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Future limits on isotropic Lorentz violation in the photon sector from UHECRs and TeV gamma rays

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Present and future ultra-high-energy-cosmic-ray facilities (e.g., the Pierre Auger Observatory with South and North components) and TeV-gamma-ray telescope arrays (e.g., HESS or VERITAS and CTA) have the potential to set stringent indirect bounds on the nine Lorentz-violating parameters of nonbirefringent modified Maxwell theory minimally coupled to standard Dirac theory. Theoretically, the most interesting case is isotropic Lorentz violation, which is described by a single parameter [taken to vanish for the standard Lorentz-invariant theory]. It appears possible to obtain in the future an upper (lower) indirect bound on this single isotropic Lorentz-violating parameter at the $+10^{-21}$ $\left(-10^{-17}\right)$ level. Comparison is made with existing and future direct bounds from laboratory experiments. The possible physics implications of upper bounds at the 10^{-21} level are briefly discussed.

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1. Isotropic Lorentz violation in the photon sector

If Lorentz violation (LV) occurs somewhere in the theory of elementary particles and their interactions, then it can be expected to feed also into the photon sector. This makes the search of possible Lorentz-violating effects in the photon sector of substantial interest, especially as photons can be measured accurately and in a variety of physical systems.

Consider the isotropic modified Maxwell theory [1, 2] minimally coupled to the standard Dirac theory of a spin–$\frac{1}{2}$ particle with charge e and mass M. The Lagrange density of this particular modification ("deformation") of quantum electrodynamics (QED) is given by

$$\mathscr{L}_{\text{modQED}}[c,\widetilde{\kappa}_{\text{tr}},M,e] = \mathscr{L}_{\text{modMaxwell}}[c,\widetilde{\kappa}_{\text{tr}}] + \mathscr{L}_{\text{Dirac}}[c,M,e]\,, \tag{1.1a}$$

$$\mathscr{L}_{\text{modMaxwell}}[c,\widetilde{\kappa}_{\text{tr}}](x) = \tfrac{1}{2}\Big((1+\widetilde{\kappa}_{\text{tr}})\,|\mathbf{E}(x)|^2 - (1-\widetilde{\kappa}_{\text{tr}})\,|\mathbf{B}(x)|^2\Big)\,, \tag{1.1b}$$

with Cartesian spacetime coordinates $(x^\mu) = (ct,\, x^1,\, x^2,\, x^3)$ and the standard Lagrange density of a Dirac particle from the textbooks, some of which are listed in Ref. [3].

This theory is gauge-invariant, CPT–even, and power-counting renormalizable, but, for $\widetilde{\kappa}_{\text{tr}} \neq 0$, violates the Lorentz boost invariance while maintaining rotational invariance in a preferred reference frame. One possible reference frame is the one with an isotropic Cosmic Microwave Background. Here, though, the usual choice of the experimentalists is followed by employing the sun-centered celestial equatorial frame.

Two questions immediately arise. First, is the modified-QED theory theoretically consistent for all values of the parameter $\widetilde{\kappa}_{\text{tr}}$ or is there a restricted parameter domain? Second, the modified-QED theory (1.1) is formulated in a flat spacetime, but how about gravity? Very briefly, the answers are as follows. The theory (1.1) is consistent (i.e., has microcausality and unitarity) only for parameters in a restricted domain [3],

$$\widetilde{\kappa}_{\text{tr}} \in (-1,\, +1]\,. \tag{1.2}$$

As to gravity, the theory (1.1) can be coupled [4, 5] to an external gravitational field (fixed background spacetime metric) but not to a dynamic gravitational field (variable spacetime metric), as the energy-momentum tensor is generally not symmetric [4, 6].

Leaving the gravitational issue aside, return to the modified-QED theory (1.1) over a flat spacetime manifold and ask what parameter values of the single Lorentz-violating parameter $\widetilde{\kappa}_{\text{tr}}$ are *a priori* to be expected. It turns out that simple spacetime-foam models (see Sec. 6) can give positive values of order unity for this deformation parameter,

$$\widetilde{\kappa}_{\text{tr}}\Big|^{\text{naive theory}} = \mathrm{O}(1)\,. \tag{1.3}$$

This implies that already the most basic experimental tests of the effective photon theory (1.1b) have the potential to teach us something of the fundamental properties of spacetime.

