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On the relativistic Feynman-Metropolis-Teller equation of state and general relativistic white-dwarfs

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The recently formulation of the relativistic Thomas-Fermi model for compressed atoms is applied to the study of white-dwarf equilibrium configurations in the framework of general relativity. The equation of state is obtained as a function of the compression by considering each atom constrained in a Wigner-Seitz cell and it takes into account the beta equilibrium and the Coulomb interaction between the nuclei and the surrounding electrons. The consequences on the numerical value of the Chandrasekhar-Landau mass limit are presented as well as the modifications to the mass-radius relation for ^{4}He and ^{56}Fe white-dwarf equilibrium configurations.

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1. Introduction

Recently, the study of a compressed atom has been revisited in [1] by extending to special relativity the powerful global approach of Feynman, Metropolis and Teller [2], which takes into account all the Coulomb contributions duly expressed relativistically without the need of any piecewise description. The relativistic Thomas-Fermi model has been solved by imposing in addition to the electromagnetic interaction also the weak interaction between neutrons, protons and electrons. This presents some conceptual differences with respect to previous approaches and can be used in order both to validate and to establish the limitations of previous approaches. We apply here the considerations presented in [1] of a compressed atom in a Wigner-Seitz cell to the description of a non-rotating white-dwarf in general relativity.

2. The Equation of State

We use the equation of state recently obtained in [1] which generalizes to relativistic regimes the treatment of Feynman, Metropolis and Teller of the compressed atom [2]. The white-dwarf matter is arranged in a Wigner-Seitz lattice composed of cells of radius $R_{\rm ws}$ filled by a relativistic gas of Z electrons in equilibrium with a nucleus of A nucleons inside a radius $R_c < R_{\rm ws}$

$$R_c = \Delta \lambda_\pi Z^{1/3}\,, \tag{2.1}$$

being $\lambda_\pi = \hbar/(m_\pi c)$. At nuclear density we have $\Delta \approx (r_0/\lambda_\pi)(A/Z)^{1/3}$ with $r_0 \approx 1.2$ fm.

The condition of equilibrium of the electrons in each Wigner-Seitz cell is expressed by

$$E_e^F = \sqrt{c^2 (P_e^F)^2 + m_e^2 c^4} - m_e c^2 - eV(r) > 0\,, \tag{2.2}$$

where V is the Coulomb potential and E_e^F is the Fermi energy of electrons inside the compressed cell.

The distribution of protons confined within the radius R_c is assumed as constant, thus

$$n_p(r) = \frac{Z}{\frac{4}{3}\pi R_c^3}\theta(r - R_c)\,, \tag{2.3}$$

where $\theta(r - R_c)$ denotes the Heaviside function. The electron density is given by

$$n_e(r) = \frac{(P_e^F)^3}{3\pi^2\hbar^3} = \frac{\left[\hat{V}^2(r) + 2m_e c^2 \hat{V}(r)\right]^{3/2}}{3\pi^2\hbar^3 c^3}\,, \tag{2.4}$$

where $\hat{V} = eV + E_e^F$.

Introducing $x = r/\lambda_\pi$, $x_c = R_c/\lambda_\pi$, $\chi/r = \hat{V}(r)/(\hbar c)$ we obtain from the Poisson equation the relativistic Thomas-Fermi equation

$$\frac{1}{3x}\frac{d^2\chi}{dx^2} = -\frac{\alpha}{\Delta^3}\theta(x_c - x) + \frac{4\alpha}{9\pi}\left[\frac{\chi^2}{x^2} + 2\frac{m_e}{m_\pi}\frac{\chi}{x}\right]^{3/2}\,, \tag{2.5}$$

which is integrated subjected to the boundary conditions $\chi(0) = 0$, $\chi(x_{\rm ws}) \geq 0$ and $d\chi/dx|_{x=x_{\rm ws}} = \chi(x_{\rm ws})/x_{\rm ws}$.

The neutron density $n_n(r)$ is determined from the condition of beta equilibrium

$$\sqrt{c^2(P_n^F)^2+m_n^2c^4}=\sqrt{c^2(P_p^F)^2+m_p^2c^4}+m_nc^2-m_pc^2+eV(r)+E_e^F\,, \tag{2.6}$$

from which we can obtain self-consistently the onset for inverse beta-decay of a given nuclear composition (Z,A).

