

## Leptonic $D/D_s$ decays and decay constants

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We review recent measurements of leptonic decays of D mesons and compare the results on the decay constants with lattice QCD calculations.

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## 1. Introduction

We review recent measurements of leptonic decays  $D_{(s)}^+ \rightarrow \ell^+ \nu$ , where  $\ell = e, \mu, \tau$ . In the Standard model (SM) these decays proceed via tree diagram; the decay rate is given by

$$\Gamma(D_{(s)}^+ \rightarrow \ell^+ \nu) = \frac{G_F^2}{8\pi} f_{D_{(s)}^+}^2 m_\ell^2 M_{D_{(s)}^+} \left(1 - \frac{m_\ell^2}{M_{D_{(s)}^+}^2}\right)^2 |V_{cd(cs)}|^2 \quad (1.1)$$

where  $m_\ell$  is the lepton mass and  $f_{D_{(s)}^+}$  the decay constant, a quantity related to the overlap of the heavy and light quark wave functions at zero spatial separation. The experimental determination of decay constant is one of the most important tests of calculations involving nonperturbative QCD.

The experimental techniques depend on the collision energy used to produce  $D$  mesons. At CLEO-c they use either  $e^+e^-$  collisions at  $\psi(3770)$  to produce  $D^+D^-$  pairs or the process  $e^+e^- \rightarrow D_s D_s^*$  at 4170 MeV to produce  $D_s$  mesons. At Belle and BaBar the collision energy is tuned to  $\Upsilon(4S)$  resonance and the  $D$  mesons produced in the continuum process  $e^+e^- \rightarrow c\bar{c}$  have been used. A common approach in these analyzes is to fully reconstruct one  $D$  meson as “the tag” and then analyze the decay of the other  $D$  to extract exclusive or inclusive properties.

## 2. CLEO-c measurements

CLEO-c collaboration has published recently the branching fraction measurements of  $D^+ \rightarrow \mu^+ \nu$  [1],  $D_s^+ \rightarrow \mu^+ \nu$  [2] and  $D_s^+ \rightarrow \tau^+ \nu$  in three  $\tau$  lepton decay modes,  $\tau^+ \rightarrow \pi^+ \nu$  [2],  $\tau^+ \rightarrow e^+ \nu \nu$  [3] and  $\tau^+ \rightarrow \rho^+ \nu$  [4].

The  $D^+ \rightarrow \mu^+ \nu$  branching fraction was measured with the data sample of 818 pb<sup>-1</sup> collected at the  $\psi(3770)$  resonance [1]. The tag side  $D^-$  meson was reconstructed with the efficiency of 22%, using the six most dominant decay modes. In total they found 460 000 signal tags. In the signal side they required a single extra track of opposite charge to the tag  $D$  meson and with less than 300 MeV energy deposited in the electromagnetic calorimeter to identify this track as a muon. To identify the missing neutrino, they require a missing mass to be near zero. The background is found to be due the  $D^+$  decays to  $\tau^+ \nu$ ,  $\pi^0 \pi^+$ ,  $\bar{K}^0 \pi^+$  and other. They found  $149.7 \pm 12.0 \mu\nu$  signal events, if they fixed the  $\tau\nu/\mu\nu$  ratio by the SM value or  $153.9 \pm 13.5 \mu\nu$  and  $13.5 \pm 15.3 \tau\nu$  events, if they left the ratio floated. The resulting branching fraction and the decay constant are  $\mathcal{B}(D^+ \rightarrow \mu^+ \nu) = (3.82 \pm 0.32 \pm 0.09) \times 10^{-4}$  and  $f_{D^+} = (205.8 \pm 8.5 \pm 2.5)\text{MeV}$ , if the  $\tau\nu/\mu\nu$  ratio is fixed, and  $\mathcal{B}(D^+ \rightarrow \mu^+ \nu) = (3.93 \pm 0.35 \pm 0.09) \times 10^{-4}$  and  $f_{D^+} = (207.6 \pm 9.3 \pm 2.5)\text{MeV}$ , if the ratio is floating. The most precise unquenched lattice QCD calculation [5] gives  $f_{D^+} = (207 \pm 4)\text{MeV}$ , a value in a good agreement with the measurement.

