner, C. T. Sachrajda, and J. M. Zanotti, JHEP **05**, 016 (2007), arXiv:hep-lat/0703005.

[10] M. Antonelli *et al.*, Eur. Phys. J. **C69**, 399 (2010), arXiv:1005.2323.

[11] S. Li and N. H. Christ, PoS **LATTICE2008**, 272 (2008), arXiv:0812.1368.

[12] RBC-UKQCD, E. Goode and M. Lightman, to be published in the proceedings of Lattice 2010 .

[13] RBC-UKQCD, Q. Liu, (2010), arXiv:1010.3768.

[14] RBC-UKQCD, R. D. Mawhinney, to be published in the proceedings of Lattice 2010 .