

A Discussion on the Role of Fermionic Supersymmetric Partners in Lorentz-Symmetry Breaking

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In this work, we present two possible venues to accommodate the K_F -type Lorentz-symmetry violating Electrodynamics in an $N = 1$ -supersymmetric framework. A chiral and a vector superfield are chosen to describe the background that signals Lorentz-symmetry breaking. In each case, the $K_{\mu\nu\kappa\lambda}$ -tensor is expressed in terms of the components of the background superfield that we choose to describe the breaking. We also present in detail the actions with all fermionic partners of the background that determine $K_{\mu\nu\kappa\lambda}$.

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1. Introduction

The Standard Model Extension (SME) is the natural framework to investigate properties of Lorentz-violation in physical systems involving possible extensions of Higgs mechanism. The SME is by-now a very stimulating research area and a great deal of results have been attained which include bounds on the photon mass [1], radiative corrections [2], systems with fermions [3], neutrinos [4], topological defects [5], topological phases [6], cosmic rays [7], particle decays [8], and other relevant aspects [9], [10]. The SME has also been used as a set-up to propose Lorentz-symmetry violation [11] and CPT- probing experiments [12], which have amounted to the imposition of stringent bounds on the Lorentz-symmetry violating (LV) coefficients.

To take into account how this violation is implemented, in the fermion sector of the SME, for example, there are two CPT-odd terms, $v_\mu \bar{\psi} \gamma^\mu \psi$, $b_\mu \bar{\psi} \gamma_5 \gamma^\mu \psi$, where v_μ, b_μ are the LV backgrounds [13], [14]. A similar study has also been developed for the case of a non-minimal coupling with the background, with new outcomes [15]. Atomic and optical physics are other areas in which Lorentz-symmetry violation has been intensively studied. Indeed, there are several works examining Lorentz-violation in electromagnetic cavities and optical systems [16], [17], which contributed to establish upper bounds on the LV coefficients.

The appearance of a more complex background, with the existence of an anisotropic vacuum at Planck scale has drawn the attention of particle and field theorists in recent years. Taking strings as the fundamental entities at this level, the idea that Lorentz-symmetry and CPT invariance might be spontaneously broken in some string theories became highly stimulating [21, 18]. Very recently, the surprising result that CPT-violating neutrino mass-squared difference would be an order of magnitude less than the current upper bound on CPT-violation in the sector of quarks and charged leptons has come to give more support to the proposal of Lorentz-symmetry violation [19], [20]. Actually, the idea of a CPT violation in the neutrino sector had been contemplated by Colladay, Kostelecky and Mewes [21].

Our main motivation to consider a supersymmetric scenario in connection with the violation of Lorentz-symmetry is based on fact that we adopt the viewpoint, according to [18], that the breaking of Lorentz-symmetry may be triggered by the vacuum condensation of a given tensor field. Nevertheless, at this scale where Lorentz-symmetry takes place, we may still have supersymmetry (SUSY) or, even if SUSY had already been broken down by some mechanism, supersymmetric partners are present and SUSY imprints are not lost. So, by adopting this scenario, we argue that Lorentz-symmetry violation, as originated at a more fundamental level in the string framework, cannot be dissociated from SUSY. We do not have however elements to decide whether SUSY breaking has occurred before Lorentz-symmetry violation, though the breaking of the latter signals the violation of the former. However, we would like to point out that, in the work of Ref. [32], Katz and Shadmi, present an interesting discussion in the rôle of vacuum expectation values that violate Lorentz symmetry as a possible to realise SUSY breaking by F - and D - terms in the MSSM.

To implement effective Lorentz-symmetry-violating actions in supersymmetric models, one may adopt a superspace formulation with Lorentz and, in some cases, CPT invariances violated by a fixed background in Wess-Zumino-like models [22]. In the gauge sector SUSY-preserving theories where discussed in [23]. Also in the gauge sector, the proposed [25] establishes a supersymmetric minimal extension for the Chern-Simons-like term [24] preserving the usual $(1 + 3)$ -dimensional

SUSY algebra. The breaking of SUSY will follow the very same route to Lorentz-symmetry breaking: the statement that v_μ is a constant shows that SUSY is also broken by the fermionic SUSY partner of v_μ .

Besides the CPT-odd terms, in the gauge sector, we have the CPT-even part, which is represented by a tensor $K^{\mu\nu\alpha\beta}$ that presents the same symmetries of the Riemann tensor, as well as an additional double-traceless condition [26]. In this scenario, we present two possibilities of constructing a supersymmetric version for the K -type models.

So, we propose to carry out the supersymmetric extension to the bosonic action below:

$$S = -\frac{1}{4} \int d^4x K_{\mu\nu\kappa\lambda} F^{\mu\nu} F^{\kappa\lambda}. \quad (1.1)$$

The “tensor” $K_{\mu\nu\kappa\lambda}$ it’s CPT even, i. e., it does not violate the CPT-symmetry. Despite CPT violation implies violation of Lorentz invariance [30], the reverse is not necessarily true. The action above is Lorentz-violating in the sense that the “tensor” $K_{\mu\nu\kappa\lambda}$ has a non-zero vacuum expectation value. That “tensor” presents the following symmetries:

$$K_{\mu\nu\kappa\lambda} = K_{[\mu\nu][\kappa\lambda]}, K_{\mu\nu\kappa\lambda} = K_{\kappa\lambda\mu\nu}, K_{\mu\nu}{}^{\mu\nu} = 0, \quad (1.2)$$

as usually appears in the literature, we can reduce the degrees of freedom take into account the ansatz [26]:

$$K_{\mu\nu\kappa\lambda} = \frac{1}{2} (\eta_{\mu\kappa} \tilde{\kappa}_{\nu\lambda} - \eta_{\mu\lambda} \tilde{\kappa}_{\nu\kappa} + \eta_{\nu\lambda} \tilde{\kappa}_{\mu\kappa} - \eta_{\nu\kappa} \tilde{\kappa}_{\mu\lambda}), \quad (1.3)$$