The pressure at the cell boundary is then a function of the compression level determined by the Wigner-Seitz cell dimensionless radius $x_{\rm ws}$ and therefore, a non-analytic pressure-density relation is derived (see [1] for details). In Fig. 1 we show the electron number density inside a Wigner-Seitz cell for a selected compressed atom of Iron. The electron distribution around the nucleus is well different from the one given by the uniform approximation of the electron fluid. It is worth to note that the electron density at the Wigner-Seitz cell boundary is smaller than the one given by the uniform electron density case (see Fig. 1). Then, for a given density, the pressure obtained from the self-consistent relativistic Feynman-Metropolis-Teller treatment is necessarily smaller than the one obtained in the uniform electron density case.

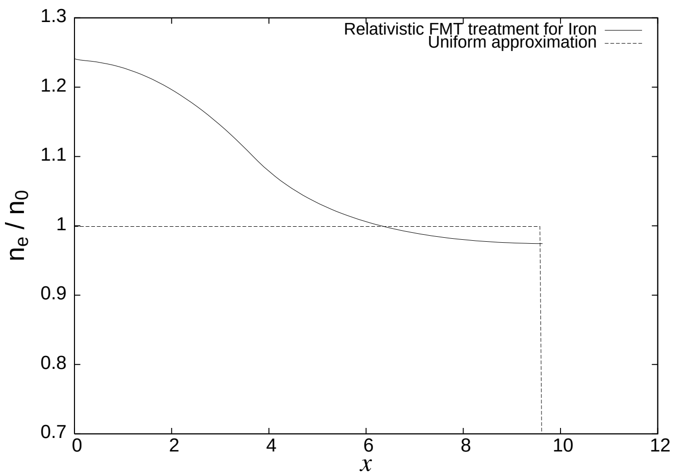

Figure 1: The electron number density n_e in units of the average electron number density $n_0=3Z/(4\pi R_{\rm ws}^3)$ is plotted as a function of the dimensionless radial coordinate $x=r/\lambda_\pi$ for $x_{\rm ws}=9.7$ in both the relativistic Feynman-Metropolis-Teller approach and the uniform approximation respectively for Iron. The electron distribution for different levels of compression and for different nuclear compositions can be found in [1].

3. General relativistic structure equations

Outside each Wigner-Seitz cell the system is electrically neutral, thus no overall electric field exists [1]. Therefore, the equation of state can be used to calculate the structure of the star through the Einstein equations. Introducing the spherically symmetric metric

$$ds^2=e^{\nu(r)}c^2dt^2-e^{\lambda(r)}dr^2-r^2d\theta^2-r^2\sin^2\theta d\varphi^2\,, \tag{3.1}$$

the Einstein equations can be written as

$$\frac{d\nu(r)}{dr} = \frac{2G}{c^2}\frac{4\pi r^3 P(r)/c^2 + M(r)}{r^2\left[1-\frac{2GM(r)}{c^2 r}\right]}, \tag{3.2}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \frac{\mathcal{E}(r)}{c^2}, \tag{3.3}$$

$$\frac{dP(r)}{dr} = -\frac{1}{2}\frac{d\nu(r)}{dr}\left[\mathcal{E}(r)+P(r)\right], \tag{3.4}$$

where $e^{\lambda(r)} = 1-2GM(r)/(c^2 r)$, $\mathcal{E}(r)$ is the energy-density and $P(r)$ is the total pressure.

4. Mass and radius of general relativistic stable white-dwarfs

In Figs. 2 and 3 we show the total mass of ^{4}He and ^{56}Fe white dwarfs as a function of the central density and the total radius respectively. The results correspond to the numerical integration of the Einstein equations (3.2)–(3.4) for the equation of state obtained from the relativistic Feynman-Metropolis-Teller treatment described in Sec. 2.

