

$|V_{us}|$ and Charged Lepton Universality from τ Decays

Ian M. Nugent^{*†}

University of Victoria/TRIUMF

E-mail: inugent@triumf.ca

τ decays provide a unique opportunity to make precision measurements of the weak interaction between the first and second generation of quarks, the Cabibbo-Kobayashi-Maskawa matrix (CKM) element $|V_{us}|$, and tests charged lepton universality, the assumption that all leptons have the same coupling strength to the gauge bosons. We present the recent measurements of τ decays from BELLE and BABAR and the improvements in determining $|V_{us}|$ and testing lepton universality.

*The Xth Nicola Cabibbo International Conference on Heavy Quarks and Leptons,
October 11-15, 2010
Frascati (Rome) Italy*

^{*}Speaker.

[†]On behalf of the BABAR and BELLE Collaborations

1. τ Decay Measurements from BELLE and BABAR

The general strategy used at B-Factories to select clean sample of $\tau^+\tau^-$ events is to split the event into two hemisphere using the thrust in the CM frame. One hemisphere is required to have an isolated electron or muon from $\tau \rightarrow e\nu_\tau\bar{\nu}_e$ or $\tau \rightarrow \mu\nu_\tau\bar{\nu}_\mu$ to tag the event, while the other hemisphere, the signal hemisphere, is required to decay hadronically. Particle identification is applied to the charged tracks and selection of γ s, π^0 s and η s are applied to identify the hadronic decay mode.

The BABAR Collaboration has published a measurement of $BR(\tau^- \rightarrow K^-\pi^0\nu_\tau)$ [1]. This analysis used a lepton tag with one well reconstructed π^0 and one kaon identified in the signal hemisphere. The branching fraction was measured to be $BR(\tau^- \rightarrow K^-\pi^0\nu_\tau)=(4.16 \pm 0.03 \pm 0.18) \times 10^{-3}$.

BABAR and BELLE have both measured $\tau^- \rightarrow K_S^0\pi^-\nu_\tau$, where the BABAR result is preliminary [2] and the BELLE result is published[3]. In this analysis, the signal hemisphere was required to contain a K_S^0 reconstructed from a $\pi^+\pi^-$ pair and a bachelor pion. In the $\tau^- \rightarrow K_S^0\pi^-\pi^0\nu_\tau$ channel, the signal hemisphere was also required to contain a π^0 reconstructed from two photons[2]. BELLE measured $BR(\tau^- \rightarrow K_S^0\pi^-\nu_\tau)=(8.08 \pm 0.04 \pm 0.26) \times 10^{-3}$, while BABAR measured $BR(\tau^- \rightarrow K_S^0\pi^-\nu_\tau)=(8.40 \pm 0.04 \pm 0.23) \times 10^{-3}$ and $\tau^- \rightarrow K_S^0\pi^-\pi^0\nu_\tau=(3.42 \pm 0.06 \pm 0.15) \times 10^{-3}$.

Both the BELLE and BABAR Collaborations have measured $\tau^- \rightarrow h^-h^-h^+\nu_\tau$ where h has been identified as a pion or kaon[4, 5]. After selecting the event with a leptonic tag and three charged tracks in the signal hemisphere, particle identification was applied and a matrix technique was used to extract the branching fractions shown in Table 1. The BABAR paper also measured $BR(\tau^- \rightarrow \pi^-\phi\nu_\tau)=(3.42 \pm 0.55 \pm 0.25) \times 10^{-5}$, $BR(\tau^- \rightarrow K^-\phi\nu_\tau)=(3.39 \pm 0.20 \pm 0.28) \times 10^{-5}$ and $BR(\tau^- \rightarrow K^-K^-K^+\nu_\tau [ex. \phi]) < 2.5 \times 10^{-6}$ at 90% CL. whereas BELLE measured $BR(\tau^- \rightarrow K^-\phi\nu_\tau)=(4.06 \pm 0.25 \pm 0.26) \times 10^{-5}$ in [6]

Table 1: The measured $\tau^- \rightarrow h^-h^-h^+\nu_\tau$ branching fractions.

