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Inflation and Loop Quantum Cosmology

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On the one hand, inflation is an extremely convincing scenario: it solves most cosmological paradoxes and generates fluctuations that became the seeds for the growth of structures. It, however, suffers from a "naturalness" problem: generating initial conditions for inflation is far from easy. On the other hand, loop quantum cosmology is very successful: it solves the Big Bang singularity through a non-perturbative and background-independent quantization of general relativity. It, however, suffers from a key drawback: it is extremely difficult to test. Recent results can let us hope that inflation and LQC could mutually cure those pathologies: LQC seems to naturally generate inflation and inflation could allow us to test LQC.

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1. LQC helps inflation

Loop quantum gravity (LQG) is a tentative background-independent and non-perturbative quantization of general relativity. It relies on Ashtekar variables, namely SU(2) valued connections and conjugate densitized triads. The quantization is performed through holonomies of the connections and fluxes of the densitized triads (see, *e.g.*, [1] for excellent introductions). At the intuitive level, loop quantum cosmology (LQC) can be seen as the symmetry reduced version of LQG (although it should be underlined that the derivation of LQC from the full theory is not yet fully demonstrated, see [2] for the latest progresses on the spinfoam approach to LQC). While predictions of LQC are very close to those of the Wheeler-deWitt theory in the low density regime, there is a fundamental difference once we approach the Planck scale: the Big Bang is replaced by a Big Bounce due to huge repulsive quantum geometrical effects (see, *e.g.*, [3] for reviews). Unquestionably, this resolution of the primordial singularity problem is the most striking result of LQC. Following the pioneering works [4], many studies have confirmed this prediction in different situations (see, *e.g.*, references in [5]).

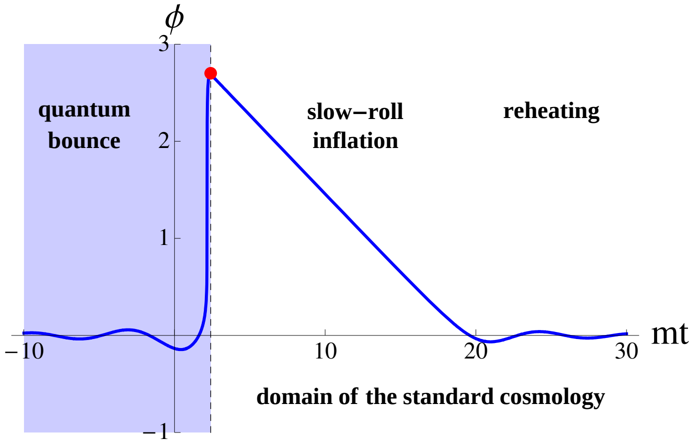

Figure 1: Evolution of a scalar field in a LQC bouncing universe. During the contraction phase the oscillations are amplified. Then a slow-roll inflation phase takes place, followed by the reheating.

Moreover, in the last years, it was realized that there are strong links between inflation and LQC (see, *e.g.*, [6] for a pioneering paper on super inflation which was followed by many others). The key feature can be understood very easily. Let's consider the simplest possible model, without any ϕ^4 potential or any other intricate feature: just a massive scale field filling the bouncing LQC universe. The Klein-Gordon equation simply reads $\ddot{\phi}+3H\dot{\phi}+m^2\phi=0$ where the second term is usually called the friction term due to the expansion of the Universe. However, during the pre-bounce stage, the Universe is contracting, the Hubble parameter is therefore negative and this becomes an anti-friction term. Otherwise stated, the field automatically climbs-up its potential: whatever the small oscillations, they are amplified. However, just after the bounce, H becomes positive (the Universe is expanding). This term is indeed now a friction term and the field is (nearly) frozen, usually high on its potential, where it was thrown away during the antifriction-bouncing phase. This is *exactly* what is needed for slow-roll inflation to occur, without any feature introduced "by hand" (see Fig. 1) ! It means that the model naturally predicts inflation. It is rather remarkable to realize that the canonical quantization of Einstein equations, applied to the Universe, could have predicted inflation far before it was understood to be necessary for cosmology.

This scenario, together with the detailed values of the parameters, is described into the details in [7]. Furthermore, it was shown in [8] that the probability for a long enough inflationary phase (say with more than 60 e-folds or so) is extremely close to 1, in sharp contrast with what was estimated for standard inflation in usual general relativity ([9]). In fact, as demonstrated in [10], it seems that this difference is mostly due to the fact that probabilities are not estimated at the same time in the cosmic history. In the LQC framework, one can use a naturally defined surface and the high probability for inflation to occur is a (quite) reliable result. My view is that it is just not (yet) possible to define a meaningful probability for inflation in the standard Big Bang paradigm. This is not without echoing the problems faced for making predictions in the multiverse...

Impressively, the bouncing scenario, as predicted in (but not only in) LQC, seems to lead generically to inflation. Inflation is just a nearly unavoidable consequence of this model which was, by no means, designed for this. Sounds good.

2. Inflation helps LQC

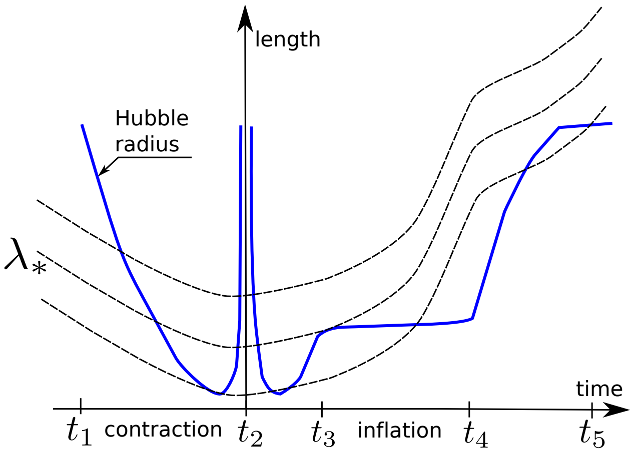

Figure 2: Evolution of the Hubble radius (solid line) and of some length scales (dashed lines). Different times are distinguished: t_1 – time when the initial conditions are set; t_2 – bounce ($H = 0$); t_3 – beginning of inflation; t_4 – end of inflation; t_5 – present epoch of dark energy domination.

