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Hidden Fine Tuning In The Quark Sector Of Little Higgs Models

Benjamín Grinstein∗†
University of California, San Diego and CERN
E-mail: bgrinstein@ucsd.edu

Randall Kelley
Harvard University
E-mail: randallkelley@physics.ucsd.edu

Patipan Uttayarat
University of California, San Diego
E-mail: puttayarat@physics.ucsd.edu

In Little Higgs models a collective symmetry prevents the higgs from acquiring a quadratically divergent mass at one loop. We have previously shown that the couplings in the Littlest Higgs model introduced to give the top quark a mass do not naturally respect the collective symmetry. We extend our previous work showing that the problem is generic: it arises from the fact that the would be collective symmetry of any one top quark mass term is broken by gauge interactions.

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∗Speaker.

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1. Fine Tuning Problem in The Littlest Higgs Model

To establish notation we briefly review elements of the Littlest Higgs [1]. It has a global symmetry $G_f = SU(5)$ that spontaneously breaks to $SO(5)$; $SU(5) \to SO(5)$ is characterized by the Goldstone boson decay constant f. The embedding of the weakly gauged subgroup $G_w = \prod_{i=1,2} SU(2)_i \times U(1)_i$ in G_f is fixed by taking the generators of $SU(2)_1$ and $SU(2)_2$ to be

$$Q_1^a = \begin{pmatrix} \frac{1}{2}\sigma^a & 0_{2\times3} \\ 0_{3\times2} & 0_{3\times3} \end{pmatrix} \qquad \text{and} \qquad Q_2^a = \begin{pmatrix} 0_{3\times3} & 0_{3\times2} \\ 0_{2\times3} & -\frac{1}{2}\sigma^{a*} \end{pmatrix}, \tag{1.1}$$

and the generators of $U(1)_1$ and $U(1)_2$ to be

$$Y_1 = \frac{1}{10}\text{diag}(3,3,-2,-2,-2) \qquad \text{and} \qquad Y_2 = \frac{1}{10}\text{diag}(2,2,2,-3,-3). \tag{1.2}$$

The vacuum manifold is characterized by a unitary, symmetric 5×5 matrix Σ. We denote by g_i (g_i') the gauge couplings associated with $SU(2)_i$ ($U(1)_i$). If one sets $g_1 = g_1' = 0$ the model has an exact global $SU(3)_u$ symmetry (acting on the upper 3×3 block of Σ), while for $g_2 = g_2' = 0$ it has a different exact global $SU(3)_d$ symmetry (acting on the lower 3×3 block). Either of these exact global $SU(3)$ would-be symmetries guarantee the Higgs remains exactly massless. Hence, the Higgs mass should vanish for either $g_1 = g_1' = 0$ or $g_2 = g_2' = 0$. The perturbative quadratically divergent correction to the Higgs mass must be polynomial in the couplings and can involve only one of the couplings at one loop order. Hence it must vanish at one loop. This is the collective symmetry mechanism that ensures the absence of 1-loop quadratic divergences in the higgs mass.

Next introduce couplings of Σ to quarks, to generate a top mass. Take the third generation doublet q_L to be a doublet under $SU(2)_1$ and a singlet under $SU(2)_2$. Introduce additional $SU(2)_1\times SU(2)_2$-singlet spinor fields: q_R, u_L and u_R. The third generation right handed singlet is a linear combination of u_R and q_R. The charges of q_L, q_R, u_L and u_R under $U(1)_1\times U(1)_2$ are, in terms of a free parameter y, $(\frac{11}{30}-y, y-\frac{1}{5})$, $(\frac{2}{3}-y, y)$, $(\frac{13}{15}-y, y-\frac{1}{5})$ and $(\frac{13}{15}-y, y-\frac{1}{5})$, respectively. If their couplings are taken to be

$$\mathscr{L}_{\text{top}} = -\lambda_1 f \bar{q}_L{}^i \varepsilon^{xy}\Sigma_{ix}\Sigma_{3y}q_R - \frac{1}{2}\lambda_1' f \bar{u}_L \varepsilon^{3jk}\varepsilon^{xy}\Sigma_{jx}\Sigma_{ky}q_R - \lambda_2 f \bar{u}_L u_R + \text{h.c.} \tag{1.3}$$

