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Vus from precision measurements of Kaons

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The experimental inputs to Vus from precision measurements in the kaon sector have reached the accuracy to probe physics beyond the Standard Model at the TeV scale. The measurements and the sensitivity for testing lepton universality and CKM unitarity are reviewed.

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1. Introduction

The leptonic and semileptonic decays of K, D and B mesons ($\to \ell\nu, \to \pi\ell\nu$, $\ell = e,\mu$) in the Standard Model (SM) proceed through W–boson exchange. Gauge coupling universality and three-generation quark mixing imply that all of them are governed by the effective coupling $G_{ij} = G_\mu V_{ij}$, where G_μ is the muon decay constant and V_{ij} are the elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix. In the SM, G_{ij} does not depend on the lepton flavor and the effective couplings, as obtained from transitions with quarks of different generations, are constrained by the unitary relation, $\sum_j |V_{ij}|^2 = 1$. Physics beyond the SM (BSM) could introduce new contributions and therefore non-trivial constraints on physics at the TeV scale are provided by precision tests of the unitarity relations [1]. Kaon physics currently plays a prominent role in testing both, quark-lepton universality, through the V_{us} entry in the CKM unitarity relation $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$, and lepton universality, through the measurement of $|V_{us}| \times f_+(0)$ and the helicity–suppressed ratio $\Gamma(K \to e\nu(\gamma))/\Gamma(K \to \mu\nu(\gamma))$. Large amount of data has been collected on the $K \to \pi\ell\nu$ semileptonic modes by several experiments, BNL-E865, KLOE, KTeV, ISTRA+, and NA48. The data stimulated a substantial progress on the theoretical inputs and most of the errors associated to radiative corrections [2, 3, 4, 5] and hadronic form factors [6] have been reduced below 1%. The unitarity test $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{\mathrm{CKM}}$ is presently probed to 6×10^{-4}. V_{us} from $K \to \pi\ell\nu$ contributes about half of the uncertainty, mostly from the hadronic matrix element. Both experimental and theoretical progress in $K_{\ell 3}$ decays are needed for further improvements on Δ_{CKM}. The kaon semileptonic decay rate is given by:

$$\Gamma(K_{l3}) = \frac{C_K^2 G_F^2 M_K^5}{192\pi^3} S_{EW} |V_{us}|^2 |f_+(0)|^2 \times I_{K,l}(\lambda)(1 + 2\Delta_K^{SU(2)} + 2\Delta_{K,l}^{EM}) \tag{1.1}$$

where $K = K^0, K^\pm$; $l = e,\mu$ and C_K are the Clebsch-Gordan coefficients, equal to $1/2$ and 1 for $K^\pm$ and K^0, respectively. The decay width $\Gamma(K_{l3})$ is determined by measuring the kaon lifetimes and the semileptonic branching fractions (BR) totally inclusive of radiation. The theoretical inputs are i) the universal short–distance electroweak correction $S_{EW} = 1.0232$, ii) the $SU(2)$-breaking term $\Delta_K^{SU(2)}$, iii) the long–distance electromagnetic contributions $\Delta_{K,l}^{EM}$ which depend on the kaon charge and lepton flavor, iv) the form factor $f_+(0) \equiv f_+^{K^0\pi^-}(0)$ describing the hadronic matrix element of $K \to \pi$ transitions, evaluated at zero momentum transfer and for neutral kaons. The form factor dependence on the momentum transfer, needed for the phase space integral $I_{K,l}(\lambda)$, is presently provided by combining measurements and dispersion relations and described by one or more parameters obtained from the decay spectra.

The kaon (pion) leptonic radiation-inclusive decays $K(\pi) \to \mu\bar{\nu}_\mu(\gamma)$ provide a precise determination of $|V_{us}|/|V_{ud}|$,

$$\frac{\Gamma(K^+_{\mu 2})}{\Gamma(\pi^+_{\mu 2})} = \frac{m_K\left(1 - \frac{m_\mu^2}{m_K^2}\right)^2}{m_\pi\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2} \frac{f_K^2 |V_{us}|^2}{f_\pi^2 |V_{ud}|^2} \frac{1 + \frac{\alpha}{\pi} C_K}{1 + \frac{\alpha}{\pi} C_\pi} \tag{1.2}$$

where f_K and f_π are the kaon and the pion decay constants; C_π and C_K parameterize the radiation-inclusive electroweak corrections accounting for bremsstrahlung emission of real photons and photon loop contributions.

