Planck Scale Cosmology and Asymptotic Safety in Resummed Quantum Gravity

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In Weinberg's asymptotic safety approach, a finite dimensional critical surface for a UV stable fixed point generates a theory of quantum gravity with a finite number of physical parameters. We argue that, in an extension of Feynman's original formulation of the theory, we recover this fixed-point UV behavior from an exact re-arrangement of the respective perturbative series. Our results are consistent with the exact field space Wilsonian renormalization group results of Reuter *et al.* and with recent Hopf- algebraic Dyson-Schwinger renormalization theory results of Kreimer. We obtain the first "first principles" predictions of the dimensionless gravitational and cosmological constants and our results support the Planck scale cosmology of Bonanno and Reuter. We conclude with an estimate for the currently observed value of the cosmological constant.

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1. Introduction

In Ref. [1], Weinberg suggested that the general theory of relativity may have a non-trivial UV fixed point, with a finite dimensional critical surface in the UV limit, so that it would be asymptotically safe with an S-matrix that depends on only a finite number of observable parameters. In Refs. [2–4], strong evidence has been calculated using Wilsonian [5] field-space exact renormalization group methods to support asymptotic safety for the Einstein-Hilbert theory. We have shown in Refs. [6, 7] that the extension of the amplitude-based, exact resummation theory of Ref. [8] to the Einstein-Hilbert theory (we call the extension resummed quantum gravity) leads to UV fixed-point behavior for the dimensionless gravitational and cosmological constants, but with the bonus that the resummed theory is actually UV finite. More evidence for asymptotic safety for quantum gravity has been calculated using causal dynamical triangulated lattice methods in Ref. [9][1]. There is no known inconsistency between our analysis and Refs. [2–4, 9]. Our results are also consistent with the results on leg renormalizability of quantum gravity in Refs. [11]. Contact with experiment is now in order.

Specifically, in Ref. [12], it has been argued that the approach in Refs. [2–4] to quantum gravity may provide a realization[2] of the successful inflationary model [14, 15] of cosmology without the need of the inflaton scalar field: the attendant UV fixed point solution allows one to develop Planck scale cosmology that joins smoothly onto the standard Friedmann-Walker-Robertson classical descriptions so that one arrives at a quantum mechanical solution to the horizon, flatness, entropy and scale free spectrum problems. In Ref. [7], using the resummed quantum gravity theory [6], we recover the properties as used in Refs. [12] for the UV fixed point with "first principles" predictions for the fixed point values of the respective dimensionless gravitational and cosmological constants. Here, we carry the analysis one step further and arrive at a prediction for the observed cosmological constant Λ in the context of the Planck scale cosmology of Refs. [12]. We comment on the reliability of the result as well, as it will be seen already to be relatively close to the observed value [16]. More such reflections, as they relate to an experimentally testable union of the original ideas of Bohr and Einstein, will be taken up elsewhere [17].

The discussion is organized as follows. In the next section we review the Planck scale cosmology presented in Refs. [12]. In Section 3 we review our results [7] for the dimensionless gravitational and cosmological constants at the UV fixed point. In Section 4, we combine the Planck scale cosmology scenario [12] with our results to predict the observed value of the cosmological constant Λ.

2. Planck Scale Cosmology

More precisely, we recall the Einstein-Hilbert theory

$$\mathcal{L}(x) = \frac{1}{2\kappa^2}\sqrt{-g}\,(R - 2\Lambda) \tag{2.1}$$

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[1]We also note that the model in Ref. [10] realizes many aspects of the effective field theory implied by the anomalous dimension of 2 at the UV-fixed point but it does so at the expense of violating Lorentz invariance.

[2]The attendant scale choice $k \sim 1/t$ used in Refs. [12] was also proposed in Ref. [13].

where R is the curvature scalar, g is the determinant of the metric of space-time $g_{\mu\nu}$, Λ is the cosmological constant and $\kappa = \sqrt{8\pi G_N}$ for Newton's constant G_N. Using the phenomenological exact renormalization group for the Wilsonian [5] coarse grained effective average action in field space, the authors in Ref. [12] have argued that the attendant running Newton constant $G_N(k)$ and running cosmological constant $\Lambda(k)$ approach UV fixed points as k goes to infinity in the deep Euclidean regime: $k^2 G_N(k) \rightarrow g_*$, $\Lambda(k) \rightarrow \lambda_* k^2$ for $k \rightarrow \infty$.