The measurements of leptonic decays of  $D_s$  meson were performed using 600 pb<sup>-1</sup> of experimental data collected at 4170 MeV, where a sizable fraction of  $e^+e^-$  collisions results in a production of  $D_s^+ D_s^{*-}$  pair. The tag side consisted of fully reconstructed  $D_s^{*-} \rightarrow D_s^- \gamma$  decays, with the  $D_s$  reconstructed in nine decay modes. The tags were selected in a missing mass recoiling against the  $D_s \gamma$ , since for the  $D_s D_s^*$  events this quantity always peaks at  $M_{D_s}$ . In total they found 44 000 signal tags.

The measurements of  $D_s^+ \rightarrow \mu^+ \nu$  and  $D_s^+ \rightarrow \tau^+ \nu$  with  $\tau^+ \rightarrow \pi^+ \nu \nu$  were done simultaneously [2]. In the signal side they require a single extra track with the opposite charge to tag side

and not being identified as an electron or a kaon. They considered two cases for the track energy deposited in the electromagnetic calorimeter: (a)  $E_{\text{cal}} < 300$  MeV, which selects 99% of  $\mu$  and 55% of  $\pi$ , and (b)  $E_{\text{cal}} > 300$  MeV which selects 1% of  $\mu$  and 45% of  $\pi$ . By fitting simultaneously the missing mass spectra of (a) and (b) they found  $235.5 \pm 13.8$   $\mu\nu$  events and  $125.6 \pm 15.7$   $\tau\nu$  ( $\tau \rightarrow \pi\nu$ ) events. The background was estimated to about 10 events in total and originates mainly from  $\tau^+ \rightarrow \pi^+\pi^0\nu$ ,  $\mu^+\nu\nu$  and  $D_s^+ \rightarrow \pi^+\pi^0\pi^0$ ,  $K^0\pi^+$ ,  $\eta\pi^+$  decays. They measured  $\mathcal{B}(D_s^+ \rightarrow \mu^+\nu) = (0.565 \pm 0.045 \pm 0.017)\%$  and  $\mathcal{B}(D_s^+ \rightarrow \tau^+\nu) = (6.42 \pm 0.81 \pm 0.18)\%$ , and found their ratio to be consistent with the SM prediction. The corresponding results for the decay constant are  $f_{D_s^+} = (257.6 \pm 10.3 \pm 4.3)\text{MeV}$  ( $\mu^+\nu$ ) and  $f_{D_s^+} = (278.0 \pm 17.5 \pm 4.4)\text{MeV}$  ( $\tau^+\nu$ ).

For the measurement of  $D_s^+ \rightarrow \tau^+\nu$ ,  $\tau^+ \rightarrow \rho^+\nu$  [4] they use an additional observable, an extra energy in the calorimeter ( $E_{\text{extra}}$ ) to discriminate efficiently between signal and background, since for the  $\tau\nu$  decays the missing mass spectrum doesn't peak at zero. The signal side consisted of an event with exactly one charged pion forming  $\rho^+ \rightarrow \pi^+\pi^0$ . By fitting the missing mass distribution in the first two  $E_{\text{extra}}$  bins they found  $155.2 \pm 16.5$  and  $43.7 \pm 11.3$  events. They measured  $\mathcal{B}(D_s^+ \rightarrow \tau^+\nu) = (5.52 \pm 0.57 \pm 0.21)\%$  and  $f_{D_s^+} = (257.8 \pm 13.3 \pm 5.2)\text{MeV}$ .

CLEO-c also measured  $D_s^+ \rightarrow \tau^+\nu$  branching fraction using  $\tau^+ \rightarrow e^+\nu\nu$  decays [3]. Here they selected only the three cleanest tag modes ( $D_s^- \rightarrow \phi\pi^-$ ,  $D_s^- \rightarrow K^-K^{*0}$ ,  $D_s^- \rightarrow K^-K_s^0$ ) and the events with exactly one electron in the signal side. Since there are three unseen neutrinos in the signal side, the missing mass is not good signal discriminant. Instead, they used  $E_{\text{extra}}$  below 400 MeV to discriminate against the main background arising from  $D_s$  semileptonic decays. They found  $180.6 \pm 15.9$  signal decays; they measured  $\mathcal{B}(D_s^+ \rightarrow \tau^+\nu) = (5.30 \pm 0.47 \pm 0.22)\%$  and  $f_{D_s^+} = (252.5 \pm 11.1 \pm 5.2)\text{MeV}$ .

