PROCEEDINGS OF SCIENCE

On the structure of the crust of neutron stars

Riccardo Belvedere, Jorge A. Rueda, Remo Ruffini and S.-S. Xue
Dipartimento di Fisica and ICRA, Sapienza Università di Roma, P.le Aldo Moro 5, I–00185 Rome, Italy
ICRANet, P.zza della Repubblica 10, I–65122 Pescara, Italy
E-mail: riccardo.belvedere@icra.it, jorge.rueda@icra.it, ruffini@icra.it, xue@icra.it

We calculate the mass and the thickness of neutron star crusts corresponding for different neutron star core mass-radius relations. The system of equilibrium equations, taking into account quantum statistics, electro-weak, and strong interactions, is formulated within the framework of general relativity in the non-rotating spherically symmetric case. The core is assumed to be composed of interacting degenerate neutrons, protons and electrons in beta equilibrium. The strong interaction between nucleons is modeled through sigma-omega-rho meson exchange in the context of the extended Walecka model.

25th Texas Symposium on Relativistic Astrophysics - TEXAS 2010
December 06-10, 2010
Heidelberg, Germany

PoS(Texas 2010)270

1. Introduction

In absence of any external field, thermodynamic equilibrium demands in addition to the constancy of the temperature the constancy of the particle chemical potential throughout the configuration. In presence of an external field, such a condition becomes [1] $\mu_0 + U =$ constant, where U denotes the external potential and μ_0 is the free-particle chemical potential. The extension of these equilibrium conditions to the case of general relativity were obtained by O. Klein [2], who investigated the thermodynamic equilibrium conditions of a self-gravitating one-component fluid of non-interacting neutral particles in spherical symmetry. The generalization of the Klein's equilibrium conditions to the case of a multicomponent fluid of non-interacting neutral particles was given by T. Kodama and M. Yamada [3]. E. Olson and M. Bailyn [4] went one step further obtaining the equilibrium conditions for a self-gravitating multicomponent fluid of charged particles taking into account the Coulomb interaction. The generalization of all these works when strong interactions are present has been recently accomplished by D. Pugliese et al. [5] assuming nuclear matter composed of interacting degenerate neutrons, protons and electrons in beta equilibrium. A general relativistic Thomas-Fermi treatment of nuclear matter within the framework of quantum statistics and of the general relativistic field theory for the gravitational, the electromagnetic and the hadronic fields has been there constructed. The constancy of the general relativistic Fermi energy of particles

$$E_n^F = \sqrt{g_{00}}\mu_n + g_\omega \omega - g_\rho \rho, \tag{1.1}$$

$$E_p^F = \sqrt{g_{00}}\mu_p + g_\omega \omega + g_\rho \rho + eV, \tag{1.2}$$

$$E_e^F = \sqrt{g_{00}}\mu_e - eV, \tag{1.3}$$

throughout the entire configuration has been there demonstrated in complete generality. Here g_{00} is the 00 component of the metric tensor, μ_i is the particle chemical potential and we adopt units with $\hbar = c = 1$. The nuclear interaction is introduced through the Walecka model (or quantum hadrodynamical model) [6, 7], in which the strong interaction is modeled by the exchange of the sigma, omega and rho meson-fields. The Coulomb potential is denoted by V and e stands for the fundamental charge. σ is an isoscalar meson field that provides the attractive 'long' range part of the nuclear force, ω is a massive vector field that provides the repulsive 'short' range part of the nuclear force, and ρ is the massive isovector field, that accounts for the isospin contribution. The coupling constants g_s, g_ω and g_ρ and the meson masses m_σ, m_ω and m_ρ are fixed by fitting experimental properties of nuclei.

The request of the constancy of the general relativistic Fermi energy for all particle-species brings to a neutron star equilibrium configurations quite different with respect to the ones traditional constructed. Some comments are here appropriate. In the construction of neutron star configurations has been traditionally assumed what is called local charge neutrality condition $n_e(r) = n_p$, namely that the electron and proton number densities are exactly the same at each point of the configuration. It has been recently showed that such a condition violates the above conditions of equilibrium of particles [8] and therefore instead of local charge neutrality, global charge neutrality $N_e = N_p$ has been imposed there, being N_e and N_p the total number of electrons and protons respectively.