$$\tilde{\kappa}_{\mu\nu} = \kappa (\xi_\mu \xi_\nu - \eta_{\mu\nu} \xi^\alpha \xi_\alpha / 4), \quad (1.4)$$

$$\kappa = \frac{4}{3} \tilde{\kappa}^{\mu\nu} \xi_\mu \xi_\nu, \quad (1.5)$$

where $\tilde{\kappa}^{\mu\nu}$ is a traceless “tensor”. Using the ansätze (1.3), (1.4), in expression (1.1), we obtain,

$$S = \frac{\kappa}{4} \int d^4x \left\{ \frac{1}{2} \xi_\mu \xi_\nu F^\mu{}_\kappa F^{\kappa\nu} + \frac{1}{8} \xi_\lambda \xi^\lambda F_{\mu\nu} F^{\mu\nu} \right\}. \quad (1.6)$$

The supersymmetrization of the action (1.6), instead of (1.1), does not yield higher-spin components in the background dictated by SUSY. To realize the supersymmetrization, we have two possible routes: to achieve supersymmetrisation of (1.1) with $K_{\mu\nu\kappa\lambda}$ given by (1.3), without the particular expression (1.4) for $\tilde{\kappa}_{\mu\nu}$, we get that the background superfield must be chiral. On the other hand, if we are to supersymmetrise (1.1) with $K_{\mu\nu\kappa\lambda}$ given by (1.3) and $\tilde{\kappa}_{\mu\nu}$ given by (1.4), we conclude that a vector superfield must be used to accommodate the background vector, ξ^λ . The content of partners is richer in this case than in the case of the chiral background. The first route accommodate the chiral superfield as $\theta \sigma^\mu \bar{\theta} \partial_\mu S$, while the second, is carried out by a vector superfield accommodate this “vector” background as $\theta \sigma^\mu \bar{\theta} \xi_\mu$.

This work is organized as follows: In Section 2, we start with the model of Lorentz breaking proposed as $K_{\mu\nu\kappa\lambda}$ by Kosteleký. In this Section, we first study the possibility that the background tensor, $K_{\mu\nu\kappa\lambda}$, be originated from a background chiral superfield, which imposes a special form -

not yet discussed in the literature - for the K - tensor. Then in Section 3, we proceed by investigating the possibility that $K_{\mu\nu\kappa\lambda}$ originates from a (real) vector superfield, which exactly reproduces (1.1) with the K - tensor defined by means of (1.3), and (1.4). Finally, some Concluding Remarks are stated in Section 4.

2. First Proposal: Lorentz-symmetry Breaking by a Chiral Supermultiplet

Based on the work of Ref. [25], we take the idea that the background vector could originate from a chiral supermultiplet, Ω . As we shall see, this imposes on ξ_μ the constraint $\xi_\mu = \partial_\mu S$, where S is a complex scalar. Indeed, as it will become clear at the end of calculations, this choice of SUSY supermultiplet yields an interesting form for $K_{\mu\nu\kappa\lambda}$, completely fixed by a complex scalar field. We then show that the superspace action below shall accomplish the task of yielding the component field extension of the action of eq. (1.6).

Adopting covariant superspace-superfield formulation, we propose the following minimal extension for:

$$S = \kappa \int d^4x d^2\theta d^2\bar{\theta} \left\{ (D^\alpha \Omega) W_\alpha (\bar{D}_{\dot{\alpha}} \bar{\Omega}) \bar{W}^{\dot{\alpha}} + h.c. \right\}. \quad (2.1)$$

The superfields W_a , V , Ω and the SUSY-covariant derivatives D_a , $\bar{D}_{\dot{a}}$ hold the definitions:

$$D_a = \frac{\partial}{\partial \theta^a} + i \sigma_{a\dot{a}}^\mu \bar{\theta}^{\dot{a}} \partial_\mu, \quad (2.2)$$

$$\bar{D}_{\dot{a}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{a}}} - i \theta^a \sigma_{a\dot{a}}^\mu \partial_\mu; \quad (2.3)$$

from $\bar{D}_{\dot{b}} W_a(x, \theta, \bar{\theta}) = 0$, and $D^a W_a(x, \theta, \bar{\theta}) = \bar{D}_{\dot{a}} \bar{W}^{\dot{a}}(x, \theta, \bar{\theta})$, it follows that

$$W_a(x, \theta, \bar{\theta}) = -\frac{1}{4} \bar{D}^2 D_a V. \quad (2.4)$$

Its θ -expansion reads as below:

$$\begin{aligned} W_a(x, \theta, \bar{\theta}) = & \lambda_a(x) + i \theta^b \sigma_{b\dot{a}}^\mu \bar{\theta}^{\dot{a}} \partial_\mu \lambda_a(x) - \frac{1}{4} \bar{\theta}^2 \theta^2 \square \lambda_a(x) + \\ & + 2 \theta_a D(x) - i \theta^2 \bar{\theta}^{\dot{a}} \sigma_{a\dot{a}}^\mu \partial_\mu D(x) + \sigma^{\mu\nu}{}_a{}^b \theta_b F_{\mu\nu}(x) \\ & - \frac{i}{2} \sigma^{\mu\nu}{}_a{}^b \sigma_{b\dot{a}}^\alpha \theta^2 \bar{\theta}^{\dot{a}} \partial_\alpha F_{\mu\nu}(x) - i \sigma_{a\dot{a}}^\mu \partial_\mu \bar{\lambda}^{\dot{a}}(x) \theta^2, \end{aligned} \quad (2.5)$$

and $V = V^\dagger$. The Wess-Zumino gauge choice is taken as usually done:

