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Measurement of Scattering Lengths from Ke4 and K3π Decays

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Kaon decays are an ideal laboratory to study strong interaction dynamics in the low energy regime. In the last few years it has become possible to study low energy scattering with improved precision thanks to the high statistics measurements of $K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu$ (Ke4) and $K^{\pm} \rightarrow \pi^{0}\pi^{0}\pi^{\pm}$ (K3π) decays performed by the NA48/2 experiment at the CERN SPS. The analysis of more than one million $K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu$ (Ke4) rare decays allows a model independent approach to the study of low energy π-π scattering close to threshold providing an accurate test of Chiral Perturbation Theory (ChPT) predictions. This result is combined with the independent and complementary NA48/2 result obtained in the analysis of about 60 millions $K^{\pm} \rightarrow \pi^{0}\pi^{0}\pi^{\pm}$ (K3π), leading to an experimental measurement of a_0 and a_2, the isospin 0 and 2 S-wave π-π scattering lengths, of unprecedented precision. This result is in perfect agreement with the ChPT prediction.

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1. Introduction

In the low-energy regime, the perturbative description of the strong interaction breaks down, as the strong coupling constant becomes of O(1). Chiral Perturbation Theory (ChPT) is an effective theory, which circumvents this problem by making use of the chiral symmetry of the theory in the limit of vanishing quark masses. Spontaneous breaking of the chiral symmetry generates eight pseudo-scalar Goldstone bosons, among them pions and kaons. They obtain their small but non-zero masses by the additional symmetry breaking of non-vanishing quark masses. The values of the isospin 0 and 2 S-wave scattering lengths a_0 and a_2 are directly connected with the pion mass, as shown by Weinberg [1] in the sixties. ChPT has been particularly powerful in describing π-π scattering at low energy and over the past forty years calculations at leading order and the two subsequent orders have converged towards very precise values of the underlying constants of the theory. In the framework of ChPT, the scattering lengths can accurately be predicted to $a_0 m_{\pi+} = 0.220 \pm 0.005$ and $a_2 m_{\pi+} = -0.044 \pm 0.010$ [2]. Thus, precise experimental measurements of the scattering lengths are a crucial test of ChPT.

Traditionally, measurements have been performed in the semileptonic rare decay $K^{\pm} \to \pi^{+}\pi^{-} e^{\pm}\nu$ (Ke4). An early measurement by the Geneva-Saclay experiment analyzed 30,000 events [3]. More recently, the BNL experiment E865 has measured $(a_0 - a_2)m_{\pi+} = 0.258 \pm 0.013$ from about 400,000 Ke4 events [4]. Another recent determination of the scattering lengths has been carried out by the DIRAC experiment at CERN from the lifetime of pionium atoms obtaining $(a_0 - a_2)m_{\pi+} = 0.264^{+0.020}_{-0.011}$ from an analysis of a part of their data (6,500 events) [5].

Here, the most recent results, based on two different approaches, from the NA48/2 Collaboration are reported .

Thanks to the very high statistics (1.1×10^6 Ke4 events) we have determined a_0 and a_2 in semileptonic Ke4 decays, with a greatly improved precision with respect to previous measurements.

In addition in the $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$ hadronic decay mode (60×10^6 events), a Wigner-cusp from rescattering in the decay amplitude was discovered at $m(\pi^0\pi^0) = 2m_{\pi+}$. This allows a determination of a_0 and a_2 with a completely new method and with similar precision.

The combination of both measurements provides one of the most stringent tests of Chiral Perturbation Theory.

2. The NA48/2 experiment at CERN

The main goal of the NA48/2 experiment was to search for direct CP violation in decays of charged kaons into three pions. Thanks to the very high statistics accumulated many other interesting measurements have been performed with unprecedented precision.

The NA48/2 experiment beamline [6] was designed to deliver simultaneously K^+ and K^- produced from SPS primary 400 GeV/c protons. Charged particles with momentum 60 ± 3 GeV/c are selected by an achromatic system of dipole magnets, which splits the two beams in the vertical plane and then recombines them on a common axis. The ratio of K^+ and K^- fluxes is about 1.8.

The NA48 detector [7] consists of a magnetic spectrometer, made of four drift chambers and a dipole magnet in the middle, to track charged particles. The momentum resolution is 1.4% for 20 GeV/c charged tracks. The spectrometer is followed by a plastic scintillator hodoscope used to provide fast trigger signals and precise time measurements for charged particles. Then there is a high resolution liquid krypton electromagnetic calorimeter used for photon detection and particle identification. It is an almost homogeneous ionization chamber, 27 X_0 deep, segmented transversally into 13,248 cells $2 \times 2\text{cm}^2$ each, with good time resolution and energy resolution of about 1% for 20 GeV electrons and photons. Further downstream there is a hadron calorimeter and a muon detector.