Due to the neutrality of the crust (see e.g. [9]), global charge neutrality must be guaranteed at the edge of the crust. The Coulomb potential energy inside the core of the neutron star is found to be $\sim m_\pi c^2$ and related to this there is an internal electric field of order $\sim 10^{-14} E_c$ where $E_c = m_e^2 c^3/(e\hbar)$ is the critical field for vacuum polarization. At the core-crust boundary the continuity of all general relativistic particle Fermi energies guarantee a self-consistent matching of the core and the crust. As a consequence of such boundary conditions a core-crust transition surface of thickness $\gtrsim \hbar/(m_e c)$ is developed in which an overcritical electric field appears [10]. The neutron and proton densities decrease sharply there due to the nuclear surface tension and the electron density also decreases fast and match continuously the electron density at the edge of the crust. The continuity of the electron Fermi energy puts stringent limits on the density we might have at the edge of the crust: the variation of electron chemical potential at the core-crust boundary $\mu_e(\text{core}) - \mu_e(\text{crust})$ must be necessarily of order $eV \sim m_\pi c^2$. Therefore a suppress of the so-called inner crust of the neutron star is possible if $\mu_e(\text{crust}) \sim \mu_e(\text{core}) - eV \lesssim 25$ MeV, which is approximately the value of the electron chemical potential at the neutron drip point (see e.g. [11]). We analyze here the structure of neutron star crusts composed only by what is currently known as outer crust, namely a crust with edge density $\sim 4.3 \times 10^{11}$ g/cm^3. In this article we focus on the structure of the crust, however, it is worth to recall that to each of these configurations a core-crust transition surface with a very rich electrodynamical structure is associated.

We obtain the mass and the radius of the core of neutron stars for the NL3 [12], NL-SH [13], TM1 [14] and TM2 [15] parameterizations of the Walecka model. For such models we construct the corresponding crusts using two selected equations of state for the crust matter. The first one assumes a uniform free-electron fluid model and therefore neglects the Coulomb interactions between the degenerate electrons and the nuclear component of fixed charge to mass ratio Z/A; the second one is due to G. Baym et al. [16] which takes into account the Coulomb interaction between point-like nuclei with a uniform fluid of degenerate electrons and the nuclear masses are obtained from experimental data. The aim of this article is to compare and contrast the mass and the thickness of the crust obtained with these two different EoS and with the different parameterizations of the Walecka model.

2. Equilibium Equations of the Core

We consider non-rotating neutron stars. Introducing the non-rotating spherically symmetric spacetime metric

$$ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2, \tag{2.1}$$

The zero-covariant component of the conserved currents within the mean-field approximation are given by

$$J_0^{ch} = n_{ch} u_0 = (n_p - n_e) e^{\nu/2}, \tag{2.2}$$

$$J_0^{\omega} = n_b u_0 = (n_n + n_p) e^{\nu/2}, \tag{2.3}$$

$$J_0^{\rho} = n_3 u_0 = (n_p - n_n) e^{\nu/2}, \tag{2.4}$$

where u^α denotes the four-velocity and n_b, n_s are the baryon and scalar densities, $n_3 = n_p - n_n$ being n_i the particles densities of the i-specie.

The Einstein-Maxwell-Dirac equations are given by

$$e^{-\lambda(r)}\left(\frac{1}{r^2}-\frac{\lambda'}{r}\right)-\frac{1}{r^2}=-8\pi G T_0^0, \tag{2.5}$$

$$e^{-\lambda(r)}\left(\frac{1}{r^2}+\frac{\nu'}{r}\right)-\frac{1}{r^2}=-8\pi G T_1^1, \tag{2.6}$$

$$V''+V'\left[\frac{2}{r}-\frac{(\nu'+\lambda')}{2}\right]=-e^{\lambda}eJ_{ch}^0, \tag{2.7}$$

$$\partial_\sigma U(\sigma)+g_s n_s=0, \tag{2.8}$$

$$g_\omega J_\omega^0-m_\omega^2\omega=0, \tag{2.9}$$

$$g_\rho J_\rho^0-m_\rho^2\rho=0, \tag{2.10}$$

$$e^{\nu/2}\mu_e-eV=\text{constant}, \tag{2.11}$$

$$e^{\nu/2}\mu_p+eV+C_\omega n_B+C_\rho n_3=\text{constant}, \tag{2.12}$$

$$\mu_n-\mu_p-\mu_e-2C_\rho n_3=0, \tag{2.13}$$

where $C_i=(g_i/m_i)^2$ and the components of the energy-momentum tensor are: $T_0^0=\mathscr{E}+(E^2/8\pi)+U_\sigma+(1/2)C_\omega n_b^2+(1/2)C_\rho n_3^2$, $T_1^1=-P+(E^2/8\pi)+U_\sigma-(1/2)C_\omega n_b^2+(1/2)C_\rho n_3^2$, where $\mathscr{E}$ and P are the total energy-density and pressure, $U_\sigma=(1/2)m_\sigma^2\sigma^2+(1/3)g_2\sigma^3+(1/4)g_3\sigma^4$ is the self interaction scalar field potential, quartic-order polynom for a renormalizable theory [17, 18, 19], being g_2 and g_3 the third and fourth order constants of the self-scalar interactions.