Decay Mode	BELLE	BABAR
$BR(\tau^- \rightarrow \pi^-\pi^-\pi^+\nu_\tau [ex. K_S^0])$	$(8.42 \pm 0.01^{+0.26}_{-0.25}) \times 10^{-2}$	$(8.83 \pm 0.01 \pm 0.13) \times 10^{-2}$
$BR(\tau^- \rightarrow K^-\pi^-\pi^+\nu_\tau [ex. K_S^0])$	$(3.30 \pm 0.01^{+0.16}_{-0.17}) \times 10^{-3}$	$(2.73 \pm 0.02 \pm 0.09) \times 10^{-3}$
$BR(\tau^- \rightarrow K^-\pi^-K^+\nu_\tau)$	$(1.55 \pm 0.010^{+0.06}_{-0.05}) \times 10^{-3}$	$(1.346 \pm 0.010 \pm 0.036) \times 10^{-3}$
$BR(\tau^- \rightarrow K^-K^-K^+\nu_\tau)$	$(3.29 \pm 0.17^{+0.19}_{-0.20}) \times 10^{-5}$	$(1.58 \pm 0.13 \pm 0.12) \times 10^{-5}$

Recently, the BELLE collaboration has published a paper on precision measurements of τ decays containing an η [7]. In this paper, the signal hemisphere and π^0 and η are reconstructed from two photons, where the $\tau^- \rightarrow K^-\eta\nu_\tau$ used the additional η to $\pi^-\pi^+\pi^0$ decay mode. Using a fit of the η invariant mass, BELLE measured $BR(\tau^- \rightarrow K^-\eta\nu_\tau)=(1.48 \pm 0.05 \pm 0.09) \times 10^{-4}$, $BR(\tau^- \rightarrow K^-\pi^0\eta\nu_\tau)=(4.6 \pm 1.1 \pm 0.4) \times 10^{-5}$, and $BR(\tau^- \rightarrow K_S^0\pi^-\eta\nu_\tau)=(4.4 \pm 0.7 \pm 0.2) \times 10^{-5}$. For the $\tau^- \rightarrow K^*(892)^-\eta\nu_\tau$ decays the η s are selected within a mass window, and a fit is applied to the $K_S^0\pi^-$ and $K^-\pi^0$ invariant mass to extract $BR(\tau^- \rightarrow K^*(892)^-\eta\nu_\tau)=(1.34 \pm 0.12 \pm 0.09) \times 10^{-4}$.

The BABAR Collaboration has recently published measurements of the one prong decay channels: $\tau \rightarrow e\nu_\tau\bar{\nu}_e$; $\tau \rightarrow \mu\nu_\tau\bar{\nu}_\mu$; $\tau \rightarrow \pi\nu_\tau$ and $\tau \rightarrow K\nu_\tau$ [8]. This was a blind analysis that used a

$\tau^- \rightarrow \pi^- \pi^- \pi^+ \nu_\tau$ decay to tag, instead of the typical leptonic tag, and required an isolated signal track in the signal hemisphere that was topology consistent with coming from a one prong τ decay. A high purity selection is achieved by applying a tight particle identification criteria to suppress cross-feed between channels and other τ backgrounds. The backgrounds, cross-feed and particle identification are carefully estimated using a variety of Monte Carlo and data control samples. To reduce systematics that are common between channels, *BABAR* measured the $BR(\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu)/BR(\tau \rightarrow e \nu_\tau \bar{\nu}_e) = (0.9796 \pm 0.0016 \pm 0.0035)$, $BR(\tau \rightarrow \pi \nu_\tau)/BR(\tau \rightarrow e \nu_\tau \bar{\nu}_e) = (0.5945 \pm 0.0014 \pm 0.0061)$, $BR(\tau \rightarrow K \nu_\tau)/BR(\tau \rightarrow e \nu_\tau \bar{\nu}_e) = (0.03882 \pm 0.00032 \pm 0.00056)$ and $BR(\tau \rightarrow K \nu_\tau)/BR(\tau \rightarrow \pi \nu_\tau) = (0.06531 \pm 0.00056 \pm 0.00093)$. Using the world average $BR(\tau \rightarrow e \nu_\tau \bar{\nu}_e) = (17.82 \pm 0.05) \times 10^{-2}$, the branching fractions $BR(\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu) = (17.46 \pm 0.09) \times 10^{-2}$, $BR(\tau \rightarrow \pi \nu_\tau) = (10.59 \pm 0.11) \times 10^{-2}$ and $BR(\tau \rightarrow K \nu_\tau) = (6.92 \pm 0.12) \times 10^{-3}$ were determined.