An overview of experimental data is presented in the following sections.

2. The branching fractions

The most recent measurements of the kaon semileptonic branching fractions have an accuracy of fractions of per cent. The first was published in year 2003 by the E865 experiment at Brookhaven (BNL), that was realized to search for the lepton-violating (LV) decay $K^+ \to \pi^+\mu^+e^-$. The other measurements of the BR($K \to \pi l \nu$) have been obtained by KTeV at Fermilab, NA48 at CERN, and KLOE at Frascati, that were designed to study CP violation in kaon decays. The E865 experiment at the BNL Alternating Gradient Syncrotron (AGS) measured the $K^+_{e3(\gamma)}$ branching ratio. The result [7], BR($K^+_{e3(\gamma)}$) = (5.13$\pm$0.02$_{stat}$$\pm$0.09$_{syst}$$\pm$0.04$_{norm}$), was 2.3–$\sigma$ greater than the world average from previous measurements dating back more than 25 years. In the limit of isospin conservation, the semileptonic decay width ratio of charged and neutral kaons is 2:1, and the isospin-breaking effect gives a 2% correction. Since the experimental ratio of previous measurements was already higher than expected, the 5% increase in the BR($K^+_{e3(\gamma)}$) pointed to an SU(2) breaking correction at the 6% level, definitely greater than the theoretical estimate, $\Delta^K_{SU(2)} = (\Delta^{K^+}_{SU(2)} - \Delta^{K^0}_{SU(2)}) =$ (2.3$\pm$0.2)% [3]. Evidence that the puzzle could be solved from the experimental side came with the KTeV measurement of the K_L semileptonic decays, published in year 2004. The new measurement, in fact, showed that also in the neutral sector the semileptonic branching ratio $K^0_{e3(\gamma)}$ was understimated by 5%. The KTeV experiment at Fermilab provided for the first time the measurements of all of the main K_L decay channels [8]. KTeV obtained the absolute branching fractions from the relative ratios, $K_{\mu 3}/K_{e3}$, K_{+-0}/K_{e3}, K_{000}/K_{e3}, K_{+-}/K_{e3}, K_{00}/K_{000}, exploiting the condition on $\sum BR_i$ = 1 - BR(K$_L \to rare$) = 99.93%. Few per mil accuracy on the semileptonic branching ratios, BR($K_{e3(\gamma)}$) = 0.4067$\pm$0.0011 and BR($K_{\mu 3(\gamma)}$) = 0.2701$\pm$0.0009, was achieved. The KTeV results led to major adjustements of all of the K_L decay modes but K_{+-0} and $K_{\mu 3}$, and of several rare K_L branching fractions for which the main K_L decay modes were used as normalization. Few months later, the NA48 experiment at CERN confirmed the KTeV result on the BR(K$_L \to \pi e \nu$). NA48 measured [9] the fraction of K$_{e3}$ events normalized to the K_L decays with two charged tracks in the final state, obtaning BR(K$_{e3(\gamma)}$) = 0.4010$\pm$0.0045, dependent on the world average of the BR(K$_{2trk}$). In year 2006 KLOE published [10] the results of the analysis of all the main K$_L$ decay modes, including K$_{e3}$, K$_{\mu 3}$, K_{+-0}, and K_{000}. At the ϕ–factory, where kaon pairs are produced from ϕ decays, the measurement of the absolute branching fractions is possible by tagging K_L beams from the reconstruction of K_S decays. Due to the long lifetime, about 46% of K_L's decays before reaching the calorimeters. The fraction depends on the K_L lifetime and to less extent on the cross section of the hadronic interactions. Since, analogously to the KTeV analysis, the measured branching fractions represent 99.64% of all of the K_L decays, the dependence and the uncertainty from the knowledge of the K_L lifetime was eliminated by imposing $\sum BR_i$ = 0.9964. The results, BR($K_{e3(\gamma)}$) = 0.4007$\pm$0.0015 and BR($K_{\mu 3(\gamma)}$) = 0.2698$\pm$0.0015, are well in agreement with the other precision measurements.