$$V_{WZ} = \theta \sigma^\mu \bar{\theta} A_\mu(x) + \theta^2 \bar{\theta} \bar{\lambda}(x) + \bar{\theta}^2 \theta \lambda(x) + \theta^2 \bar{\theta}^2 D, \quad (2.6)$$

so the action (2.1) is gauge-invariant. The background superfield is so chosen to be a chiral one. Such a constraint restricts the maximum spin component of the background to be an $s = \frac{1}{2}$ component-field, showing up as a SUSY-partner for a spinless dimensionless scalar field. Also, one should notice that Ω has dimension of mass to -1 . The superfield expansion for Ω then reads:

$$\begin{aligned} \bar{D}_{\dot{\alpha}}\Omega(x, \theta, \bar{\theta}) &= 0, \\ \text{consequently } \Omega(x, \theta, \bar{\theta}) &= S(x) + \sqrt{2}\theta\zeta(x) + i\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}S(x) + \\ &+ \theta^2G(x) + \frac{i}{\sqrt{2}}\theta^2\bar{\theta}\bar{\sigma}^{\mu}\partial_{\mu}\zeta(x) - \frac{1}{4}\bar{\theta}^2\theta^2\Box S(x). \end{aligned} \quad (2.7)$$

With its SUSY transformations given by

$$\begin{aligned} \delta S &= \sqrt{2}\varepsilon^{\alpha}\zeta_{\alpha}, \\ \delta\zeta_{\alpha} &= \sqrt{2}G\varepsilon_{\alpha} + i\sqrt{2}\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\varepsilon}^{\dot{\alpha}}\partial_{\mu}S, \\ \delta G &= i\sqrt{2}\bar{\varepsilon}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}\partial_{\mu}\zeta_{\alpha}. \end{aligned} \quad (2.8)$$

Notice that, if we wish to have $\partial_{\mu}S$ constant (as we shall get in the sequel, a constant $\partial_{\mu}S$ give us a constant $K_{\mu\nu\kappa\lambda}$), so that S depends linearly on x^{μ} , SUSY is also broken by the background, as the expression for $\delta\zeta$ shows: if we apply a SUSY transformation on an Ω -superfield for which $S \neq 0$, $\zeta = 0$, $G = 0$, then $\delta\zeta \neq 0$ whenever $\partial_{\mu}S \neq 0$. So, to realise the breaking of Lorentz-symmetry in terms of an Ω with a constant $\partial_{\mu}S$, then SUSY is not an invariance of the background Ω . However, as it stands, we have an effective model which may descend from a more fundamental theory in which SUSY might be spontaneously broken. At this stage, we have no commitment with any specific mechanism for SUSY breaking.

The component-wise counterpart for the action (2.1) is as follows¹:

$$S = \kappa \int d^4x d^2\theta d^2\bar{\theta} \left\{ (D^{\alpha}\Omega)W_{\alpha}(\bar{D}_{\dot{\alpha}}\bar{\Omega})\bar{W}^{\dot{\alpha}} + h.c. \right\} = S_{boson} + S_{fermion} + S_{coupled}, \quad (2.9)$$

$$\begin{aligned} S_{boson} &= \kappa \int d^4x \left\{ -16 \left[\frac{1}{4}\partial_{\lambda}S\partial_{\mu}S^{*}(F^{\mu}{}_{\kappa}F^{\kappa\lambda} + F^{\lambda}{}_{\kappa}F^{\kappa\mu}) + \frac{1}{8}\partial_{\lambda}S\partial^{\lambda}S^{*}F_{\mu\nu}F^{\mu\nu} \right] + \right. \\ &+ 2D\varepsilon^{\lambda\mu\tau\rho}\partial_{\lambda}S\partial_{\mu}S^{*}F_{\tau\rho} - 2i\partial_{\lambda}S\partial_{\mu}S^{*}\varepsilon^{\lambda\tau\rho\kappa}F_{\tau\rho}F^{\mu}{}_{\kappa} + 8iD\partial_{\lambda}S\partial_{\mu}S^{*}F^{\lambda\mu} + \\ &\left. - 2D\partial_{\lambda}S\partial_{\mu}S^{*}\varepsilon^{\lambda\mu\nu\kappa}F_{\nu\kappa} + 8D^2\partial_{\mu}S\partial^{\mu}S^{*} + 16D^2|G|^2 + h.c. \right\}, \end{aligned} \quad (2.10)$$

$$\begin{aligned} S_{fermion} &= \kappa \int d^4x \left\{ \frac{1}{2}\partial_{\lambda}\zeta\sigma^{\mu}\partial_{\mu}\bar{\zeta}\lambda\sigma^{\lambda}\bar{\lambda} + \frac{1}{2}\partial_{\lambda}\zeta\sigma^{\mu}\bar{\lambda}\lambda\sigma^{\lambda}\partial_{\mu}\bar{\zeta} + 2\partial_{\mu}\zeta\partial^{\mu}\lambda\bar{\zeta}\bar{\lambda} + \right. \\ &- \frac{1}{2}\partial_{\lambda}\zeta\sigma^{\lambda}\partial_{\mu}\bar{\zeta}\lambda\sigma^{\mu}\bar{\lambda} - 2\partial_{\lambda}\zeta\sigma^{\lambda}\bar{\sigma}^{\mu}\partial_{\mu}\lambda\bar{\zeta}\bar{\lambda} - \frac{1}{2}\lambda\sigma^{\lambda}\bar{\sigma}^{\mu}\partial_{\lambda}\zeta\bar{\zeta}\partial_{\mu}\bar{\lambda} + \\ &\left. - \frac{1}{2}\zeta\lambda\bar{\zeta}\Box\bar{\lambda} - \zeta\lambda\partial_{\mu}\bar{\zeta}\bar{\sigma}^{\mu}\sigma^{\tau}\partial_{\tau}\bar{\lambda} + \frac{1}{2}\zeta\lambda\partial_{\mu}\bar{\lambda}\bar{\sigma}^{\nu}\sigma^{\mu}\partial_{\nu}\bar{\zeta} + \right. \end{aligned}$$