2. Cabibbo-Kobayashi-Maskawa matrix element $|V_{us}|$ from τ

The CKM matrix element $|V_{us}|$ can be determined from τ decays using multiple techniques. One technique to extract $|V_{us}|$ from τ is to use flavor breaking difference with Finite Energy Sum Rules to extract $|V_{us}|$:

$$\frac{R_{\tau, \text{strange}}}{|V_{us}|^2} - \frac{R_{\tau, \text{nonstrange}}}{|V_{ud}|^2} = \delta R_{\tau, \text{SU3 breaking}}$$

where $R_{\tau, \text{strange}} = \Gamma(\tau^- \rightarrow X_{\text{strange}} \nu_\tau)/\Gamma(\tau^- \rightarrow e \nu_\tau \bar{\nu}_e)$ is the strange hadronic width, $R_{\tau, \text{nonstrange}} = \Gamma(\tau^- \rightarrow X_{\text{nonstrange}} \nu_\tau)/\Gamma(\tau^- \rightarrow e \nu_\tau \bar{\nu}_e)$ is the nonstrange hadronic width and $\delta R_{\tau, \text{SU3 breaking}}$ is the theoretical SU(3) flavor breaking correction determined using Operator Product Expansion (OPE). Using the HFAG values shown in Table 2, which combine the recent measurements from BELLE and *BABAR* with previous τ results, one obtains $|V_{us}| = 0.2188 \pm 0.0023$. The FESR approach for extracting $|V_{us}|$ has theoretical errors which range from 0.23%-0.47% depending on the FESR weight. This is the smallest theoretical error of the τ decay methods and smaller than the theoretical error from K_{l3} decays (0.58%)[9] and K_{l2} decays (0.5%)[10]. If the strange branching fractions and spectral functions are updated with the data currently available at BELLE and *BABAR* this method has the potential for making the most precise measurement of $|V_{us}|$ [11].

Another method which can be used to determine $|V_{us}|$ from τ decays is from the ratio:

$$\frac{BR(\tau \rightarrow K \nu_\tau)}{BR(\tau \rightarrow \pi \nu_\tau)} = \frac{f_K^2 |V_{us}|^2 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2}{f_\pi^2 |V_{ud}|^2 \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2} (1 + \delta_{LD}),$$

where $f_K/f_\pi = 1.189 \pm 0.007$ [12] is determined from lattice QCD, $|V_{ud}|$ [13], and the long-distance correction $\delta_{LD} = (0.03 \pm 0.44)\%$ is estimated [14] using corrections to $\tau \rightarrow h \nu_\tau$ and $h \rightarrow \mu \nu_\mu$ [15, 16]. This method is orthogonal to the inclusive sum of strange τ decays approach and has a theoretical uncertainty of 0.5%. Using this method, *BABAR* [8] determined $|V_{us}| = 0.2255 \pm 0.0024$. This value is consistent with CKM unitarity [13] and 2.5σ higher than $|V_{us}|$ from the inclusive sum of strange τ decays.

A third method for determining $|V_{us}|$ from τ decays is from the $BR(\tau^- \rightarrow K^- \nu_\tau)$ directly.

$$BR(\tau^- \rightarrow K^- \nu_\tau) = \frac{G_F^2 f_K^2 |V_{us}|^2 m_\tau^3}{16\pi\hbar} \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{EW},$$

Table 2: The current status of the branching fraction for the strange τ decays.

