PROCEEDINGS OF SCIENCE

Moving NRQCD

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Theoretical predictions for exclusive semileptonic decays of heavy mesons are essential for determining CKM matrix elements and constraining new physics.
On the lattice, one must treat the heavy quark in an effective theory, as $1/M \ll a$. At high recoil, discretization errors from the large 3-momentum of the final meson in the rest frame of the heavy meson are significant.
A technique to avoid these errors is to give the heavy meson a significant "external" momentum in the lattice frame. The decay meson momentum is then not large on the lattice and a wider q^2 region can be covered. While boosting to a moving frame *increases* the discretization error for the quarks in a B meson by $\propto (\gamma^2-1)\Lambda^2_{\rm QCD}$, the discretization error of the final-state light meson behaves like $\propto \left(\frac{1}{2}|\boldsymbol{p}_F|\right)^2$, and using $q^2 = (p_B - p_F)^2$, one can pick an optimal v to minimize the total error.
The key fact in applying the boosted frame technique is that almost all of the heavy meson momentum is carried by the heavy quark whilst the dynamics of the heavy quark inside the heavy meson remain non-relativistic. This leads to a generalization of Non-Relativistic Quantum Chromodynamics (NRQCD) to the case of moving heavy quarks, called moving NRQCD (mNRQCD).
We have carried out extensive perturbative and non-perturbative tests of the formalism, which show that the decay constants of both heavy-light and heavy-heavy mesons can be calculated with small systematic errors (although at the cost of increasing statistical errors) up to much larger momenta than with NRQCD.

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Relating the Dirac spinor Ψ to the Pauli spinors ψ_v and ξ_v for quarks and antiquarks in a frame boosted with the velocity $\boldsymbol{v}$ by a field transformation

$$\Psi(x) = S(\Lambda)\, T_{\rm FWT}\, e^{-imu\cdot x\hat{\gamma}^0}\, A_{D_t}\, \frac{1}{\sqrt{\gamma}} \begin{pmatrix} \psi_v(x) \\ \xi_v(x) \end{pmatrix}$$

with the spinorial Lorentz boost $S(\Lambda)$, the Foldy-Wouthuysen-Tani transformation $T_{\rm FWT}$, and a field transformation A_{D_t} designed to remove unwanted temporal derivatives from the Lagrangian gives the (continuum, Minkowski-space) Lagrangian [1]

$$\mathscr{L} = \psi_v^\dagger \Big[\underbrace{iD_0 + i\boldsymbol{v}\cdot\boldsymbol{D} + \frac{\boldsymbol{D}^2 - (\boldsymbol{v}\cdot\boldsymbol{D})^2}{2\gamma m}}_{=H_0} + \underbrace{\frac{g}{2\gamma m}\boldsymbol{\sigma}\cdot\boldsymbol{B}' + \mathscr{O}(1/m^2)}_{\delta H} \Big] \psi_v + \begin{pmatrix} \psi_v \to \xi_v \\ m \to -m \end{pmatrix},$$

where the leading $\mathscr{O}(1/m^3)$ term is retained in δH to ensure the proper power counting for heavy-heavy states; on the lattice, additional terms have to be added for $\mathscr{O}(a^4, \alpha_s a^2)$ improvement. With partial exponentiation to avoid the well-known parabolic instability, this yields the lattice Lagrangian

$$\begin{aligned} \mathscr{L}_{\psi_v}(\boldsymbol{x},\tau) &= \psi_v^+(\boldsymbol{x},\tau)\left[\psi_v(\boldsymbol{x},\tau) - K(\tau)\psi_v(\boldsymbol{x},\tau-a)\right] \\ K(\tau) &= \left(1-\tfrac{\delta H|_\tau}{2}\right)\left(1-\tfrac{H_0|_\tau}{2n}\right)^n U_4^\dagger(\tau-a)\left(1-\tfrac{H_0|_{\tau-a}}{2n}\right)^n\left(1-\tfrac{\delta H|_{\tau-a}}{2}\right). \end{aligned}$$

Note that H_0 is bounded from below at all v, as opposed to the case of HQET in a moving frame. Also note that for $v = 0$, mNRQCD reduces to ordinary NRQCD.

We have computed the wavefunction renormalization constant Z_ψ, vacuum energy shift E_0, mass renormalization constant Z_m, and frame-velocity renormalization constant Z_v at the one-loop level [1]; a two-loop calculation is in progress. Since the full mNRQCD action is very complicated, we employ automated methods (HiPPy/HPsrc [2]) for this purpose. We have also calculated the one-loop matching coefficients for the vector and tensor currents needed for radiative and semileptonic B decays [3].

Going beyond perturbation theory, we have tested non-perturbative improvement [3] to restore cubic symmetry, and have shown that the decay constants of both heavy-light and heavy-heavy mesons can be calculated with small systematic errors up to much larger momenta than with NRQCD [1].

In what is intended as the central application of mNRQCD, form factors for B and B_s decays, including $B \to K^{(*)}\ell\ell$ and $B \to \pi\ell\nu$, are currently being measured [4] towards low q^2. Preliminary results suggest that a somewhat larger range in q^2 may become accessible by using mNRQCD.

References

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