In summary, CLEO-c collaboration measured the branching fractions of  $D_s^+ \rightarrow \mu^+\nu$  and  $D_s^+ \rightarrow \tau^+\nu$  with consistent results in three different  $\tau$  decay modes. The decay constants extracted from  $\mu\nu$  and  $\tau\nu$  branching fractions are consistent to each other. They also report the weighted average of their decay constant measurements:  $f_{D_s^+} = (259.0 \pm 6.2 \pm 3.0)\text{MeV}$ .

### 3. Belle measurement

Belle measured  $D_s^+ \rightarrow \mu^+\nu$  branching fraction using the charm production at  $\Upsilon(4S)$  via a process  $e^+e^- \rightarrow c\bar{c}$  [6]. They used events of the type  $D^{\pm,0}K^{\pm,0}XD_s^{*\pm}$ , where  $X$  denotes additional particles from fragmentation, assumed to be pions or pions and a photon. The tag side consists of  $D$  and  $K$  mesons; the neutral or charged  $D$  meson was reconstructed in decay modes of the type  $D \rightarrow Kn\pi$ ,  $n = 1, 2, 3$ , where  $K$  and  $\pi$  can be charged or neutral. The total branching fraction amounted to about 25%. The signal side consists of  $D_s^{*\pm} \rightarrow D_s^+\gamma$ . They used the recoil mass against  $DKX\gamma$  to count all  $D_s$  meson decays and the recoil mass against  $DKX\gamma\mu$  to count  $D_s \rightarrow \mu\nu$  decays.

By using  $548 \text{ fb}^{-1}$  of experimental data they obtained  $\mathcal{B}(D_s^+ \rightarrow \mu^+\nu) = (0.644 \pm 0.076 \pm 0.057)\%$ , a result consistent with the CLEO-c measurement. The corresponding decay constant is  $f_{D_s^+} = (275 \pm 16 \pm 12)\text{MeV}$ .

### 4. BaBar measurements

The most recent measurements of leptonic  $D_s$  decays are from BaBar experiment [7]. They

**Table 1:** Summary of recent  $f_{D_s^+}$  measurements. Units are MeV.

mode	CLEO-c	Belle	BaBar
$\mu^+ \nu$	$257.6 \pm 10.3 \pm 4.3$	$275 \pm 16 \pm 12$	$265.7 \pm 8.4 \pm 7.7$
$\tau_{\pi\nu}^+ \nu$	$278.0 \pm 17.5 \pm 4.4$		
$\tau_{\rho\nu}^+ \nu$	$257.8 \pm 13.3 \pm 5.2$		
$\tau_{e\nu\nu}^+ \nu$	$252.6 \pm 11.2 \pm 5.6$		$247 \pm 13 \pm 17$
$\tau_{\mu\nu\nu}^+ \nu$			$243 \pm 12 \pm 14$
average	$259.0 \pm 6.2 \pm 3.0$	$275 \pm 16 \pm 12$	$258.6 \pm 6.4 \pm 7.5$

used their final data set ( $521 \text{ fb}^{-1}$ ) and the same method as Belle, but they measured also  $D_s^+ \rightarrow \tau^+ \nu$  in  $\tau \rightarrow e\nu\nu$  and  $\tau \rightarrow \mu\nu\nu$  decay modes. The signal yields in the latter cases were extracted from  $E_{\text{extra}}$  distributions.