Decay Mode	Branching Fraction (%)	BELLE	BABAR
$BR(\tau \rightarrow K \nu_\tau)$	0.697 ± 0.010		[8]
$BR(\tau^- \rightarrow K^- \pi^0 \nu_\tau)$	0.431 ± 0.015		[1]
$BR(\tau^- \rightarrow K^- \pi^0 \pi^0 \nu_\tau)$ (ex. K^0)	0.060 ± 0.022		
$BR(\tau^- \rightarrow K^- \pi^0 \pi^0 \pi^0 \nu_\tau)$ (ex. K^0, η)	0.039 ± 0.022		
$BR(\tau^- \rightarrow K^0 \pi^- \nu_\tau)$	0.831 ± 0.018	[3]	[2]
$BR(\tau^- \rightarrow K^0 \pi^- \pi^0 \nu_\tau)$	0.350 ± 0.015		[17]
$BR(\tau^- \rightarrow K^0 \pi^- \pi^0 \pi^0 \nu_\tau)$	0.031 ± 0.023		
$BR(\tau^- \rightarrow K^0 h^- h^- h^+ \nu_\tau)$	0.029 ± 0.020		
$BR(\tau^- \rightarrow K^- \pi^- \pi^+ \nu_\tau)$ (ex. K^0)	0.294 ± 0.007	[4]	[5]
$BR(\tau^- \rightarrow K^- \pi^- \pi^+ \pi^0 \nu_\tau)$ (ex. K^0, η)	0.078 ± 0.012		
$BR(\tau^- \rightarrow K^- \eta \nu_\tau)$	0.016 ± 0.001	[7]	
$BR(\tau^- \rightarrow K^- \eta \pi^0 \nu_\tau)$	0.005 ± 0.001	[7]	
$BR(\tau^- \rightarrow K^0 \eta \pi^- \nu_\tau)$	0.009 ± 0.001	[7]	
$BR(\tau^- \rightarrow K^- K^- K^+ \nu_\tau)$	0.0024 ± 0.0008	[4]	[5]
$BR(\tau^- \rightarrow K^- K^0 K^0 \nu_\tau)$ from $\tau^- \rightarrow K^- K^- K^+ \nu_\tau \times \frac{\phi \rightarrow K^0 K^0}{\phi \rightarrow K^- K^+}$	0.0015 ± 0.0001		
Total	$2.91(55) \pm 0.05(10)$		
Branching Fractions from HFAG constrained fit [18] $\chi^2/\text{d.o.f.}=135.2/115$ CL=9.6%			

Using the kaon decay constant $f_K = 157 \pm 2 \text{ MeV}$ [12], and $S_{EW} = 1.0201 \pm 0.0003$ [19], *BABAR* determined $|V_{us}| = 0.2193 \pm 0.0032$. This method has a theoretical uncertainty of 1.27%.

Figure 1 shows a summary of $|V_{us}|$ measurements, including the HFAG values of $|V_{us}|$ for the three methods mentioned above.

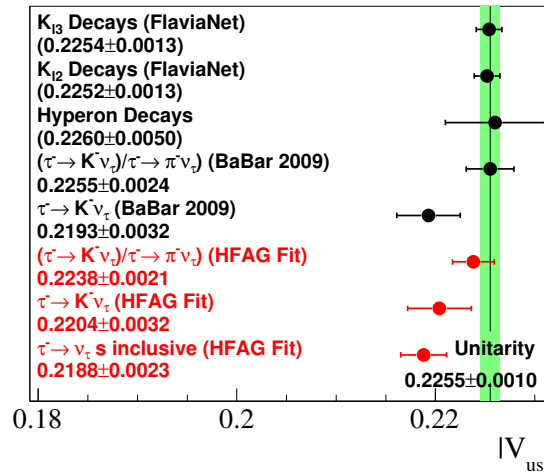


Figure 1: A summary of $|V_{us}|$ measurements. The FlaviaNet measurements come from [10], the hyperon decays come from [20], the *BABAR* τ measurements come from [8] and the HFAG results come from [18].

3. Charged Lepton Universality

Charged lepton universality $\mu - e$ can be tested using

$$\left(\frac{g_\mu}{g_e}\right)_\tau^2 = \frac{BR(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau) f(m_e^2/m_\tau^2)}{BR(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) f(m_\mu^2/m_\tau^2)},$$

where $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$, assuming that the neutrino masses are negligible [21]. The *BABAR* paper [8] gives $\left(\frac{g_\mu}{g_e}\right)_\tau = 1.0036 \pm 0.0020$, yielding a new world average from τ decays of 1.0018 ± 0.0014 [18]. Figure 2 shows the current status of $\mu - e$ lepton universality.

