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Decay constants and sigma terms from the lattice.

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Thanks to the recent developments both in our understanding of lattice simulations and in computer power, accurate predictions can now be made in nonperturbative QCD, with all sources of error under control. I review recent results of the Budapest-Marseille-Wuppertal lattice collaboration. The first concern the pion and kaon leptonic decay constants, which can be combined with experiment to determine CKM matrix elements and check the unitarity of this matrix's first row. The second pertain to the nucleon sigma terms, which are important for calculating the sensitivity to dark matter of direct search experiments. For the latter, preliminary results based on a subset of our new dataset will be presented. I will emphasize how we are able to control systematic errors in these calculations.

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1. Lattice QCD

Lattice QCD is not a model for the strong interactions, but real world QCD, provided appropriate limits are taken. Taking these limits leads to uncertainties in the final result, that have to be properly estimated. These uncertainties can be broadly classified in two groups.

First we have the statistical uncertainty of our result. This is associated with the numerical evaluation of the Euclidean path integral by Monte Carlo methods. There are standard ways to assess the statistical accuracy of a result (see for example [1] and references therein).

Second there are systematic uncertainties associated with how continuum QCD is reached from computations in a finite volume, at non-zero lattice spacing and, usually, at values of the up and down quark masses larger than in the real world.

To obtain results in the physical limit requires extrapolations in these different simulation parameters. In general there are different ways of performing each of the previous extrapolations. A well estimated systematic uncertainty should include the resulting variations.

2. Meson decay constants and physics beyond the standard model

The weak interaction is the only known interaction that allows flavour changing decays. The strength of this flavour changing is contained in the CKM matrix. Checking the predicted unitarity of this matrix is one of the traditional searches for physics beyond the standard model. Weak decays in nature are usually observed as decays of hadrons. The theoretical amplitude of these decays include a non perturbative QCD factor that can be computed in lattice QCD.

In particular we will use the ratio of decay constants of the π and K mesons. With experimental input concerning these decays, one can compute the ratio of CKM elements $|V_{us}|/|V_{ud}|$. This ratio, together with the value of $|V_{ud}|$ obtained from nuclear β-decay, and the most precise value available today for $|V_{ub}|$ can be used to check the first row unitarity relation of the CKM matrix.

We determine this ratio of decay constants F_K/F_π by using a total of 20 lattice simulations (that of [2]) corresponding to different values of the quark masses of the lattice spacing and of the lattice size. We perform the chiral extrapolation using a total of 7 different functional forms given by $SU(2)$ and $SU(3)$ χPT and regular expansions. Moreover we impose two different pion mass cuts (that affect the number of simulation points included in the fit). Cutoff effects are partially cancelled in the ratio of decay constants, and therefore small in our data. We parametrise them as being absent, of $\mathscr{O}(a)$ and of $\mathscr{O}(a^2)$, the three options compatible with our data. Finally we use 18 different time fitting intervals for the correlators to quantify the possible unwanted contributions of excited estates, and two different physical inputs to set the scale.

In total we have 1512 fitting procedures that weighted with the fit quality produce a distribution of possible values of F_K/F_π. We use the median of this distribution as our final result, and the 68% confidence interval as a measure of the systematic uncertainty of our computation. The final result is

$$\left.\frac{F_K}{F_\pi}\right|_{\text{phys}} = 1.192(7)_{\text{stat}}(6)_{\text{syst}} \tag{2.1}$$

and the first row unitarity relation of the CKM matrix is well observed with no signal of physics beyond the standard model. More details about the computation can be found in the original work and references therein [3].

3. Nucleon sigma terms and dark matter detection

According to observations at galaxy as well as cosmological scales, there should exist a large portion of matter that does not interact electro-magnetically nor strongly with ordinary matter. The standard model has no particle candidate to account for these observations, and therefore direct detection of dark matter (DM) particles is direct evidence of new physics.

Direct detection of DM is performed by measuring the interaction of dark matter particles with heavy nuclei. Since the fundamental interaction would be between a quark inside a nucleon and a dark matter particle, the cross section of the effective interaction DM-nucleus has a component that depends on the quark content of the nucleon which is given by the nucleon sigma terms.

The nucleon sigma terms are not directly measurable quantities. Their value is not precisely known, and determinations rely on effective field theory approaches. A precise knowledge of the values of these quantities is fundamental to understand direct DM searches [4].

On the other hand these are the typical non perturbative QCD quantities that can be accurately computed on the lattice. One possibility to compute the sigma terms trough the Feynman-Hellman theorem. This theorem relates the sigma term with the dependence of the nucleon mass with respect to the quark masses.

Here we will present some preliminary results (the interested reader should consult [5] and references therein) for the nucleon sigma terms. We perform the chiral extrapolation 2 different functional based on Taylor/Padè expansions around a regular point. Moreover we impose two different pion mass cuts. Cutoff effects and finite volume effects have been checked to be smaller than the statistical accuracy of the data. Finally we use 144 different time fitting intervals for the correlators to quantify the possible unwanted contributions of excited estates. The scale is set through the nucleon. Our preliminary results are

$$\sigma_{\pi N} = 49(10)_{\rm stat}(11)_{\rm sys}\ {\rm MeV} \tag{3.1}$$

$$\sigma_{\bar{s}sN} = 49(37)_{\rm stat}(26)_{\rm sys}\ {\rm MeV} \tag{3.2}$$

References

[1] B. A. Berg, *Markov Chain Monte Carlo Simulations And Their Statistical Analysis: With Web-based Fortran Code*. World Scientific Publishing Company, October, 2004.

[2] S. Durr *et. al.*, *Ab Initio Determination of Light Hadron Masses*, *Science* **322** (2008) 1224–1227, [arXiv:0906.3599].

[3] S. Durr *et. al.*, *The ratio FK/Fpi in QCD*, *Phys. Rev.* **D81** (2010) 054507, [arXiv:1001.4692].

[4] J. R. Ellis, K. A. Olive, and C. Savage, *Hadronic Uncertainties in the Elastic Scattering of Supersymmetric Dark Matter*, *Phys. Rev.* **D77** (2008) 065026, [arXiv:0801.3656].

[5] S. Durr et al, *Sigma term and strange content of the nucleon*, *PoS* **LAT2010** (2010) [arXiv:1012.1208].