¹the ref. [31] has been used in our calculations

$$\begin{aligned}
& + \frac{1}{2} \partial_\mu \zeta \sigma^\mu \bar{\sigma}^\nu \lambda \bar{\zeta} \partial_\nu \bar{\lambda} - \frac{1}{2\sqrt{2}} \zeta \square \lambda \bar{\zeta} \bar{\lambda} - \frac{1}{2} \zeta \partial_\nu \lambda \partial_\mu \bar{\lambda} \bar{\sigma}^\mu \sigma^\nu \bar{\lambda} + \\
& - \frac{1}{2} \zeta \partial_\nu \lambda \partial_\mu \bar{\zeta} \bar{\sigma}^\nu \sigma^\mu \bar{\lambda} + \zeta \partial_\mu \lambda \bar{\zeta} \partial^\mu \bar{\lambda} - 2\zeta \sigma^\mu \partial_\mu \bar{\lambda} \bar{\zeta} \bar{\sigma}^\nu \partial_\nu \lambda + h.c. \Big\}, \quad (2.11)
\end{aligned}$$

$$\begin{aligned}
S_{coupled} = \kappa \int d^4x \Big\{ & -4iD\zeta \sigma^\mu \partial_\mu \bar{\zeta} - 2\sqrt{2}iDG^* \zeta \sigma^\mu \partial_\mu \bar{\lambda} + 2\sqrt{2}D\partial_\nu \lambda \sigma^\nu \bar{\sigma}^\mu \zeta \partial_\mu S^* + \\
& + 2D\zeta \sigma^\nu \partial_\mu \bar{\zeta} F_V^\mu + iD\varepsilon^{\tau\rho\mu\alpha} \zeta \sigma_\alpha \partial_\mu \bar{\zeta} F_{\tau\rho} + \sqrt{2}G^* \zeta \sigma^\mu \partial_\nu \bar{\lambda} F_\mu^\nu + \\
& + \frac{i}{\sqrt{2}} G^* \varepsilon^{\tau\rho\mu\alpha} \zeta \sigma_\alpha \partial_\mu \bar{\lambda} F_{\tau\rho} + \sqrt{2}i\zeta \sigma^\tau \bar{\sigma}^\nu \partial_\nu \lambda \partial_\mu S^* F_\tau^\mu + \\
& - \frac{1}{\sqrt{2}} \varepsilon^{\tau\rho\mu\alpha} \zeta \sigma_\alpha \bar{\sigma}^\nu \partial_\mu S^* \partial_\nu \lambda F_{\tau\rho} - 4\sqrt{2}iG^* D\zeta \sigma^\mu \partial_\mu \bar{\lambda} + \\
& + 2\sqrt{2}D\zeta \partial_\mu \lambda \partial^\mu S^* - \frac{i}{\sqrt{2}} \varepsilon^{\mu\nu\kappa\tau} \zeta \partial_\tau \lambda \partial_\mu S^* F_{\nu\kappa} + \frac{1}{2\sqrt{2}} \varepsilon^{\mu\nu\kappa\tau} \zeta \partial_\tau \lambda \partial_\mu S^* F_{\nu\kappa} + \\
& - 4iD^2 \bar{\zeta} \bar{\sigma}^\mu \partial_\mu \zeta - 2D\bar{\zeta} \bar{\sigma}^\nu \partial_\mu \zeta F_V^\mu + iD\varepsilon^{\nu\kappa\mu\alpha} \bar{\zeta} \bar{\sigma}_\alpha \partial_\mu \zeta F_{\nu\kappa} + \\
& + 2\sqrt{2}iDG^* \partial_\mu \zeta \sigma^\mu \bar{\lambda} + 2D\partial_\mu \zeta \sigma^\tau \bar{\zeta} F_\tau^\mu + iD\varepsilon^{\tau\rho\mu\alpha} \partial_\mu \zeta \sigma_\alpha \bar{\zeta} F_{\tau\rho} + \\
& + 2\partial_\mu \zeta \sigma^\tau \bar{\sigma}^{\nu\kappa} \bar{\zeta} F_{\nu\kappa} F_\tau^\mu + i\varepsilon^{\tau\rho\mu\alpha} \partial_\mu \zeta \sigma_\alpha \bar{\sigma}^{\nu\kappa} \bar{\zeta} F_{\tau\rho} F_{\nu\kappa} + \sqrt{2}\partial_\mu \zeta \sigma^\tau \bar{\lambda} F_\tau^\mu + \\
& + \frac{i}{\sqrt{2}} \varepsilon^{\tau\rho\mu\alpha} \partial_\mu \zeta \sigma_\alpha \bar{\lambda} F_{\tau\rho} - 2\sqrt{2}iGD\lambda \sigma^\mu \partial_\mu \bar{\zeta} + 2i|G|^2 \lambda \sigma^\mu \partial_\mu \bar{\lambda} + \\
& - 2i\partial_\nu \lambda \sigma^\nu \bar{\sigma}^\mu \lambda \partial_\mu S^* + 4\sqrt{2}iGD\partial_\mu \lambda \sigma^\mu \bar{\zeta} - 2\sqrt{2}iGD\bar{\zeta} \bar{\sigma}^\mu \partial_\mu \lambda \bar{\zeta} + \\
& - \sqrt{2}G\bar{\zeta} \bar{\sigma}^\mu \partial_\tau \lambda F_\mu^\tau + \frac{i}{\sqrt{2}} G\varepsilon^{\mu\nu\tau\alpha} \bar{\zeta} \bar{\sigma}_\alpha \partial_\tau \lambda F_{\mu\nu} - 2i|G|^2 \bar{\lambda} \bar{\sigma}^\mu \partial_\mu \lambda + \\
& + 2\sqrt{2}D\partial_\mu S \bar{\zeta} \partial^\mu \bar{\lambda} + \sqrt{2}i\partial_\mu (\bar{\lambda} \bar{\zeta}) \partial_\lambda S F^{\lambda\mu} - \frac{1}{\sqrt{2}} \varepsilon^{\mu\lambda\tau\rho} \partial_\mu (\bar{\lambda} \bar{\zeta}) \partial_\lambda S F_{\tau\rho} + \\
& - 2\sqrt{2}D\bar{\lambda} \partial_\mu \bar{\zeta} \partial^\mu S + 2\sqrt{2}D\bar{\zeta} \bar{\sigma}^\mu \sigma^\nu \partial_\nu \bar{\lambda} \partial_\mu S - \sqrt{2}i\bar{\zeta} \bar{\sigma}^\nu \sigma^\mu \partial_\mu \bar{\lambda} F_{\nu\lambda} \partial^\lambda S + \\
& - \frac{1}{\sqrt{2}} \varepsilon^{\nu\kappa\lambda\alpha} \bar{\zeta} \bar{\sigma}_\alpha \sigma^\mu \partial_\mu \bar{\lambda} F_{\nu\kappa} \partial_\lambda S + 2G^* \bar{\lambda} \bar{\sigma}^\mu \sigma^\nu \partial_\nu \bar{\lambda} \partial_\mu S - 2\sqrt{2}\partial_\mu S \partial^\mu D\bar{\zeta} \bar{\lambda} + \\
& + \sqrt{2}D\partial_\nu \zeta \sigma^\nu \bar{\sigma}^\mu \lambda \partial_\mu S^* - \frac{i}{\sqrt{2}} \bar{\sigma}^\lambda \partial_\lambda \zeta \lambda \sigma^\nu \partial_\mu S^* F_V^\mu + \frac{1}{2\sqrt{2}} \varepsilon^{\mu\nu\kappa\alpha} \lambda \sigma_\alpha \bar{\sigma}^\lambda \partial_\lambda \zeta \partial_\mu S^* F_{\nu\kappa} + \\
& - \frac{1}{2} \partial_\lambda \zeta \sigma^\lambda \bar{\lambda} \lambda \sigma^\mu \partial_\mu \bar{\zeta} + \sqrt{2}D\lambda \sigma^\lambda \bar{\sigma}^\mu \partial_\lambda \zeta \partial_\mu S^* - \frac{i}{\sqrt{2}} \partial_\lambda \zeta \sigma^\nu \bar{\sigma}^\lambda \lambda \partial_\mu S^* F_V^\mu + \\
& - \frac{1}{2\sqrt{2}} \varepsilon^{\mu\nu\kappa\alpha} \partial_\lambda \zeta \sigma_\alpha \bar{\sigma}^\lambda \lambda \partial_\mu S^* F_{\nu\kappa} + h.c. \Big\}. \quad (2.12)
\end{aligned}$$