In Fig. 1 we show the results of the integration of the above equations for the NL3 [12], NL-SH [13], TM1 [14] and TM2 [15] parameterizations of the Walecka model.

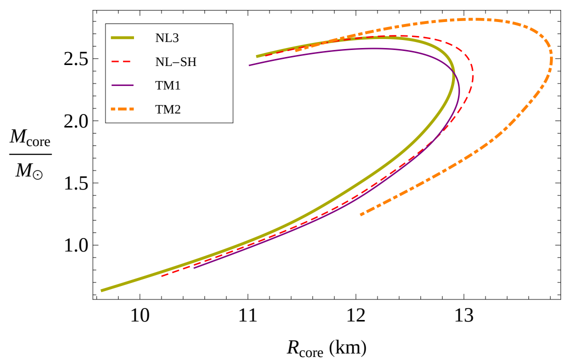

Figure 1: Mass-radius relation of neutron star cores for selected parameterizations of the Walecka model.

3. Equilibium Equations of the Crust

Due to the neutrality of the crust (see e.g. Rotondo et al. [9]) the structure equations to be integrated in the crust are just the Tolman-Oppenheimer-Volkoff equations

$$\frac{dP}{dr}=-\frac{G(\mathscr{E}+P)(m+4\pi r^3 P)}{r^2(1-\frac{2Gm}{r})}, \tag{3.1}$$

$$\frac{dm}{dr}=4\pi r^2\mathscr{E}, \tag{3.2}$$

where $m = m(r)$ is the mass enclosed at the radius r.

For a given core, we integrate the above equations for two different models of the crust. The first one is based on the uniform approximation for the electron fluid. Here the electrons are considered as a fully degenerate free-gas described by a Fermi-Dirac statistic. Therefore no Coulomb interaction either between nuclei and electrons or between electrons is included here. In this first case the electromagnetic interactions are not taken into account and then it leads to an energy-density given by $\mathscr{E} = \xi m_n n_e$, were n_e is the number density of electrons and $\xi = A/Z$. The pressure is given by the pressure of degenerate relativistic electrons.

The second model is based on the EoS by G. Baym, C. Pethick and P. Sutherland (BPS). In this case the crust is divided into Wigner-Size cells; each one of these cells is composed by a point-like nucleus of charge $+Ze$ with A nucleons, surrounded by a uniformly distributed cloud of Z fully-degenerate electrons. The Coulomb interaction in this case is easily computed due to the assumption of uniformity of the electrons. The sequence of the equilibrium nuclides present at each density between 10^4 and 4.3×10^{11} g/cm^3 in the BPS EoS is obtained by looking by the nuclear composition that minimizes the energy. Then in this case the EoS is given by

$$P = P_e + \frac{1}{3} W_L n_N \tag{3.3}$$

$$\frac{E_{tot}}{n_b} = \frac{W_N + W_L}{A} + \frac{E_e(n_b Z/A)}{n_b} \tag{3.4}$$

where $W_N(A,Z)$ is the total energy of an isolated nucleus, including rest mass of the nucleons but not including any electron energy, W_L is the lattice energy (total Coulomb energy) per nucleus, E_e is the electron energy and the baryon chemical potential is given by $\mu = (W_N + \frac{4}{3} W_L + Z\mu_e)/A$.

4. Crusts comparison

In Figs. 2 and 3 we show the mass and the thickness of the crusts obtained from the numerical integration of the structure equations for the two described EoS and for selected cores.

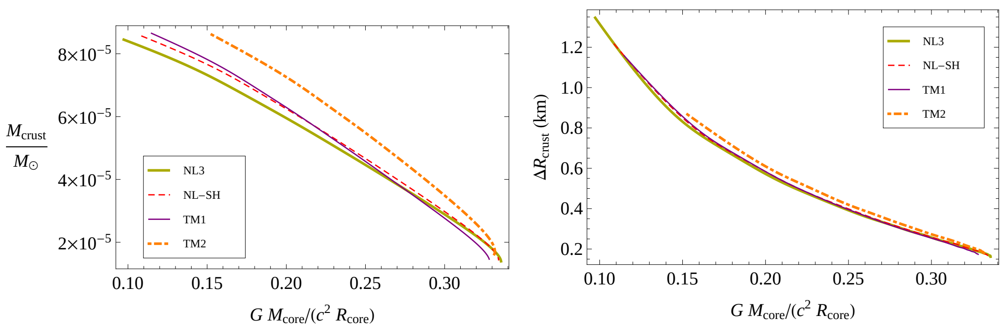

Figure 2: Mass (left panel) and thickness (right panel) of the crust for the EoS without Coulomb interaction as a function of the compactness of the core for selected parameterizations of the Walecka model.