2. Existing direct laboratory bounds

The first direct laboratory bound was obtained in 1938 by Ives and Stilwell at Bell Labs, USA, with the following approximate result [7]:

$$|\widetilde{\kappa}_{\text{tr}}| \lesssim 10^{-2}\,. \tag{2.1}$$

Over the years, this difficult experiment has been improved steadily. The two most recent results were obtained at the Max-Planck-Institut für Kernphysik (MPIK) in Heidelberg, Germany [8] and at the University of Western Australia (UWA) in Perth, Australia [9], giving, respectively, the following two–σ bounds:

$$|\widetilde{\kappa}_{\text{tr}}| < 2 \times 10^{-7}, \tag{2.2a}$$

$$|\widetilde{\kappa}_{\text{tr}}| < 3 \times 10^{-8}. \tag{2.2b}$$

In principle, this last direct laboratory bound can be improved by 4 orders of magnitude if cryogenic resonators are used [10, 11]. For completeness, also another type of laboratory bound [12] will be mentioned in the next section.

3. Indirect bounds from particle astrophysics

Following up on an early suggestion by Beall [13] $\big($and a later one by Coleman and Glashow [14]$\big)$, it is possible to obtain tight <u>indirect</u> bounds via particle astrophysics [15, 16]. The basic idea is remarkably simple [13, 14]:

(a) With modified dispersion relations, new decay channels appear which are absent in the standard relativistic theory.

(b) This leads to rapid energy loss of particles with energies above threshold [a generic LV parameter 'κ' typically gives a threshold energy $E_{\text{thresh}}(\kappa) \to \infty$ for $|\kappa| \to 0$].

(c) Observing these particles implies that they necessarily have energies at or below threshold [$E \leq E_{\text{thresh}}(\kappa)$], which, in turn, gives bounds on the LV parameters ('κ') of the theory.

In modified QED theory (1.1) with $\widetilde{\kappa}_{\text{tr}} \in (-1, 1]$, exact tree-level decay rates have been calculated for two processes [15], which occur for $\widetilde{\kappa}_{\text{tr}} \neq 0$ because of the difference between the maximum attainable velocity c of the Dirac particle and the photon velocity $v_\gamma = c\sqrt{(1-\widetilde{\kappa}_{\text{tr}})/(1+\widetilde{\kappa}_{\text{tr}})}$. The resulting bounds will be called 'indirect,' because they do not directly rely on the propagation properties of the photon but on indirect mass-shell effects, as is made clear by point (a) above.[1]

The first process, vacuum-Cherenkov radiation for $\widetilde{\kappa}_{\text{tr}} > 0$ (Fig. 1–left), is found to have the energy threshold

$$E_{\text{thresh}}^{(\text{a})} = M\sqrt{\frac{1+\widetilde{\kappa}_{\text{tr}}}{2\,\widetilde{\kappa}_{\text{tr}}}} = \frac{1}{\sqrt{2}}\,\frac{M}{\sqrt{\widetilde{\kappa}_{\text{tr}}}} + \mathrm{O}\Big(M\sqrt{\widetilde{\kappa}_{\text{tr}}}\Big), \tag{3.1a}$$

where the charged particle can be a proton p or heavy nucleus N, each, in first approximation, considered as a charged pointlike Dirac particle with mass $M = M_p$ or $M = M_N$. The vacuum-Cherenkov decay rate depends, of course, on the value of the electric charge of the Dirac particle ($|e|$ or $Z_N\,|e|$) but the energy threshold does not. In fact, the energy threshold (3.1a) simply follows from energy-momentum conservation. Still, it is important to know the radiation rate above threshold, in particular, to make sure that it is not strongly suppressed.

[1] Indirect bounds can also be obtained in the laboratory. For isotropic modified Maxwell theory, a remarkable bound [12] has been obtained from the apparent absence of nonstandard synchrotron-radiation losses at the Large Electron Positron (LEP) collider of CERN. This indirect laboratory bound will be listed in the summary table below.

Figure 1: Left: Vacuum-Cherenkov radiation $p^+ \to p^+\gamma$. Right: Photon decay $\gamma \to e^+e^-$.