We notice that, in trying to supersymmetrise the $\xi_\mu \xi_\nu F_K^\mu F^{K\nu}$ -term, we automatically get the supersymmetrisation of the term $\xi_\lambda \xi^\lambda F^2$. This is not a simple coincidence, but it can naturally be expected from an analysis of the irreducible representations of $SO(1,3)$. We conclude that, in the case the background superfield is chiral, we are able to write down the supersymmetric version of (1.1) with $K_{\mu\nu\kappa\lambda}$ given by [26]:

$$K_{\mu\nu\kappa\lambda} = -16 (\eta_{k\mu} \bar{\kappa}_{\nu\lambda} - \eta_{\mu\lambda} \bar{\kappa}_{\nu\kappa} - \eta_{\nu\lambda} \bar{\kappa}_{\mu\kappa} - \eta_{k\nu} \bar{\kappa}_{\mu\lambda}), \quad (2.13)$$

with,

$$\tilde{\kappa}_{\mu\nu} = \kappa \left\{ \left(\frac{\partial_\mu S \partial_\nu S^* + \partial_\nu S \partial_\mu S^*}{2} \right) - \partial_\lambda S \partial^\lambda S^* \eta_{\mu\nu} / 4 \right\}, \quad (2.14)$$

and

$$K^{\mu\nu}{}_{\mu\nu} = 0. \quad (2.15)$$

Concluding, this special form for $\tilde{\kappa}$ is a natural consequence of the assumption that a chiral superfield carries the background that breaks Lorentz-symmetry.

It is worthy to notice that $\tilde{\kappa}$, and consequently K , depend exclusively on the scalar component S . No ζ -condensate and no G -dependence appear in the expression for the K -tensor. Since the action (2.9) is quadratic in Ω , one might in principle expect that tensor bilinears in ζ could show up as contributions to $K_{\mu\nu\kappa\lambda}$. However, the K -tensor depends exclusively on the gradient of S . It becomes clear that a constant $\partial_\mu S$ ensures the constancy of $K_{\mu\nu\kappa\lambda}$, as we had already anticipated. In a particular case, where the background fields $\partial_\mu S \neq 0$, $\zeta = 0$, and $G = 0$, we have that the auxiliary field, D , is given by (we suppose the supersymmetric version of the $F_{\mu\nu}^2$ -term is added up),

$$D = - \frac{8\kappa i \partial_\mu S \partial_\nu S^* F^{\mu\nu}}{16 + 32\kappa \partial_\mu S \partial^\mu S^*}. \quad (2.16)$$

It is interesting to comment that, by virtue of (2.16), which is valid in the conditions above for the background, the breaking of Lorentz-symmetry, fixes the auxiliary field to be non-trivial, even, if the gauge potential superfield (V) is not coupled to matter. The background, as (2.16) shows, determines D by the gauge field strength, $F_{\mu\nu}$. However, if supersymmetric matter happens to be minimally coupled to the gauge field, then (2.16) indicates that charged scalar particles (selectrons, for example) may acquire a magnetic dipole moment given in terms of the vector $\vec{\mu} = \vec{v} \times \vec{v}^*$, where $\vec{v} \equiv \vec{\nabla} S$. This investigation is being pursued and we shall soon report on it [29].