From Figs. 2 and 3 it can be seen that we obtain systematically crusts with smaller mass and smaller thickness when Coulomb interactions are taken into account. The results are in line with

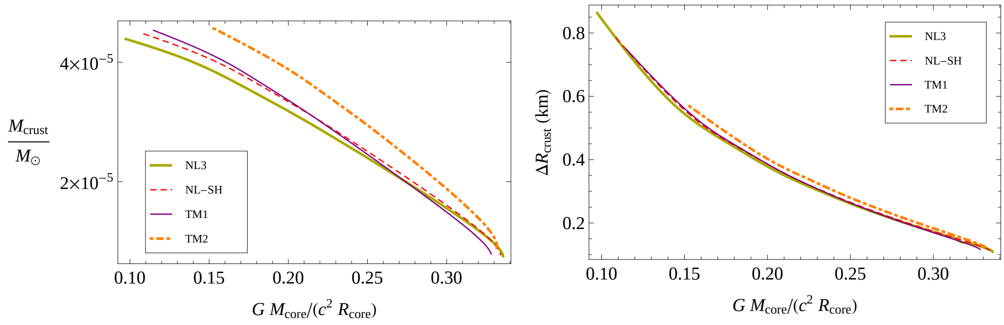

Figure 3: Mass (left panel) and thickness (right panel) of the crust for the BPS EoS as a function of the compactness of the core for selected parameterizations of the Walecka model.

the recent findings obtained by M. Rotondo et al. [20], where the mass-radius relation of white-dwarfs has been calculated using an equation of state that generalizes to relativistic regimes the Feynman-Metropolis-Teller model for compressed atoms [9].

In Table 1 we show the sequence of equilibrium nuclides present in the crust of the neutron star in the case of the BPS EoS for two selected neutron star cores: $M_{\rm core} = 2.558 M_\odot$, $R_{\rm core} = 12.795$ km and $M_{\rm core} = 1.354 M_\odot$ and $R_{\rm core} = 11.766$ km.

Equilibrium Nuclei Below Neutron Drip						
Nucleus	Z	ρ_{max}(g cm^{-3})	$\Delta\, R_1$ (km)	R.A.1(%)	$\Delta\, R_2$ (km)	R.A.2(%)
^{56}Fe	26	8.1×10^{6}	0.0165	7.566×10^{-7}	0.0064	6.969×10^{-7}
^{62}Ni	28	2.7×10^{8}	0.0310	0.00010	0.0121	0.00009
^{64}Ni	28	1.2×10^{9}	0.0364	0.00057	0.0141	0.00054
^{84}Se	34	8.2×10^{9}	0.0046	0.00722	0.0017	0.00683
^{82}Ge	32	2.2×10^{10}	0.0100	0.02071	0.0039	0.01983
^{80}Zn	38	4.8×10^{10}	0.1085	0.04521	0.0416	0.04384
^{78}Ni	28	1.6×10^{11}	0.0531	0.25635	0.0203	0.25305
^{76}Fe	26	1.8×10^{11}	0.0569	0.04193	0.0215	0.04183
^{124}Mo	42	1.9×10^{11}	0.0715	0.02078	0.0268	0.02076
^{122}Zr	40	2.7×10^{11}	0.0341	0.20730	0.0127	0.20811
^{120}Sr	38	3.7×10^{11}	0.0389	0.23898	0.0145	0.24167
^{118}Kr	36	4.3×10^{11}	0.0101	0.16081	0.0038	0.16344

Table 1: ρ_{max} is the maximum density at which the nuclide is present;$\Delta\, R_1$, $\Delta\, R_2$ and R.A.1(%), R.A.2(%) are rispectively the thickness of the layer where a given nuclide is present and their relative abundances in the outer crust for two different case, one with core mass$= 2.558 M_\odot$ and core radius$= 12.795$ km, and one with core mass$= 1.354 M_\odot$ and core radius$= 11.766$ km.