In year 2007, NA48/2 published the measurements of the branching fractions of the semileptonic decays of charged kaons normalized to the K$\to \pi^\pm\pi^0$ ($K_{2\pi}$) mode [11]. The experiment took data in year 2003–2004 and 2007–2008 mostly to measure the CP–violating asymmetry in the Dalitz–plot of the $K^\pm \to \pi^\pm\pi^+\pi^-$ decays. The analysis of the semileptonic decays was based on the events accumulated with a dedicated run in year 2003. The K$^+$ and the K$^-$ samples were separately processed obtaining consistent results on BR($K_{e3,\mu 3}$)/BR($K_{2\pi}$), of 4 per mil accuracy, about

2–σ greater and 5 times more precise than the world average at the time. The branching fractions, BR($K^{\pm}_{e3(\gamma)}$) = 0.0517±0.0004 and BR($K^{\pm}_{\mu3(\gamma)}$) = 0.0343±0.0002, with 0.8% and 0.6% relative precision, confirmed the BR($K^{+}_{e3(\gamma)}$) measurement of the BNL-E865 collaboration. The results from KLOE on the semileptonic decays of charged kaons, published in year 2008 [12], were based on the analysis of four independent samples collected in year 2003-2004, giving consistent results to few per cent precision level. The samples were selected by tagging K$^+$ (K$^-$) beams with the reconstruction of the two–body decays, $K^{\pm}_{2\pi}$ and $K^{\pm}_{\mu2}$. The averaged results from the four samples, BR($K^{\pm}_{e3(\gamma)}$) = (4.965 ± 0.053)% and BR($K^{\pm}_{\mu3(\gamma)}$) = (3.233 ± 0.040)%, have an accuracy of one per cent, dominated by the statistics of the control samples used to correct for data-MC discrepancies affecting efficiency evaluation.

KLOE, exploiting the possibility to tag the K_S decays, has also measured the BR(K_Se3) with 1.3% precision [13]. The analysis, especially focused on the control of the background from dominant $K_S \to \pi^+\pi^-$ decays, led to BR(K_Se3)/BR($K_S\pi^+\pi^-$) = (1.019 ± 0.013)×10^{-3}. The branching fractions for both charges, π^+e^- and π^-e^+, have been separately determined exploiting the excellent time resolution of the calorimeter. The ratios of the branching fractions were used together with i) the BR($K_S \to \pi^+\pi^-$)/BR($K_S \to \pi^0\pi^0$) measured by KLOE [14], ii) the assumption of lepton universality, and iii) the constraint $\sum BR_i$ = 1, to extract BR(K$_S e3(\gamma)$) = (7.028 ± 0.092)×10^{-4}.

3. The lifetimes

The best measurements of the K_S lifetime have been published in year 2002 by the NA48 [15] and KTeV [16] experiments. The relative precision reached, better than one per mil, exploited most of the experimental solutions envisaged to take under control the systematics for the measurement of Re ε'/ε to 10^{-4} accuracy. Independent analyses, on the neutral, $K_S \to \pi^0\pi^0$, and the charged, $K_S \to \pi^+\pi^-$, decay modes have been carried out giving consistent results whose average was (89.60 ± 0.07)× 10^{-12} s from NA48 data analysis [15], and (89.65 ± 0.07)× 10^{-12} s from the KTeV experiment.

KLOE obtained the K_L lifetime from the analysis of the spatial distribution of the dominant neutral K_L decay, $K_L \to \pi^0\pi^0\pi^0$. The result, to per mil precision, was τ_L = (50.92±0.30) ns, in agreement with the indirect determination of the lifetime, τ_L = (50.72±0.37) ns obtained from the measurements of all of the dominant K_L decays by imposing $\sum BR_i$ = 0.9964, as reported in Sec.2.

The charged kaon decay lifetime has been measured by KLOE with two independent methods based on the time resolution of the calorimeter and on the momentum resolution of the drift chamber. The first analysis, relying on photon time-of-flight measurements, was conducted on a sample of $K^{\pm} \to \pi^{\pm}\pi^0$ tagged by $K_{\mu2}$ decays. The proper kaon decay time is evaluated from the arrival time on the calorimeter of the photons from π^0 decays. The other method is based on the measurement of the charged vertex produced by $K^{\pm}$ decays in the drift chamber, tagged by the reconstruction of $K_{\mu2}$ in the opposite emisphere. The charged kaon lifetime, from the average of the fit results of the proper time distributions obtained for positive and negative kaon samples, and from the two independent methods, is (12.347 ± 0.030) ns.

From the comparison of the kaon semileptonic widths in the electron and muon decay channels, the lepton universality is probed to 5 per mil level, the same precision reached with τ decays.