3. Second Proposal: Lorentz-symmetry Breaking from a Vector Supermultiplet

Adopting covariant superspace-superfield formulation, we propose the following minimal extension of (1.6) for:

$$S = \kappa \int d^4x d^2\theta d^2\bar{\theta} \left\{ (D^\alpha \Xi) W_\alpha (\bar{D}_{\dot{\alpha}} \Xi) \bar{W}^{\dot{\alpha}} + h.c. \right\}. \quad (3.1)$$

W_α is the superfield strength of the gauge supermultiplet V_{WZ} , as given in eq. (1.1). Ξ is the so-called vector superfield, whose θ - expansion is as follows:

$$\Xi(\theta, \bar{\theta}) = C + \theta\chi + \bar{\theta}\bar{\chi} + \theta^2 M + \bar{\theta}^2 M^* + \theta\sigma^\mu\bar{\theta}\xi_\mu + \theta^2\bar{\theta}\bar{\psi} + \bar{\theta}^2\theta\psi + \theta^2\bar{\theta}^2 B. \quad (3.2)$$

In the special case of constant background component fields, the SUSY transformations simplify and acquire the form below:

$$\begin{aligned}
\delta C &= \varepsilon^\alpha \chi_\alpha + \bar{\varepsilon}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}}, \\
\delta \chi_\alpha &= 2M \varepsilon_\alpha + \sigma_{\alpha\dot{\alpha}}^\mu \bar{\varepsilon}^{\dot{\alpha}} \xi_\mu, \\
\delta \bar{\chi}^{\dot{\alpha}} &= -\varepsilon^\alpha \sigma_{\alpha\dot{\beta}}^\mu \varepsilon^{\dot{\beta}\dot{\alpha}} \xi_\mu + 2M^* \bar{\varepsilon}^{\dot{\alpha}}, \\
\delta M &= \bar{\varepsilon}_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}}, \\
\delta M^* &= \varepsilon^\alpha \chi_\alpha, \\
\delta \xi^\mu &= \varepsilon^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \bar{\psi}^{\dot{\alpha}} - \bar{\varepsilon}_{\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \psi_\alpha, \\
\delta \bar{\psi}^{\dot{\alpha}} &= 2\bar{\varepsilon}^{\dot{\alpha}} B, \\
\delta \psi_\beta &= 2\varepsilon_\beta B, \\
\delta B &= 0.
\end{aligned} \tag{3.3}$$

In the general case, B transforms as a total derivative; only for a constant ψ -background, we get $\delta B = 0$. Notice that we are taking the full θ -expansion for Ξ . Nothing like a Wess-Zumino gauge can be taken, for Ξ is a fixed background and is not a gauge super-potential.

The superaction in component fields is given by,

$$S = \kappa \int d^4x d^2\theta d^2\bar{\theta} \left\{ (D^\alpha \Xi) W_\alpha (\bar{D}_{\dot{\alpha}} \Xi) \bar{W}^{\dot{\alpha}} + h.c. \right\} = S_{boson} + S_{fermion} + S_{coupled}; \tag{3.4}$$

$$\begin{aligned}
S_{boson} &= \kappa \int d^4x \left\{ 2 \left(\frac{1}{2} \xi_\lambda \xi_\mu F^\mu{}_\kappa F^{\kappa\lambda} + \frac{1}{8} \xi_\lambda \xi^\lambda F_{\mu\nu} F^{\mu\nu} \right) + \right. \\
&\quad \left. + \frac{i}{4} \xi_\lambda \xi_\mu \varepsilon^{\lambda\tau\rho\kappa} F_{\tau\rho} F^\mu{}_\kappa - 2D^2 \xi_\lambda \xi^\lambda + 8|M|^2 D^2 + h.c. \right\},
\end{aligned} \tag{3.5}$$

where we can readily identify the action (1.1) with $K_{\mu\nu\kappa\lambda}$ given as in (1.3), and (1.4).

$$\begin{aligned}
S_{fermion} &= \kappa \int d^4x \left\{ -i\psi\sigma^\mu\partial_\mu\bar{\lambda}\bar{\chi}\bar{\lambda} + \psi\lambda\bar{\psi}\bar{\lambda} + \frac{i}{2}\partial_\mu\lambda\sigma^\mu\bar{\psi}\bar{\chi}\bar{\lambda} + \psi\lambda\partial_\mu\lambda\sigma^\mu\bar{\chi} + \right. \\
&\quad \left. - \frac{i}{2}\lambda\sigma^\mu\bar{\psi}\bar{\chi}\partial_\mu\bar{\lambda} + \lambda^2\bar{\psi}\bar{\lambda} + i\chi\lambda\partial_\mu\lambda\sigma^\mu\bar{\psi} - i\chi\lambda\psi\sigma^\mu\partial_\mu\bar{\lambda} + h.c. \right\},
\end{aligned} \tag{3.6}$$