The contact with cosmology then proceeds as follows. Using a phenomenological connection between the momentum scale k characterizing the coarseness of the Wilsonian graininess of the average effective action and the cosmological time t, $k(t) = \frac{\xi}{t}$ for $\xi > 0$, the authors in Refs. [12] show that the standard cosmological equations admit of the following extension: $(\frac{\dot{a}}{a})^2 + \frac{K}{a^2} = \frac{1}{3}\Lambda + \frac{8\pi}{3}G_N\rho$, $\dot{\rho} + 3(1+\omega)\frac{\dot{a}}{a}\rho = 0$, $\dot{\Lambda} + 8\pi\rho\dot{G_N} = 0$, $G_N(t) = G_N(k(t))$, and $\Lambda(t) = \Lambda(k(t))$ for the density ρ and scale factor $a(t)$ with the Robertson-Walker metric representation as $ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right)$ so that $K = 0, 1, -1$ correspond respectively to flat, spherical and pseudo-spherical 3-spaces for constant time t. The equation of state is $p(t) = \omega\rho(t)$ where p is the pressure.

Using the UV fixed points for g_* and λ_*, the authors in Refs. [12] show that the extended cosmological system given above admits, for $K = 0$, a solution in the Planck regime where $0 \leq t \leq t_{\text{class}}$, with t_{class} a "few" times the Planck time t_{Pl}, which joins smoothly onto a solution in the classical regime, $t > t_{\text{class}}$, which coincides with standard Friedmann-Robertson-Walker phenomenology but with the horizon, flatness, scale free Harrison-Zeldovich spectrum, and entropy problems all solved purely by Planck scale quantum physics. We now review the results in Refs. [7] for these UV limits and show how to use them to predict the current value of Λ.

3. g_* and λ_* in Resummed Quantum Gravity

We start with the prediction for g_*, which we already presented in Refs. [6, 7]. We have shown in Refs. [6] that the large virtual IR effects in the respective loop integrals for the scalar propagator in quantum general relativity can be resummed to the *exact* result $i\Delta'_F(k)|_{\text{resummed}} = \frac{ie^{B''_g(k)}}{(k^2 - m^2 - \Sigma'_s + i\varepsilon)}$ for $B''_g(k) = \frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{m^2}{m^2+|k^2|}\right)$, where this form holds for the UV regime, so that the resummed scalar propagator falls faster than any power of $|k^2|$. An analogous result [6] holds for m=0. As Σ'_s, the residual self-energy function, starts in $\mathcal{O}(\kappa^2)$, we may drop it in calculating one-loop effects. It follows that, when the respective analogs of $i\Delta'_F(k)|_{\text{resummed}}$ are used for the elementary particles, all quantum gravity loop corrections are UV finite [6].

When we use our resummed propagator results, as extended to all the particles in the SM Lagrangian and to the graviton itself, the denominator of the graviton propagator becomes [6] (M_{Pl} is the Planck mass) $q^2 + \Sigma^T(q^2) + i\varepsilon \cong q^2 - q^4\frac{c_{2,eff}}{360\pi M_{Pl}^2}$, for $c_{2,eff} = \sum_{\text{SM particles } j} n_j I_2(\lambda_c(j)) \cong 2.56 \times 10^4$ with I_2 given in Refs. [6] and with $\lambda_c(j) = \frac{2m_j^2}{\pi M_{Pl}^2}$. n_j is the number of effective degrees of freedom [6] of particle j of mass m_j. We take the SM masses as explained in Refs. [6, 7] following Refs. [16, 18–20]. We also note that from Ref. [21] it also follows that the value of n_j for the graviton and its attendant ghost is 42. We thus identify (we use G_N for $G_N(0)$) $G_N(k) = G_N/(1 + \frac{c_{2,eff}k^2}{360\pi M_{Pl}^2})$ so that $g_* = \lim_{k^2 \rightarrow \infty} k^2 G_N(k^2) = \frac{360\pi}{c_{2,eff}} \cong 0.0442$, a pure property of the known world.