τ - μ universality is tested with

$$\left(\frac{g_\tau}{g_\mu}\right)_h^2 = \frac{BR(\tau \rightarrow h \nu_\tau)}{BR(h \rightarrow \mu \nu_\mu)} \frac{2m_h m_\mu^2 \tau_h}{(1 + \delta_h) m_\tau^3 \tau_\tau} \left(\frac{1 - m_\mu^2/m_h^2}{1 - m_\mu^2/m_\tau^2}\right)^2,$$

where the radiative corrections are $\delta_\pi = (0.16 \pm 0.14)\%$ and $\delta_K = (0.90 \pm 0.22)\%$ [15]. *BABAR* [8] determined $\left(\frac{g_\tau}{g_\mu}\right)_{\pi(K)} = 0.9856 \pm 0.0057$ (0.9827 ± 0.0086) and $\left(\frac{g_\tau}{g_\mu}\right)_h = 0.9850 \pm 0.0054$, where the world averaged mass, lifetime values and meson decay rates were taken from [22]. A summary of τ - μ universality tests are shown in Figure 2. It is interesting to note, that the measurements of $\left(\frac{g_\tau}{g_\mu}\right)_h$ for both kaon and pions are low compared to the HFAG values from the constrained fit and Standard Model expectation.

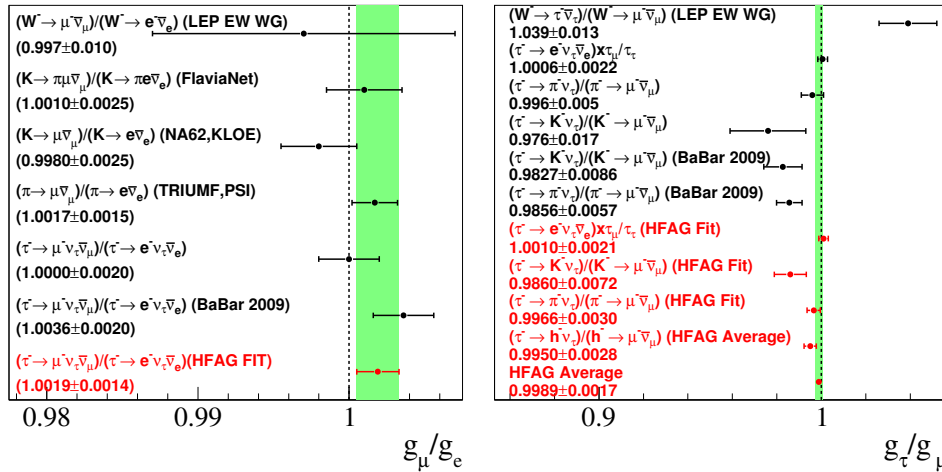


Figure 2: (left) The current status of g_μ/g_e lepton universality measurements. The HFAG average is the weighted average of previous τ results with the recent *BABAR* g_μ/g_e measurement. (right) The current status of g_τ/g_μ lepton universality measurements. The HFAG values are presented for: g_τ/g_μ determined from $\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) \times \tau_\mu / \tau_\tau$ using the HFAG fit for $\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau$; $(g_\tau/g_\mu)_\pi$ using the HFAG fit value for $\tau^- \rightarrow \pi^- \nu_\tau$; $(g_\tau/g_\mu)_K$ using the HFAG fit value for $\tau^- \rightarrow K^- \nu_\tau$; the average of the HFAG $(g_\tau/g_\mu)_h$ values; and the HFAG average for τ decays.

References

- [1] B. Aubert *et al.* (BaBar Collaboration). Measurement of the $\tau^- \rightarrow K^- \pi^0 \nu_\tau$ branching fraction. *Phys. Rev.*, D76:051104, 2007.