4. The semileptonic form factors

The dependence of the semileptonic form factors from the momentum transfer, $\tilde{f}_+$(t) and $\tilde{f}_0$(t), is determined by the analysis of the kinematical distributions of the semileptonic decays. The $K_L \to \pi e \nu$ mode gives access to $\tilde{f}_+$(t) only because of the suppression of the contribution proportional to $\tilde{f}_0$(t) by a factor of $(m_e/m_\pi)^2$. The form factor measurements are also affected by the parameterization obtained from i) Taylor coefficients of the power expansion, or ii) K*(892) pole dominance assumption, or from iii) dispersive relations as proposed in Ref.[17, 18]. The latter parameterization of the vector form factor [18] gives, $\tilde{f}_+(t) = \exp\left[\frac{t}{m_\pi^2}(\Lambda_+ + H(t))\right]$, where $H(t)$ is obtained from $K-\pi$ scattering data and $\tilde{f}_+(0) = 1$; $\tilde{f}'_+(0) = \Lambda_+/m_\pi^2$ is the first–order coefficient of the Taylor expansion of the vector form factor. The pole–fit parameterization leads within experimental accuracy first– and second– order coefficients of the Taylor expansion consistent with those obtained from the dispersive relation. The dispersive approach is particurarly interesting in the case of the scalar form factor $\tilde{f}_0$(t),

$$\tilde{f}_0(t) = \exp\left(\frac{t}{\Delta_{K\pi}} \ln(C - G(t))\right), \tag{4.1}$$

where $\Delta_{K\pi} = (m_K^2 - m_\pi^2)$, $C = \tilde{f}_0(\Delta_{K\pi})$, $\tilde{f}_0(t) = 1$, and $G(t)$ is derived from $K-\pi$ scattering data [17]. At $t_{CT} \equiv \Delta_{K\pi}$, the Callan–Treiman theorem [19, 20] implies

$$\tilde{f}_0(t_{CT}) = \frac{f_K}{f_\pi}\frac{1}{f_+(0)} + \Delta_{CT}, \tag{4.2}$$

where $\Delta_{CT} \sim \mathscr{O}(m_{u,d}/4\pi F_\pi)$ has been estimated from ChPT at NLO in the limit of isospin conservation, $\Delta_{CT} = (-3.5 \pm 8) \times 10^{-3}$ [21]. The theorem fixes the value of the scalar form factor at t_{CT} to the ratio of the meson decay constants $f_K/f_\pi \times 1/f_+(0)$, thus providing the way to experimentally test the precision LQCD determination of such a combination of golden quantities, irrespectively from CKM unitarity.

The most recent measurements of the K_{e3} and $K_{\mu 3}$ Dalitz–plot distributions have been published by ISTRA+ [22], KTeV [23, 24], NA48 [25, 26], and KLOE [27, 28]. The form factor slope and curvature from charged kaon decays, $K^-_{\mu 3}$ and K^-_{e3}, have been obtained in 2004 by ISTRA+ [29], at the 70–GeV protosyncrotron of the IHEP, in Protvino. The experiment, devoted to high–statistics studies of semileptonic kaon decays, performed the analyses of both $K^-_{\mu 3}$ [30] and K^-_{e3} [22] channels on the basis of about half and one million selected events, from a dedicated run in year 2001. The results on the slopes of both vector and scalar form factors, are obtained by a fit to the E_μ–E_π distribution, giving λ'_+ = 0.0215±0.0060, λ''_+ = 0.0010±0.0010, and λ'_0 = 0.0160±0.0021.

The KTeV data analysis published in year 2004 was based on same data sample as for the K_L dominant branching fractions, with 1.9 and 1.5 million selected events from K^0_{e3} and $K^0_{\mu 3}$ decays, respectively. The results, averaged on the two modes taking into account the error matrix, are: $\lambda'_+ = (20.6 \pm 1.8)\ \times 10^{-3}, \lambda''_+ = (3.2 \pm 0.7)\ \times 10^{-3}$, and $\lambda_0 = (13.7 \pm 1.3)\ \times 10^{-3}$, where the comparable statistical and systematic uncertainties have been added in quadrature. The KTeV analysis has been revised in year 2009 [23] to determine the parameters of the dispersive approach [17, 18] obtaining Λ_+ = 0.02509±0.00055 and ln$C$ = 0.1915±0.0122.