The second process, photon decay for $\widetilde{\kappa}_{\text{tr}} < 0$ (Fig. 1–right), has a similar energy threshold:

$$E_{\text{thresh}}^{(b)} = 2M\sqrt{\frac{(1-\widetilde{\kappa}_{\text{tr}})}{-2\,\widetilde{\kappa}_{\text{tr}}}} = \sqrt{2}\,\frac{M}{\sqrt{-\widetilde{\kappa}_{\text{tr}}}} + \mathrm{O}\Big(M\sqrt{-\widetilde{\kappa}_{\text{tr}}}\Big), \tag{3.1b}$$

where the charged particles in the final state can be an electron and a positron, each considered as a charged pointlike Dirac particle with mass $M = M_e$.

Both decay rates and corresponding energy thresholds are well-behaved for parameter values in the domain (1.2).

4. Existing indirect earth-based bounds from particle astrophysics

The absence of vacuum-Cherenkov radiation for a particular ultra-high-energy-cosmic-ray (UHECR) event [17] from the Pierre Auger Observatory (Auger, for short) with $E_{\text{prim}} = (212 \pm 53)$ EeV implies $E_{\text{prim}} < E_{\text{thresh}}^{(a)}(\widetilde{\kappa}_{\text{tr}})$. Formula (3.1a), then, gives the following indirect two–σ upper bound [15]:

$$\widetilde{\kappa}_{\text{tr}} \;<\; +0.6 \times 10^{-19}\,, \tag{4.1a}$$

for a conservative mass value $M = M_{\text{Fe}} = 52$ GeV.

Similarly, the absence of photon decay for gamma-ray events [18] from HESS with $E_\gamma = (30 \pm 5)$ TeV implies $E_\gamma < E_{\text{thresh}}^{(b)}(\widetilde{\kappa}_{\text{tr}})$. Formula (3.1b), then, gives the following indirect two–σ lower bound [15]:

$$\widetilde{\kappa}_{\text{tr}} \;>\; -0.9 \times 10^{-15}\,, \tag{4.1b}$$

for photon decay into an electron-positron pair with an individual particle mass $M = M_e = 511$ keV. A similar bound can perhaps be obtained with appropriate gamma-ray events from VERITAS [19].

Equation (4.1a) or (4.1b) depends only on the inferred travel length of a meter or more for the primary at the top of the Earth's atmosphere and on the energy of this primary. As such, each is an earth-based bound, not an "astrophysical" bound. This type of bound does not depend on the precise (astronomical) origin of the primary nor on the actual distance between the source and the Earth. Specifically, bounds (4.1a)–(4.1b) rely on having detected primaries traveling over a few meters in the Earth's atmosphere and having measured their energy reliably. (The previous statements on these bounds are somewhat repetitive, but they hopefully dispel the considerable confusion in the literature about the nature of these indirect bounds.)

5. Future indirect earth-based bounds from particle astrophysics

According to (3.1), the bounds from Sec. 4 scale as $(M/E)^2$. This scaling behavior invites the following considerations.

In the future, it may be possible to obtain an appropriate Auger sample of UHECR events with

$$E_{\text{prim}} \stackrel{?}{=} (25 \pm 5)\ \text{EeV}\,, \quad M_{\text{prim}} \stackrel{?}{=} M_p \;=\; 0.938\ \text{GeV}\,. \tag{5.1a}$$

Similarly, it may be possible to obtain an appropriate Cherenkov Telescope Array [20] (CTA) sample of TeV-gamma-ray events with

$$E_\gamma \stackrel{?}{=} (3.0 \pm 0.5) \times 10^2\ \text{TeV}\,. \tag{5.1b}$$

These data samples would allow us to improve the previous two–σ bounds (4.1a)–(4.1b) by 2 orders of magnitude [16],

$$-\,0.9 \times 10^{-17} \stackrel{?}{<} \widetilde{\kappa}_{\text{tr}} \stackrel{?}{<} +1.0 \times 10^{-21}\,, \tag{5.2}$$

again with $M = M_e = 0.511$ MeV for the lower bound. The question marks in (5.2) are a reminder that the samples (5.1) are not yet available.