$$\begin{aligned}
S_{coupled} &= \kappa \int d^4x \left\{ -\frac{3}{4}i\chi\lambda\xi_\mu\partial^\mu D - \frac{1}{2}\chi\lambda\xi_\mu\partial_\nu F^\nu{}_\mu + \right. \\
&\quad - \frac{i}{2}\chi\lambda\varepsilon^{\psi\nu\kappa\mu}\xi_\mu\partial_\psi F_{\nu\kappa} + 4\chi\lambda BD + D\chi\sigma^\mu\bar{\chi}\partial_\nu F^\nu{}_\mu + \\
&\quad + \frac{i}{4}D\varepsilon^{\psi\nu\kappa\alpha}\chi\sigma_\alpha\bar{\chi}\partial_\psi F_{\nu\kappa} - 6iDM^*\chi\sigma^\mu\partial_\mu\bar{\lambda} + iD\partial_\nu\lambda\sigma^\nu\bar{\sigma}^\mu\chi\xi_\mu + \\
&\quad \left. - 4D^2\chi\psi - \frac{i}{2}\chi\sigma^\tau\bar{\chi}F_{\tau\rho}\partial_\nu F^{\nu\rho} + \frac{1}{4}\varepsilon^{\tau\rho\kappa\alpha}\chi\sigma_\alpha\bar{\chi}F_{\tau\rho}\partial_\nu F^\nu{}_\kappa + \right\}
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{4}\varepsilon^{\lambda\nu\kappa\rho}\chi\sigma^\tau\bar{\chi}F_{\tau\rho}\partial_\lambda F_{\nu\kappa} + \frac{i}{8}\varepsilon^{\psi\nu\kappa\alpha}\varepsilon^{\tau\rho}{}_{\alpha\beta}\chi\sigma^\beta\bar{\chi}F_{\tau\rho}\partial_\psi F_{\nu\kappa} + \\
& + M^*\chi\sigma^\tau\partial_\nu\bar{\lambda}F_\tau{}^\nu + \frac{i}{2}M^*\varepsilon^{\tau\rho\mu\alpha}\chi\sigma_\alpha\partial_\mu\bar{\lambda}F_{\tau\rho} + \frac{1}{2}\chi\sigma^\tau\bar{\sigma}^\nu\partial_\nu\lambda\xi_\mu F_\tau{}^\mu + \\
& + \frac{i}{4}\varepsilon^{\tau\rho\mu\alpha}\sigma_\alpha\bar{\sigma}^\nu\partial_\nu\lambda\xi_\mu F_{\tau\rho} - 2D\chi\sigma^{\tau\rho}\lambda F_{\rho\tau} + \frac{1}{2}\chi\partial_\mu\lambda\bar{\chi}\partial^\mu\bar{\lambda} + \\
& + i\chi\partial_\mu\lambda\xi^\mu - \bar{\chi}\bar{\sigma}^\nu\chi\partial_\mu DF_{\nu}{}^\mu + \frac{i}{2}\varepsilon^{\nu\kappa\mu\alpha}\bar{\chi}\bar{\sigma}_\alpha\chi\partial_\mu DF_{\nu\kappa} + \\
& + iMD\partial_\mu\lambda\sigma^\mu\bar{\chi} + \frac{i}{2}M\lambda\sigma^\kappa\bar{\chi}\partial_\mu F^\mu{}_\kappa - \frac{1}{4}M\varepsilon^{\mu\nu\kappa\alpha}\lambda\sigma_\alpha\bar{\chi}\partial_\mu F_{\nu\kappa} + \\
& - i|M|^2\lambda\sigma^\mu\partial_\mu\bar{\lambda} + iM\partial_\nu\lambda\sigma^\nu\bar{\sigma}^\mu\lambda\xi_\mu - 4MD\lambda^2 + \\
& - \frac{1}{2}\bar{\chi}\bar{\sigma}^\nu\partial_\mu\lambda F_{\nu}{}^\mu + \frac{i}{4}\varepsilon^{\nu\kappa\mu\alpha}\bar{\chi}\bar{\sigma}_\alpha\partial_\mu\lambda F_{\nu\kappa} + \frac{i}{2}M^*\partial_\mu\lambda\sigma^\mu\bar{\lambda} + \\
& + i\lambda\sigma^\mu\partial_\nu\bar{\lambda}\xi^\nu\xi_\mu - \frac{i}{2}\lambda\sigma^\mu\partial_\mu\bar{\lambda}\xi_\lambda\xi^\lambda - \frac{1}{2}\varepsilon^{\lambda\nu\mu\alpha}\lambda\sigma_\alpha\partial_\nu\bar{\lambda}\xi_\lambda\xi_\mu + \\
& + \frac{1}{2}\varepsilon^{\nu\kappa\lambda\alpha}\lambda\sigma_\alpha\bar{\psi}\xi_\lambda F_{\nu\kappa} - B\lambda\sigma^\mu\bar{\lambda}\xi_\mu + \bar{\chi}\partial_\nu\bar{\lambda}\xi_\mu F^{\mu\nu} + \\
& + \frac{i}{2}\varepsilon^{\tau\rho\nu\lambda}\bar{\chi}\partial_\nu\bar{\lambda}\xi_\lambda F_{\tau\rho} - \frac{i}{2}\bar{\lambda}\bar{\sigma}^\rho\lambda\xi_\lambda F^\lambda{}_\rho - \frac{1}{4}\varepsilon^{\lambda\tau\rho\alpha}\bar{\lambda}\bar{\sigma}_\alpha\lambda\xi_\lambda F_{\tau\rho} + \\
& - iD\partial_\mu\bar{\lambda}\bar{\sigma}^\mu\sigma^\lambda\bar{\chi}\xi_\lambda + \frac{1}{2}\partial_\mu\bar{\lambda}\bar{\sigma}^\mu\sigma^\kappa\bar{\chi}\xi_\lambda F^\lambda{}_\kappa + \frac{i}{4}\varepsilon^{\lambda\nu\kappa\alpha}\partial_\mu\bar{\lambda}\bar{\sigma}^\mu\sigma_\alpha\bar{\chi}\xi_\lambda F_{\nu\kappa} + \\
& - iM^*\partial_\mu\bar{\lambda}\bar{\sigma}^\mu\sigma^\lambda\bar{\lambda}\xi_\lambda - \frac{1}{2}\bar{\chi}\bar{\lambda}\xi_\lambda\partial_\nu F^{\lambda\nu} - \frac{i}{4}\bar{\chi}\bar{\lambda}\varepsilon^{\tau\rho\nu\lambda}\xi_\lambda\partial_\nu F_{\tau\rho} + \\
& - \frac{1}{2}D^2\bar{\chi}\bar{\psi} + D\bar{\chi}\bar{\sigma}^{\nu\kappa}\bar{\psi}F_{\nu\kappa} - 2M^*D\bar{\psi}\bar{\lambda} - \frac{i}{2}\psi\sigma^\tau\bar{\lambda}\xi_\mu F_\tau{}^\mu + \\
& + \frac{1}{4}\varepsilon^{\tau\rho\mu\alpha}\psi\sigma_\alpha\bar{\lambda}\xi_\mu F_{\tau\rho} - B\lambda\sigma^\mu\bar{\lambda}\xi_\mu + 4BD\bar{\chi}\bar{\lambda} + h.c. \Big\}. \tag{3.7}
\end{aligned}$$