They obtained the following branching fractions, which are consistent with the CLEO-c and Belle results:  $\mathcal{B}(D_s^+ \rightarrow \mu^+ \nu) = (0.602 \pm 0.038 \pm 0.034)\%$ ,  $\mathcal{B}(D_s^+ \rightarrow \tau_{\tau \rightarrow e\nu\nu}^+ \nu) = (5.07 \pm 0.52 \pm 0.68)\%$  and  $\mathcal{B}(D_s^+ \rightarrow \tau_{\tau \rightarrow \mu\nu\nu}^+ \nu) = (4.91 \pm 0.47 \pm 0.54)\%$ . The results for the decay constant are consistent to each other:  $f_{D_s^+} = (265.7 \pm 8.4 \pm 7.7)\text{MeV}$  ( $\mu\nu$ ),  $f_{D_s^+} = (247 \pm 13 \pm 17)\text{MeV}$  ( $\tau_{e\nu\nu}\nu$ ) and  $f_{D_s^+} = (243 \pm 12 \pm 14)\text{MeV}$  ( $\tau_{\mu\nu\nu}\nu$ ). They provide also an error-weighted average of above results,  $f_{D_s^+} = (258.6 \pm 6.4 \pm 7.5)\text{MeV}$ , which is consistent and very competitive to CLEO-c average.

## 5. Comparison with lattice calculations

The measured values of  $D_s$  decay constant are summarized in Table 1. As we can see the measurements are consistent to each other. Recent world average of the decay constant measurements given by the HFAG group [8] is  $f_{D_s^+} = (257.3 \pm 5.3) \text{ MeV}$ . The most precise unquenched lattice QCD calculations are from the HPQCD collaboration; their latest result is  $f_{D_s^+} = (248.0 \pm 2.5) \text{ MeV}$  [9], which is in agreement with the recent world average of measurements within 1.6 standard deviations.

## 6. Future prospects

Today's experimental uncertainties of decay constants are about two times larger than the uncertainties of the most precise lattice QCD calculations. The decay constants are known to 4.3% ( $f_{D^+}$ ) and 2% ( $f_{D_s}$ ) precision, while the uncertainties of lattice predictions are estimated to 2% and 1%, respectively. The relative errors of single measurements of  $f_{D_s}$  from Table 1 range from 4.3% to 8.6%, depending on the decay mode and the experimental conditions. None of the measurement is dominated by the irreducible systematic error, thus the experimental uncertainties can be further reduced by analyzing larger data sets.

In the near future one can expect an updated measurement of  $f_{D_s}$  from the Belle collaboration with their final data set of  $1 \text{ ab}^{-1}$ . As shown by BaBar, a competitive measurement in  $D_s^+ \rightarrow \tau^+ \nu$

decays is also possible at B-factories; including also this decay mode the new Belle measurement can reach the sensitivity of 3% - 4%.

At BES-III they plan to accumulate a total of  $20 \text{ fb}^{-1}$  of data at  $\psi(3770)$  and a smaller  $D_s^+ D_s^-$  sample at 4030 MeV or  $D_s^+ D_s^{*-}$  sample at 4170 MeV [10]. They estimate the precision of  $D^+ \rightarrow \mu^+ \nu$  and  $D_s^+ \rightarrow \mu^+ \nu$  branching fraction measurements with these data samples to 2%. The corresponding decay constant precisions are estimated to 1.5% for  $f_{D^+}$  and 1.3% for  $f_{D_s}$ .

Similar experimental precision for  $f_{D_s}$  can be expected at the future super B factories; At Belle II [11] they estimate, that with  $5 \text{ fb}^{-1}$  of data the branching fraction of  $D_s^+ \rightarrow \mu^+ \nu$  can be measured with an accuracy of around 4% corresponding to a  $\sim 2\%$  error on  $f_{D_s}$ ; at larger data sets the precision will be dominated by systematical errors.

## 7. Conclusions

We gave a review of recent measurements of leptonic  $D_{(s)}^+$  decays and decay constants. Several years ago, when the first high precision lattice QCD result became available [5], the discrepancy of more than 3 standard deviations with the measured  $f_{D_s}$  caused some excitement. Since then the new measurements with significantly improved precision have been reported and also new unquenched lattice calculations have been performed resulting in a higher than previous decay constant value. The new lattice result is consistent with the world average of measurements. In the future we can expect further improvements in experimental and theoretical precisions.

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