then only when $\lambda_1' = \lambda_1$ (and $\lambda_2 = g_1 = g_1' = 0$) do we obtain the global $SU(3)_u$ symmetry of the collective symmetry mechanism. In Ref. [2] we pointed out that the relation $\lambda_1' = \lambda_1$, assumed throughout the little higgs literature, is unnatural. Indeed, if Λ is the cutoff of the theory, then the renormalization group gives

$$\frac{\lambda_1(\mu)}{\lambda_1'(\mu)} = \frac{\lambda_1(\Lambda)}{\lambda_1'(\Lambda)}\left(\frac{g_1'(\mu)}{g_1'(\Lambda)}\right)^{(2-3y)/b}, \tag{1.4}$$

where b is the one-loop coefficient of the β-function of g_1'. Moreover, this running must occur in the UV completion as well, and there are additional corrections from matching at Λ. So there is no natural way of justifying $\lambda_1'(\Lambda) = \lambda_1(\Lambda)$. We refer to this as the hidden fine tuning problem. How bad is it? There is now a quadratically divergent contribution to the higgs mass, $\delta m_h^2 = \frac{12}{16\pi^2}(\lambda_1^2 - \lambda_1'^2)\Lambda^2$. This requires a tuning $\delta\lambda_1 \approx \frac{1}{24}\frac{m_h^2}{f^2} \sim 0.04\%$ for $m_h = 114$ GeV, or a $\Delta = 2400$

naturalness measure[3]. Running is a 1-loop effect and $\lambda_1 - \lambda_1'$ contributes to mass at 1-loop. Enthusiasts of the model may argue that therefore the actual correction to the higgs mass is a 2-loop effect. But in the absence of fine tuning at Λ it is really a 1-loop effect. Moreover, numerically the effect is large: $\delta\lambda_1 < 4 \times 10^{-4}$ is needed, while 1-loop is $1/16\pi^2 \approx 63 \times 10^{-4}$. Note also, from Eq. (1.4), that while $y = 2/3$ gives no 1-loop logarithmic running, one cannot ignore finite, non-logarithmic corrections. We computed the logarithmic corrections because they are universal. But there is no reason to expect that the running above Λ plus the matching at Λ will keep $\lambda_1' = \lambda_1$ even when the special value $y = \frac{2}{3}$ is chosen.

Can one impose a symmetry in the underlying UV theory that enforces $\lambda_1' = \lambda_1$ to high accuracy in spite of the fact that the symmetry is broken by gauge interactions? Let us look at a more familiar example. Consider $SU(3)$ as an approximate flavor symmetry of QCD. This is a natural symmetry, in the sense that it appears automatically because all quarks are light compared to the chiral symmetry breaking scale, regardless of the relative magnitude of the masses. In the absence of fine tuning, flavor-symmetry breaking interactions in a phenomenological Lagrangian take the most general form consistent with gauge invariance. Short of an accidental tuning the only alternative is a perturbative UV completion. This however involves fundamental scalars that Little Higgs theories set out to avoid.

2. A No-go theorem

Consider a model with global "flavor" symmetry group G_f. It is assumed to break spontaneously, $G_f \to H$. There is a weakly gauged subgroup $G_w \subset G_f$, and $G_w \to G_{\rm EW}$ under $G_f \to H$ (where $G_{\rm EW}$ stands for the SM's electroweak group). We assume further that among the pseudo-goldstone bosons in G/H there is a higgs doublet, h.

In general the gauge group has a product structure, $G_w = \prod G_i$. For each G_i we assume there is a collective symmetry group, G_i^c, that commutes with G_i and that induces non-linear shifts in h. This assumption requires that each of the G_i has (four) generators that are not orthogonal to the generators of $G_{\rm EW}$ (other gauged factors that have no direction along the electroweak group are of no interest here).