The average nuclear composition in the crust can be obtained by calculating the contribution of each nuclear composition to the mass of the crust with respect to the total crust mass. For the two different cores $M_{\rm core} = 2.558 M_\odot$, $R_{\rm core} = 12.795$ km and $M_{\rm core} = 1.354 M_\odot$, $R_{\rm core} = 11.766$ km (see Fig. reffig:fig4), we obtain as average nuclear composition $^{105}_{35}$Br. The corresponding crusts with fixed nuclear composition $^{105}_{35}$Br for the two chosen cores are calculated neglecting Coulomb interactions (i.e. using the first EoS). The mass and the thickness of these crusts with fixed $^{105}_{35}$Br are

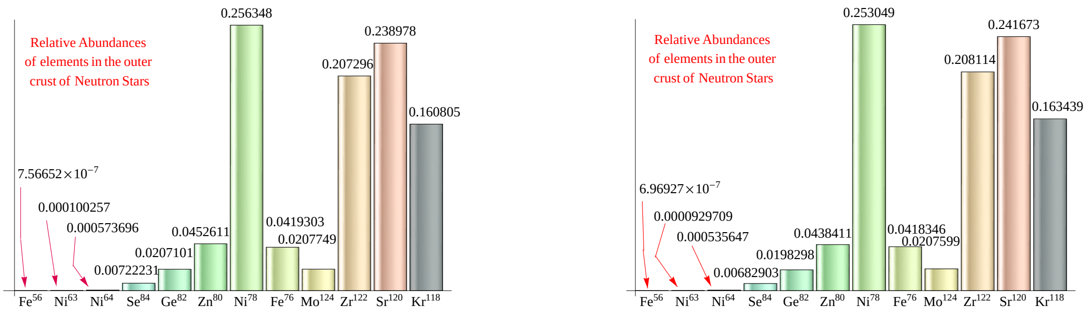

Figure 4: Relative abundances of chemical elements in the crust for the two cores analyzed in Table 1

different with respect to the ones obtained using the full BPS EoS leading to such average nuclear composition. For the two selected examples we obtain that the mass and the thickness of the crust with average $^{105}_{35}$Br are, respectively, 18% larger and 5% smaller with respect to the obtained with the corresponding BPS EOS.

References

[1] L. D. Landau and E. M. Lifshitz, Statistical Physics Part 1, Pergamon (1980).

[2] O. Klein, Rev. Mod. Phys. **21**, 531 (1949).

[3] T. Kodama and M. Yamada, Prog. Theor. Phys. **47**, 444 (1972).

[4] E. Olson and M. Bailyn, Phys. Rev. D **12**, 3030 (1975).

[5] D. Pugliese, Jorge A. Rueda, R. Ruffini and S.-S. Xue, submitted to the Proceedings of the 25th Texas Symposium on Relativistic Astrophysics (2010).

[6] H. P. Duerr, *Phys. Rev* **103**, 469 (1956).

[7] J. D. Walecka, *Annals. Phys* **83**, 491(1974).

[8] Jorge A. Rueda, M. Rotondo, R. Ruffini and S.-S. Xue, submitted to Phys. Rev. Lett. (2011).

[9] M. Rotondo, Jorge A. Rueda, R. Ruffini and S.-S. Xue, Phys. Rev. C, in press.

[10] D. Pugliese, Jorge A. Rueda, R. Ruffini and S.-S. Xue, to be submitted (2011).

[11] G. Baym, H. A. Bethe and C. J. Pethick, Nucl. Phys. A **175**, 225 (1971).

[12] G. A. Lalazissis, J. König, *Phys. Rev. C* **55**, 540 (1997).

[13] M. M. Sharma, M. A. Nagarajan and P. Ring, *Phys. Lett. B* **312**, 337 (1993).

[14] Y. Sugahara, H. Toki, *Nucl. Phys. A* **579**, 557 (1994).

[15] D. Hirata, H. Toki and I. Tanihata, *Nucl. Phys. A* **589**, 239 (1995).

[16] G. Baym, C. Pethick and P. Sutherland, *Ap. J.* **170**, 299 (1971).

[17] T. D. Lee, G. Wick, *Phys. Rev. D* **9**, 2291 (1974).

[18] T. D. Lee, *Rev. Mod. Phys.* **47**, 267 (1975).

[19] T. D. Lee, M. Margulies, *Phys. Rev. D* **11**, 1591 (1975).

[20] Jorge A. Rueda, M. Rotondo, R. Ruffini and S.-S. Xue, submitted to Phys. Rev. D (2011).