The very exciting news is that, the other way round, inflation –which can now be considered as quite natural– can help us testing quantum gravity. Loop Quantum Gravity is an appealing scenario. However, as most other attempts (including of course string theory) to quantize gravity, it suffers from the lack of Planck-scale experiments. Measuring directly areas and volumes at the required accuracy (to probe the discrete spectrum) is just technically impossible. Looking for evidences of a violation of the Lorentz invariance is an interesting idea. But it has so far remained inconclusive and is extremely speculative at the theoretical level : there are no unambiguous prediction of any kind of Lorentz invariance violation. (As pointed out by Rovelli, one should think in terms of eigenvalues of the length operators and not in terms of usual lengths. The spectrum can remain unchanged while the expectation value varies with speed.) Cosmology is therefore probably the best –if not only– way to search for loopy effects in Nature. And inflation, because it stretches very small scales up to macroscopic lengths, is our best friend in this game.

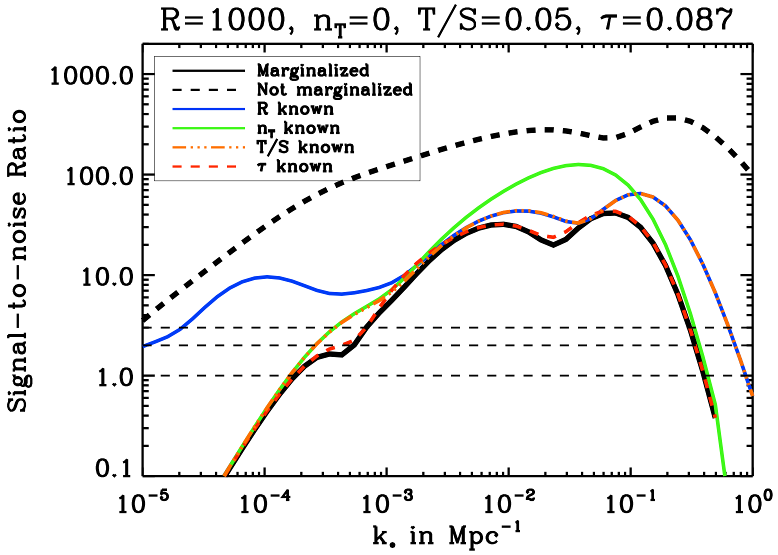

Figure 3: Signal-to-noise ratio for the detection of the bounce in the CMB B-mode for different marginalization options for a "B-POL" like mission.

If one wants to be rigorous and rely on a well establish formalism, only tensor modes are currently well understood in LQC. Primordial gravitational waves are a good candidate for testing LQC. This has been studied in several articles (see, *e.g.*, [11]). The key features can be understood easily with the plot from Fig. 2. The Hubble radius ($1/H$) is drawn in blue. The very small scales (lower black dashed curve) crosses the horizon only twice: they exit during inflation (the usual inflationary "plateau" reflects that $H \approx cst$) and re-enter later on during radiation or matter domination. This is the usual picture, leading to a nearly scale-invariant power spectrum. However, the large scales (upper black dashed curve) cross the horizon –and become frozen– in the contracting universe, therefore exhibiting the characteristic $P(k) \propto k^2$ Minkowski vacuum spectrum. The resulting B-mode power spectrum is k^2 suppressed in the IR limit and scale invariant in the UV limit. It also exhibits some oscillations between those regimes (due to causal contact at the bounce), see [7].

We have recently shown ([12]) that the transition wavenumber $k_\star$ between the standard and suppressed regimes mostly depends on the *initial conditions* at the bounce and *not* on the fundamental parameters of the theory. Basically, the wavenumber becomes large enough to be "observable" if the Universe is strongly dominated by kinetic energy at the bounce. This is consistent with the fact that backreaction is neglected in the approach. It physically means that the LQC effects can be seen if inflation did not last much more than required to solve the cosmological paradoxes. There are some arguments in favor of this, given in [13], but they have to be taken with care. Figure 3 gives the result of a Fisher analysis showing the range of $k_\star$ values that could be probed by the next-generation CMB experiments. The detectable range corresponds to a maximum value of the scalar field of the order of 3.3 M_{Pl} for $m = 10^{-6}$ M_{Pl}.

3. Inflation and LQC live (happily ?) together

My view is that the LQC-inflation paradigm is becoming "convincing". LQC (probably) generates inflation and inflation (possibly) allows us to test LQC. This is a tantalizing picture. Some important points nevertheless need to be investigated. First, scalar modes (and the resulting temperature power spectrum of the CMB) must be studied into the details. This is on the way ([14]) but computations are far from trivial as it is not straightforward to obtain an anomaly-free algebra in this case. Then, Inverse-Volume (IV) corrections should be included. All what has been said before is related to holonomy corrections only. This should not be very difficult and dramatic new effects are not expected as most of the observable features are associated with the bounce itself (which will basically remain the same with IR corrections) and not with subtle loopy corrections to the propagation of physical modes. Finally, and most importantly, inhomogeneities have to be taken into account as they are known to grow very fast during the contraction phase. This point, of course, questions the reliability of the picture.

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