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Turning now to λ_*, we use Einstein's equation $G_{\mu\nu} + \Lambda g_{\mu\nu} = -\kappa^2 T_{\mu\nu}$ in a standard notation where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$, $R_{\mu\nu}$ is the contracted Riemann tensor, and $T_{\mu\nu}$ is the energy-momentum tensor. Working with the representation $g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$ for the flat Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ we may isolate Λ in Einstein's equation by evaluating its VEV (vacuum expectation value). For any bosonic quantum field φ we use the point-splitting definition (here, $:\ :$ denotes normal ordering) $\varphi(0)\varphi(0) = \lim_{\varepsilon\to 0} \varphi(\varepsilon)\varphi(0) = \lim_{\varepsilon\to 0} T(\varphi(\varepsilon)\varphi(0)) = \lim_{\varepsilon\to 0}\{:\varphi(\varepsilon)\varphi(0): + <0|T(\varphi(\varepsilon)\varphi(0))|0>\}$ where the limit is taken with time-like $\varepsilon \equiv (\varepsilon,\vec{0}) \to (0,0,0,0) \equiv 0$ respectively. A scalar then makes the contribution [6] to Λ given by[3] $\Lambda_s = -8\pi G_N \frac{\int d^4k}{2(2\pi)^4} \frac{(2k_0^2)e^{-\lambda_c(k^2/(2m^2))\ln(k^2/m^2+1)}}{k^2+m^2} \cong -8\pi G_N [\frac{1}{G_N^2 64\rho^2}]$, where $\rho = \ln\frac{2}{\lambda_c}$, and a Dirac fermion contributes [6] -4 times Λ_s to Λ. The deep UV limit of Λ then becomes $\Lambda(k) \xrightarrow{k^2\to\infty} k^2 \lambda_*$, $\lambda_* = -\frac{c_{2,eff}}{2880} \sum_j (-1)^{F_j} n_j / \rho_j^2 \cong 0.0817$ where F_j is the fermion number of j and $\rho_j = \rho(\lambda_c(m_j))$. We see again that λ_* is a pure prediction of our known world – λ_* would vanish in an exactly supersymmetric theory. Our results for (g_*, λ_*) agree qualitatively with those in Refs. [12].

4. An Estimate of Λ

To estimate the value of Λ today, we take the normal-ordered form of Einstein's equation, $: G_{\mu\nu}: + \Lambda : g_{\mu\nu}: = -\kappa^2 : T_{\mu\nu}:$. The coherent state representation of the thermal density matrix then gives the Einstein equation in the form of thermally averaged quantities with Λ given by our result above in lowest order. Taking the transition time between the Planck regime and the classical Friedmann-Robertson-Walker regime at $t_{tr} \sim 25 t_{Pl}$ from Refs. [12], we introduce $\rho_\Lambda(t_{tr}) \equiv \frac{\Lambda(t_{tr})}{8\pi G_N(t_{tr})} = \frac{-M_{Pl}^4(k_{tr})}{64} \sum_j \frac{(-1)^F n_j}{\rho_j^2}$ and use the arguments in Refs. [23] (t_{eq} is the time of matter-radiation equality) to get the first principles estimate, from the method of the operator field, $\rho_\Lambda(t_0) \cong \frac{-M_{Pl}^4(1+c_{2,eff}k_{tr}^2/(360\pi M_{Pl}^2))^2}{64} \sum_j \frac{(-1)^F n_j}{\rho_j^2} [\frac{t_{tr}^2}{t_{eq}^2} \times (\frac{t_{eq}^{2/3}}{t_0^{2/3}})^3] \cong \frac{-M_{Pl}^2(1.0362)^2(-9.197\times 10^{-3})}{64} \frac{(25)^2}{t_0^2} \cong (2.400 \times 10^{-3} eV)^4$ where we take the age of the universe to be $t_0 \cong 13.7 \times 10^9$ yrs. In the latter estimate, the first factor in the square bracket comes from the period from t_{tr} to t_{eq} (radiation dominated) and the second factor comes from the period from t_{eq} to t_0 (matter dominated) [4]. This estimate should be compared with the experimental result [16][5] $\rho_\Lambda(t_0)|_{expt} \cong (2.368 \times 10^{-3} eV(1 \pm 0.023))^4$.

To sum up, our estimate, while it is definitely encouraging, is not a precision prediction, as possible hitherto unseen degrees of freedom have not been included and t_{tr} is not precise, yet. – We thank Profs. L. Alvarez-Gaume and W. Hollik for the support and kind hospitality of the CERN TH Division and the Werner-Heisenberg-Institut, MPI, Munich, respectively, where a part of this work was done.

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[3] We note the use here in the integrand of $2k_0^2$ rather than the $2(\vec{k}^2 + m^2)$ in Ref. [7], to be consistent with $\omega = -1$ [22] for the vacuum stress-energy tensor.

[4] The method of the operator field forces the vacuum energies to follow the same scaling as the non-vacuum excitations.

[5] See also Ref. [24] for an analysis that suggests a value for $\rho_\Lambda(t_0)$ that is qualitatively similar to this experimental result.

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