The theorem is concerned with the possibility of writing additional terms in the Lagrangian, like the terms required to give the top quark a mass. If the higgs mass is to be protected from these, then they each must have their own collective symmetry group G_Y^c, and they must remain invariant under $\prod_{i=1}^N G_i$. We will show that one cannot find a G_Y^c that commutes with $\prod_i G_i$.[1] Hence a G_Y^c invariant is a sum over terms related by G_Y^c that are independently gauge invariant. Alternatively, one could absorb in G_Y^c that part of the gauge group that does not commute with G_Y^c, gauging G_Y^c, which results in eating the higgs doublet.

Proof. That the higgs transforms linearly under the electroweak gauge group means that under $SU(2) \times U(1)$ the doublet h transforms as

$$\delta_\varepsilon h = i\varepsilon^a \frac{\sigma^a}{2} h + i\varepsilon \frac{1}{2} h, \tag{2.1}$$

[1] In Ref. [2] we argued that the generators of G_Y^c form a reducible representation of $G_{\rm EW}$, but we did not make explicit the role played by the G_i and G_i^c. The argument presented in this talk does. It thus allows us to elucidate how models like that of Kaplan and Schmaltz[4] evade the no-go theorem; see Sec. 3.

where σ^a are Pauli matrices. Under any of the collective groups $G_i^c \subset G_f$, h transforms non-linearly,

$$\delta_\eta h = \eta^m x^m + \cdots \tag{2.2}$$

where the implicit sum over m is over all generators in G_i^c, for some two component complex vectors x^m and the ellipses stand for terms at least linear in h. One can redefine the basis of generators in G_i^c so that $x^m = 0$ for $m \geq 5$ and x^m for $m = 1, \cdots, 4$ are unit vectors, with $m = 1, 3$ real and $m = 2, 4$ purely imaginary. Now the commutator,

$$(\delta_\eta \delta_\varepsilon - \delta_\varepsilon \delta_\eta)h = i\varepsilon^a \eta^m \frac{\sigma^a}{2} x^m + i\varepsilon \eta^m \frac{1}{2} x^m + \cdots, \tag{2.3}$$

is again a non-linear transformation, a linear combination of the same four generators in G_i^c that shift the higgs. In terms of the Lie algebra of G_f, denoting these generators by X^i, with[2] $i = 1, 2$ and the generators of G_{EW} by Q^a and Y, we read off

$$[Q^a, X^i] = \frac{i}{2}(\sigma^a)^{ij} X^j, \qquad [Y, X^i] = \frac{i}{2} X^i \tag{2.4}$$

We see that the G_i^c-generators of higgs shifts transform as tensors of G_{EW} with the same quantum numbers as the higgs doublet.

Let's introduce some more notation. The generators of G_f, H and G_i are denoted by $\{T^A, X^B\}$, $\{T^A\}$ and $\{Q_i^I\}$, respectively. G_{EW} has generators $\{T^1, \ldots, T^4\} = \{Q^a, Y\}$. We can always arrange the broken generators X^B so that the first four precisely correspond to the generators of the non-linear transformations on the higgs doublet, X^x, $x = 1, \ldots, 4$.[3] The X^x are not necessarily in the algebra of G_i^c but there are some unbroken generators for which $X^x + T^x$ are. The only gauged sub-groups that are relevant to our arguments are those that have a component of G_{EW}. Hence, $Q^a = \sum_{i,I} c_i^{aI} Q_i^I$. For each i a similarity transformation brings this to the form $\sum_I c_i^{aI} Q_i^I = c_i Q_i^a$ (no sum on i) so that now $Q^a = \sum_i c_i Q_i^a$ with all $c_i \neq 0$. Since $[Q_i^a, Q_j^b] = 0$ for $i \neq j$ it follows that $[Q_i^a, Q_j^b] = i\delta_{ij}\varepsilon^{abc} Q_i^c$ and $c_i = 1$ (we are assuming a common normalization for generators).