These potential future bounds, together with the existing ones, are summarized in Table 1.

6. Outlook

What do the existing and future bounds on $\widetilde{\kappa}_{\text{tr}}$ from Table 1 imply physically? Based on very general arguments (Einstein's dynamic spacetime manifold and Heisenberg's quantum-mechanical uncertainty relations), Wheeler [21] has argued that "quantum spacetime" must have a nontrivial small-scale structure. Moreover, it is to be expected that this must leave <u>some</u> remnants ("defects") in the effective classical spacetime manifold relevant over sufficiently large length scales.

Already for rather naive Swiss-cheese-type classical-spacetime models [22], it has been found that the photon propagation is modified and corresponds to an isotropic modified Maxwell theory (1.1b) with a positive deformation parameter of order

$$\widetilde{\kappa}_{\text{tr}}\,\Big|^{\text{naive theory}} = \mathrm{O}\big(\widetilde{b}^4/\widetilde{l}^4\big) \geq 0\,, \tag{6.1}$$

Type of bound	$\widetilde{\kappa}_{\text{tr}}$	Experiment + Reference(s)
Existing, direct	$\pm 10^{-8}$	Laboratory: sapphire oscillators, UWA [9]
Existing, indirect	$\pm 5 \times 10^{-15}$	Laboratory: synchrotron losses, LEP (CERN) [12]
Existing, indirect	$\big(-10^{-15}, +10^{-19}\big)$	Particle astrophysics: HESS, Auger–S [15]
Future, direct	$\pm 10^{-12}\,?$	Laboratory: cryogenic resonators [10, 11]
Future, indirect	$\big(-10^{-17}\,?, +10^{-21}\,?\big)$	Particle astrophysics: CTA, Auger–S+N [16]

Table 1: Orders of magnitude for existing and future two–σ bounds on the Lorentz-violating parameter $\widetilde{\kappa}_{\text{tr}}$ of isotropic modified Maxwell theory coupled to standard Dirac theory from laboratory and particle-astrophysics experiments (the last experiments only refer to processes occurring in the Earth's atmosphere).

for a typical defect ("hole") size $\widetilde{b}$ and a typical separation $\widetilde{l}$ between the individual defects (holes) randomly embedded in Minkowski spacetime. Equally important, the same defects do not modify the maximum velocity of the Dirac particle, at least to leading order [22]. This implies that the modified QED theory (1.1) corresponds to the effective theory of standard photons and Dirac particles propagating over a Swiss-cheese-type classical spacetime. In principle, it may be that the defects (holes) have sizes and separations related by $\widetilde{b} \lesssim \widetilde{l} \lesssim l_{\text{Planck}} \equiv \sqrt{\hbar G/c^3} \approx 1.6 \times 10^{-35}$ m or by $\widetilde{b} \lesssim \widetilde{l} \lesssim l$ if "quantum spacetime" has a new fundamental length l as argued in Ref. [23].

From (6.1), the suggestion is that the physically relevant quantity is perhaps not $\widetilde{\kappa}_{\text{tr}} \geq 0$ but rather its quartic root,

$$\left(\widetilde{\kappa}_{\text{tr}}\right)^{1/4} \geq 0\,. \tag{6.2}$$

Taking values at the boundaries of the range of Table 1, observe that, on the one hand, the number $(10^{-8})^{1/4} = 10^{-2}$ is small but not very small and, on the other hand, the number $(10^{-20})^{1/4} = 10^{-5}$ really is very small [of the same order as the ratio of the nucleus radius over the atomic radius].

Particle astrophysics thus provides a null experiment suggesting that spacetime is unexpectedly smooth [quantified as $\widetilde{b}/\widetilde{l} \lesssim 10^{-5}$ for the effective parameters $\widetilde{b}$ and $\widetilde{l}$ mentioned below (6.1)]. Perhaps this null experiment from particle astrophysics will turn out to be as important as the Michelson–Morley experiment [24], which led to Einstein's radically new concept of the relativity of simultaneity and the special theory of relativity [25]. Also in our case, it appears that radically new concepts are needed to understand the nature of what we call, for convenience, "quantum spacetime" but which may have an entirely novel content.

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