- [2] B. Aubert *et al.* (BaBar Collaboration). Measurement of $BR(\tau^- \rightarrow \bar{K}^0 \pi^- \nu_\tau)$ using the BaBar detector. *Nucl. Phys. Proc. Suppl.*, 189:193–198, 2009.
- [3] D. Epifanov *et al.* (BELLE Collaboration). Study of $\tau^- \rightarrow K_S \pi^- \nu_\tau$ decay at Belle. *Phys. Lett.*, B654:65–73, 2007.
- [4] M. J. Lee *et al.* (BELLE Collaboration). Measurement of the branching fractions and the invariant mass distributions for $\tau^- \rightarrow h^- h^+ h^- \nu_\tau$ decays. *Phys. Rev.*, D81:113007, 2010.
- [5] B. Aubert *et al.* (BaBar Collaboration). Exclusive branching fraction measurements of semileptonic tau decays into three charged hadrons, $\tau^- \rightarrow \phi \pi^- \nu_\tau$ and $\tau^- \rightarrow \phi K^- \nu_\tau$. *Phys. Rev. Lett.*, 100:011801, 2008.
- [6] K. Inami *et al.* (BELLE Collaboration). First observation of the decay $\tau^- \rightarrow \phi K^- \nu_\tau$. *Phys. Lett.*, B643:5–10, 2006.
- [7] K. Inami *et al.* (BELLE Collaboration). Precise measurement of hadronic tau-decays with an eta meson. *Phys. Lett.*, B672:209–218, 2009.
- [8] B. Aubert *et al.* (BaBar Collaboration). Measurements of Charged Current Lepton Universality and $|V_{us}|$ using Tau Lepton Decays to $e^- \bar{\nu}_e \nu_\tau$, $\mu^- \bar{\nu}_\mu \nu_\tau$, $\pi^- \nu_\tau$, and $K^- \nu_\tau$. *Phys. Rev. Lett.*, 105:051602, 2010.
- [9] P. A. Boyle *et al.* $K \rightarrow \pi$ form factors with reduced model dependence. 2010.
- [10] M. Antonelli *et al.* An evaluation of $|V_{us}|$ and precise tests of the Standard Model from world data on leptonic and semileptonic kaon decays. *Eur. Phys. J.*, C69:399–424, 2010.
- [11] K. Maltman, C. E. Wolfe, S. Banerjee, J. M. Roney, and I. Nugent. Status of the Hadronic Tau Determination of V_{us} . *Int. J. Mod. Phys.*, A23:3191–3195, 2008.
- [12] E. Follana, C. T. H. Davies, G. P. Lepage, and J. Shigemitsu. High Precision determination of the π , K , D and D_s decay constants from lattice QCD. *Phys. Rev. Lett.*, 100:062002, 2008.
- [13] J. C. Hardy and I. S. Towner. Superaligned 0^+ to 0^+ nuclear beta decays: A new survey with precision tests of the conserved vector current hypothesis and the standard model. *Phys. Rev.*, C79:055502, 2009.
- [14] S. Banerjee (BaBar Collaboration). Lepton Universality, $|V_{us}|$ and search for second class current in τ decays. 2008.
- [15] W. J. Marciano and A. Sirlin. Radiative corrections to π (lepton 2) decays. *Phys. Rev. Lett.*, 71:3629–3632, 1993.
- [16] W. J. Marciano. Precise determination of $|V_{us}|$ from lattice calculations of pseudoscalar decay constants. *Phys. Rev. Lett.*, 93:231803, 2004.
- [17] S. Paramesvaran (BaBar Collaboration). Selected topics in tau physics from BaBar. 2009.
- [18] The Heavy Flavor Averaging Group *et al.* Averages of b-hadron, c-hadron, and tau-lepton Properties. 2010.
- [19] Jens Erler. Electroweak radiative corrections to semileptonic tau decays. *Rev. Mex. Fis.*, 50:200–202, 2004.
- [20] V. Mateu and A. Pich. V_{us} determination from hyperon semileptonic decays. *JHEP*, 10:041, 2005.
- [21] Y. Tsai. Decay Correlations of Heavy Leptons in $e^+ e^- \rightarrow \text{Lepton}^+ \text{Lepton}^-$. *Phys. Rev.*, D4:2821, 1971.
- [22] C. Amsler *et al.* (Particle Data Group). Review of particle physics.