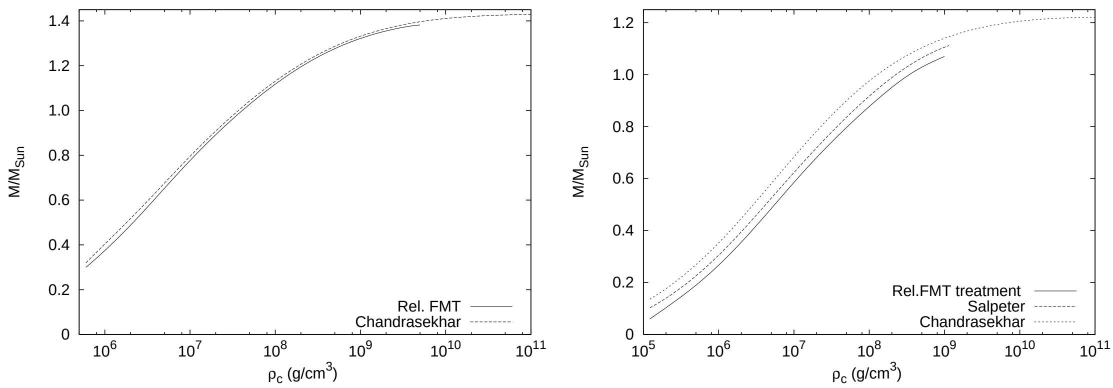

Figure 2: Mass in solar masses as a function of the central density in g cm^{-3} for ^{4}He (left) and ^{56}Fe (right) white-dwarfs. The solid curve corresponds to the general relativistic white-dwarfs using the equation of state given by the relativistic Feynman-Metropolis-Teller approach. The dashed curve corresponds to the white-dwarfs of Hamada and Salpeter [3] while, the dotted curve, are the corresponding Newtonian ^{56}Fe white-dwarfs of Chandrasekhar [4].

The value of the critical mass and the radius of white-dwarfs in our treatment and in the Hamada and Salpeter [3] treatment is a function of the nuclear composition of the star. We have given here specific examples in the limiting cases of ^{4}He and ^{56}Fe and the results of Chandrasekhar, of Salpeter and ours have been compared and contrasted (see Table 1 and Figs. 2 and 3).

References

[1] M. Rotondo, Jorge A. Rueda, R. Ruffini and S.-S. Xue, Phys. Rev. C, in press.

[2] R. P. Feynman, N. Metropolis and E. Teller, Phys. Rev. **75**, 1561 (1949).

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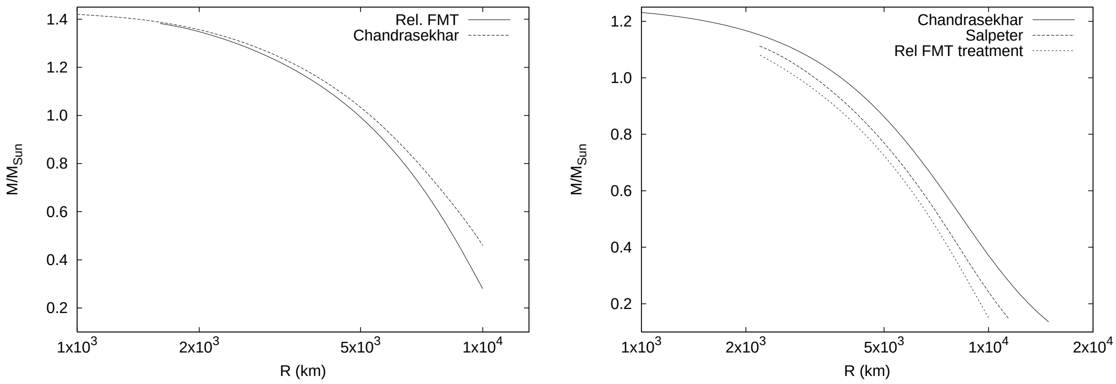

Figure 3: Mass in solar masses as a function of the radius in km for ^{4}He (left) and ^{56}Fe (right) white-dwarfs. The solid curve corresponds to the general relativistic white-dwarfs using the equation of state given by the relativistic Feynman-Metropolis-Teller approach. The dashed curve corresponds to the white-dwarfs of Hamada and Salpeter [3] while, the dotted curve, are the corresponding Newtonian ^{56}Fe white-dwarfs of Chandrasekhar [4].

	$M_{\rm crit}^{\rm Stoner}$	$M_{\rm crit}^{\rm Ch-L}$	$M_{\rm crit}^{\rm H\&S}$	$M_{\rm crit}^{\rm FMTrel}$
^{4}He	1.73	1.44	1.42	1.38
^{56}Fe	1.49	1.24	1.11	1.08

Table 1: Critical mass of ^{4} He and ^{56} Fe white-dwarfs in solar masses. $M_{\rm crit}^{\rm Stoner}$ denotes the Stoner critical mass, the Chandrasekhar-Landau limiting mass is $M_{\rm crit}^{\rm Ch-L}$ while the one of Hamada and Salpeter is denoted by $M_{\rm crit}^{\rm H\&S}$. The critical mass obtained in the present work is denoted by $M_{\rm crit}^{\rm FMTrel}$.