The terms in the superactions (2.9) and (3.4) have the same form with the factor $(D^\alpha\Omega)W_\alpha$ replaced by $(D^\alpha\Xi)W_\alpha$. Then, we obtain the right combination, i. e., $\frac{1}{2}\xi_\mu\xi_\nu F^\mu{}_\kappa F^{\kappa\nu} + \frac{1}{8}\xi_\lambda\xi^\lambda F_{\mu\nu}F^{\mu\nu}$. To get the term, $\xi_\mu\xi_\nu F^\mu{}_\kappa F^{\kappa\nu}$, we could also propose the action below:

$$S = \kappa \int d^4x d^2\theta d^2\bar{\theta} \left\{ (D^\alpha\Xi)(D_\alpha\bar{\Xi})W^\beta W_\beta + h.c. \right\}. \tag{3.8}$$

Also in this case, when we supersymmetrise the $\xi_\mu\xi_\nu F^\mu{}_\kappa F^{\kappa\nu}$ -term, automatically comes out $\xi_\lambda\xi^\lambda F^2$. So, we have a second way to build up the supersymmetric extension of the term $(\frac{1}{2}\xi_\mu\xi_\nu F^\mu{}_\kappa F^{\kappa\nu} + \frac{1}{8}\xi_\lambda\xi^\lambda F_{\mu\nu}F^{\mu\nu})$. We just mention this possible second way of working out the supersymmetrisation of (1.6) by a vector superfield background, but we do not exploit here this second possibility. This is why we do not project the superfield action (3.8) into components.

Though we formulate our model in terms of superspace and superfields, the tensor calculus of supersymmetry, we would like to point out that with a Lorentz-symmetry breaking, in the chiral case given by $\partial_\mu S \neq 0$, in the vector case parametrized by $\xi^\lambda \neq 0$, SUSY is readily seen to be broken by the background. But, as it was emphasized, we do not have elements to decide whether SUSY breaking has occurred before Lorentz-symmetry violation, though the breaking of the latter signals the violation of the former.

4. Concluding Comments

Our effort in this paper has mainly consisted in finding out possible $N = 1$ supersymmetric scenarios for the K -tensor- realized Lorentz-symmetry breaking considered in the literature (see the Refs. quoted throughout the present work).

We propose two viable descriptions. The first approach is based on a chiral superfield that accommodates the content of the background responsible for the breaking of Lorentz-symmetry. In this case, a complex scalar is the source for $K_{\mu\nu\kappa\lambda}$. The constancy of the K -tensor is ensured by the linear dependence of S on the x^μ -coordinates. An interesting question to be investigated is the analysis of the photon - photino mass splitting in terms of the background field components, specially if the fermion component ζ is a non-vanishing constant background. From the expression for $S_{coupled}$ in eq. (2.12), there result interesting terms that mix the gauge superfield components (A^μ, λ , and D) in bilinear terms where the (constant) background fields are also present, as mass parameters. The task of getting the $\langle A_\mu A_\nu \rangle - \langle A_\mu \lambda \rangle - \langle \lambda \lambda \rangle - \langle \lambda D \rangle -$ and $\langle DD \rangle$ -propagators demands special technicalities (Fierzings and spin-projection operators) and, once these tree level 2- point functions are worked out, we can read off the propagator poles and discuss in terms of the background components (S , ζ , and G). The rôle of the background fermionic condensates become clear after the propagator poles are identified.

This discussion holds through also in the cases of a vector-superfield background that carries ξ_μ , from which $K_{\mu\nu\kappa\lambda}$ is expressed. This case exhibits a richer background and fermion condensates that mix χ and Ψ (see eq. (3.7)), in addition to the χ - and Ψ -condensates become important.

So, to our mind, in either case, the important question that our study may raise concerns the influence of the Lorentz-symmetry violating background on the spectrum and which restrictions it should have so as to avoid the appearance of non-physical excitations, such as tachyons and ghosts.

Finally, a non-trivial question that remains to be addressed to is the relation between SUSY and Lorentz-symmetry breakings. We treat the latter in a supersymmetric formulation and, as we have previously commented, our backgrounds (both Ω and Ξ) are not invariant under SUSY. They simply express the fact that Lorentz-symmetry breaking does not support exact SUSY. However, by no means, we are stating that we break Lorentz-symmetry and SUSY at the same time. SUSY could have been broken before, by some more fundamental mechanism, and Lorentz-symmetry breaking takes place in an environment which still keeps the inheritance of SUSY through the whole set of supersymmetric partners. This is why approach the deviation from Lorentz-symmetry in connection with SUSY. However, it would be an interesting task- and we shall soon concentrate on this point - to build up a model such that, whenever SUSY is spontaneously broken, some tensor field (that belongs to some supermultiplet of the model coupled to the superfield responsible for the breaking of SUSY) also acquires a non-trivial vacuum expectation value and the Lorentz group simultaneously undergoes spontaneous symmetry breakdown. This sets out another possibility, with a contemporary breaking of SUSY and Lorentz-symmetry. We are considering this situation and we shall report on it elsewhere.

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