Is there a "yukawa" collective symmetry group G_Y^c that commutes with all the gauge groups? The answer is that there is none since a collective symmetry has to include X^x and therefore the algebra is going to include that of some (possibly all) of the gauged symmetries. The proof is straightforward. The generator in G_Y^c that shifts the higgs, X_Y^x, must satisfy $[Q^a, X_Y^x] = \frac{i}{2}(\sigma^a)^{xy} X_Y^y$ Using $Q^a = \sum Q_i^a$ we see that this is inconsistent with $[Q_i^a, X_Y^x] = 0$. End proof.[4]

Comments. What does this mean? If $[G_Y^c, G_i] \neq 0$ then a non-trivial invariant of G_Y^c is a sum of several terms independently invariant under G_i. To see this note that, since there is no semi-simple Lie algebra of rank 4, there must be additional generators in the algebra that contains X^x. Using the Jacobi identity we see that $\hat{X}^{xy} = [X^x, X^y]$ satisfies $[Q_i^a, \hat{X}^{xy}] = \frac{i}{2}(\sigma^a)^{yz}\hat{X}^{xz} - \frac{i}{2}(\sigma^a)^{xz}\hat{X}^{yz}$. That

[2] The index i runs over 1,2 because the hermitian matrices break into a symmetric and an antisymmetric part, corresponding to the two real and two imaginary components of x^m, and also to the real and imaginary components of the higgs doublet.

[3] These four generators are given in terms of those in Eq. 2.4 by $X^{1,2} \pm (X^{1,2})^T$.

[4] Alternatively, write an $SU(2)$ subalgebra X_Y^a of G_Y^c in terms of the generators of the G_i^c's: $X_Y^a = \sum_i c_i X_i^a$. Requiring that this commutes with Q_j^b for every j one find $c_i = 0$, all i. This is not a complete proof inasmuch as we have assumed, not proved, that one can find an $SU(2)$ subalgebra of G_Y^c that is a linear combination of generators of the G_i^c.

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is, these generators transform in a representation in the tensor product of two doublets. Continuing this way, considering commutators of the generators we have so far, we can eventually generate the complete Lie algebra and find that it breaks into sectors classified by irreducible representations under G_i. Now, any non-trivial invariant must be a product of two (combinations of) fields, one transforming in some irreducible representation R of G^c_Y and the other as the complex conjugate $\bar{R}$. The previous argument shows that under G_i the representation R breaks into a direct sum $R = r_1 \oplus r_2 \oplus \cdots$ of at least two irreducible representations of G_i. Therefore the product $R \times \bar{R}$, contains the sum of at least two invariants under G_i, one in $r_1 \times \bar{r}_1$ and another in $r_2 \times \bar{r}_2$.

3. The Kaplan-Schmaltz model

Famously, no-go theorems are most useful in showing how to avoid them. There may be some assumption one may be willing to give up. Kaplan and Schmaltz have studied a class of models for which our proof fails.[4] Their models are peculiar in that collective symmetries follow from setting the gauge coupling to zero for some fields but not for others. That is where the model evades our no-go argument.

Our proof *assumes* that for each gauge group factor G_i there is one collective symmetry group G^c_i that commutes with it. This is useful because one can consider the limit in which all other gauge couplings are set to zero and in that limit G^c_i is an exact symmetry. There is no such limit in Kaplan-Schmaltz (KM) models. In them the collective symmetry limit is obtained by judiciously ignoring certain terms in the Lagrangian, rather than by parametrically turning them off.

Specifically, for Kaplan-Schmaltz there are two different collective symmetry groups for the same gauge group factor. Neither of these commutes with the gauge group, except if one ignores the gauge couplings of a subset of fields. For example, in their simplest model $G_f = SU(3)_L \times SU(3)_R$, $H = SU(2)_L \times SU(2)_R$ and $G_w = SU(3)_V$. As it stands there is no obvious collective symmetry. But had we gauged $SU(3)_L$ only then $SU(3)_R$ would be a collective symmetry group and vice versa. Since the order parameter is $(3,1)+(1,3)$, one can accomplish this by ignoring the gauge coupling of one or the other of $(3,1)$ or $(1,3)$. There is no fine tuning in the Yukawa terms: collective symmetries are gauged $SU(3)$'s and therefore the various EW invariants are now related. Of course, if the symmetries are gauged then a higgs must be eaten. But since there are two copies of $SU(3)$, there are two doublets, one is eaten and the other is the higgs. This allows KS models to avoid the problem with quartic couplings[5] and, moreover, there is a region of parameter space where it is consistent with electroweak precision data[6].

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