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NA48 published the results of the analysis of the phase–space distributions of the $K_L \to \pi e \nu$ [26] and the $K_L \to \pi \mu \nu$ [25] in year 2004 and 2007. The accuracy of the results, $\lambda'_+ = (28.0 \pm 1.9) \times 10^{-3}$ and $\lambda''_+ = (0.2 \pm 0.4) \times 10^{-3}$, is dominated by the knowledge of the neutral kaon spectrum. The NA48 fit results of the $K^0_{\mu 3}$ events, $\lambda'_+ = (20.5 \pm 2.2) \times 10^{-3}, \lambda''_+ = (2.6 \pm 0.9) \times 10^{-3}, \lambda_0 = (9.5 \pm 1.1) \times 10^{-3}$, show a 2–$\sigma$ disagreement with the other measurements of the scalar form factor. The tension is even worst in the case of the more precise result on $\ln C$ from the dispersive parameterization that gives $\Lambda_+ = 0.0233 \pm 0.0005$ and $\ln C = 0.1438 \pm 0.0080$. KLOE measured the vector and scalar form factor parameters in year 2006 and 2007 using both semileptonic K_L channels [28, 27]. The results from K_{e3} and $K_{\mu 3}$ analyses have been averaged [27] obtaining $\lambda'_+ = (25.6 \pm 1.7) \times 10^{-3}$, $\lambda''_+ = (1.5 \pm 0.8) \times 10^{-3}$, $\lambda_0 = (15.4 \pm 2.2) \times 10^{-3}$ from Taylor expansion, and $\Lambda_+ = (25.7 \pm 0.6) \times 10^{-3}$, $\ln C = 0.204 \pm 0.024$ from dispersive parameterization of both $\tilde{f}_+(\mathrm{t})$ and $\tilde{f}_0(\mathrm{t})$.

5. Vus from semileptonic and $K_{\mu 2}$ decays

KLOE published in year 2006 the absolute measurement of the $K_{\mu 2}$ branching fraction [31], $\mathrm{BR}(K_{\mu 2(\gamma)}) = 0.6366 \pm 0.0017$, fully inclusive of radiative decays.

The measurement is used together with the $\pi^\pm \to \mu^\pm \nu(\gamma)$ decay rate [32] and the lattice calculation $f_K/f_\pi = 1.189(7)$ [33] to extract $|V_{us}|/|V_{ud}| = 0.2323(15)$ from Eq.1.2.

From K_{l3} data, the FlaviaNet Working Group on Kaon Decays has obtained $|V_{us}| = 0.2246 \pm 0.0012$ [6]. The value was used by the authors in a fit with the measurements of $|V_{us}|/|V_{ud}|$ and $|V_{ud}| = 0.97418(26)$ [34] giving $|V_{ud}| = 0.97417(26)$ and $|V_{us}| = 0.2253(9)$, from which $G_{\mathrm{CKM}} = G_{\mathrm{F}}(|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2)^{1/2} = (1.1662 \pm 0.0004) \times 10^{-5}\ \mathrm{GeV}^{-2}$ is derived, in agreement with the results from the muon lifetime measurements [35]. The sensitivity of the quark–lepton universality test is competitive and even better than the measurements from τ decays and the electroweak precision tests [1, 36].

The KLOE-2 experiment that is planned to take data at DAˇNE in year 2011-14, can improve the accuracy on the measurement of K_L, $K^\pm$ lifetimes and on the $K_S e3$ branching ratio leading to a fractional uncertainty on $|V_{us}| \times f_+(0)$ of 0.14% [37].

The presence of scalar currents from charged Higgs (H^+) exchange [38] is expected to affect $K^\pm \to \mu^\pm \nu$ decay such that the comparison of helicity-suppressed with helicity-allowed kaon modes could reveal BSM extensions of the Higgs sector. The ratio, $R_{\ell 23} = \frac{|V_{us}|(K_{\ell 2})}{|V_{us}|(K_{\ell 3})} \times \frac{V_{ud}(0^+ \to 0^+)}{V_{ud}(\pi_{\mu 2})}$ was used to bound the m_{H^+} – $\tan\beta$ plane, according to

$$R_{\ell 23} = \left| 1 - \frac{m^2_{K^+}}{m^2_{H^+}} \left(1 - \frac{m^2_{\pi^+}}{m^2_{K^+}} \right) \frac{\tan^2\beta}{1 + 0.01\tan\beta} \right| [38] \tag{5.1}$$

Experimental data on $K_{\mu 2}$ and $K_{\ell 3}$ rules out charged Higgs lighter than 260 GeV at high $\tan\beta$ ($\tan\beta > 70$), as discussed in Ref.[6]. The implications of flavour physics constraints on two–Higgs–doublet models are analyzed for instance in Ref.[39] where the interplay with direct searches at the LHC is pointed out.

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