The trigger was designed to efficiently select events with three charged tracks as well as $K^\pm \to \pi^0\pi^0\pi^\pm$ events. In total, about 2×10^9 three-tracks events and about 90×10^6 $K^\pm \to \pi^0\pi^0\pi^\pm$ events were recorded in the two years of data-taking (2003-2004).

3. π-π Scattering Lengths Measurements in Ke4 and K3π Decays

3.1 Measurement of $K^\pm \to \pi^+\pi^-e^\pm\nu$ (Ke4) decays

The Ke4 semileptonic decay is a rare decay with a branching fraction of about 4.1×10^{-5} [8] but it is simple theoretically giving access to the final state interaction of two pions in absence of any other hadron. Its amplitude depends on the two complex phases δ_S and δ_P, which are the S and P wave scattering phase shifts for isospin I = 0. In Ke4 decays, their difference $\delta = \delta_S - \delta_P$ can be measured as a function of $M_{\pi\pi}$.

The selection of the Ke4 requires three charged tracks forming a common vertex. Only one of them should be consistent with the electron hypothesis, i.e. an associated energy deposit in the calorimeter consistent with the measured track momentum. The other two tracks should have opposite signs and should not give signal in the muon detector. The total background contamination is very small, at 0.5% level, coming mainly from K3π decay with a pion misidentified as electron or with a pion decay $\pi \to e\nu$ and it is determined from data by using *wrong-sign* combinations (events suppressed by the $\Delta S = \Delta Q$ rule). In the selection a restricted kaon momentum range [54,66] GeV/c is applied in order to minimize the background contamination.

The 1.1 million event total sample is distributed over a grid of $(10\times5\times5\times5\times12 = 15,000)$ equally populated boxes in the five independent Cabibbo-Maksymowicz variables [9] space that describes the decay kinematics: the dipion and dilepton invariant masses s_π and s_e and the angles θ_π and θ_e of π^+ and e^+ with respect to the $\pi\pi$ and $e\nu$ directions in the $\pi\pi$ and $e\nu$ rest frames, respectively, and the angle ϕ between the $\pi\pi$ and $e\nu$ decay planes. For each bin in $M_{\pi\pi}$, comparing data and MonteCarlo simulation, ten independent five parameters fits are performed. This allows to determine the axial and vectorial form factors in terms of partial S and P wave expansions which do not depend upon any particular model. The GEANT3 [10] MonteCarlo sample has 25 times more statistics than the data sample. The agreement between data and MonteCarlo distributions is very good for each of the variables used to define the form factors. The Roy equations [11], based on analicity and unitarity, are used to fit the form factors q^2 dependence. Through the Roy equations, after an extrapolation from $\pi\pi$ scattering data at higher energy, it is possible to correlate the δ phase shift to the scattering lengths a_0 and a_2.

The unprecedented precision of NA48/2 result triggered theoretical work to determine the effect of isospin symmetry breaking on phase shift $\delta = \delta_S - \delta_P$. Effects from isospin symmetry breaking ($m_{\pi+} \neq m_{\pi0}$ and $m_d \neq m_u$) do have a significant impact on the value of the phase shift

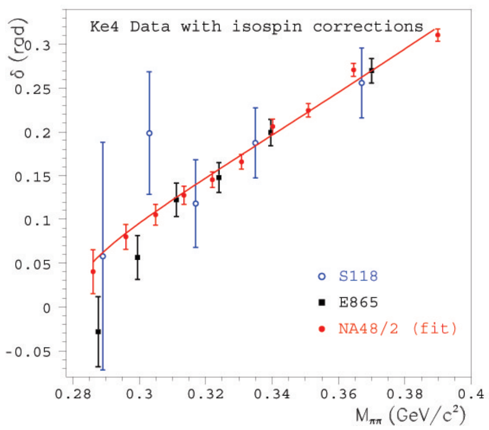

Figure 1: Measurements of the phase difference $\delta_s - \delta_p$ corrected for isospin mass effects as a function of $M_{\pi\pi}$ in Ke4 available results. The precision in the 10 points measured by NA48/2 exceeds previous measurements. The line shows the best fit to a_0 and a_2 of NA48/2 alone.

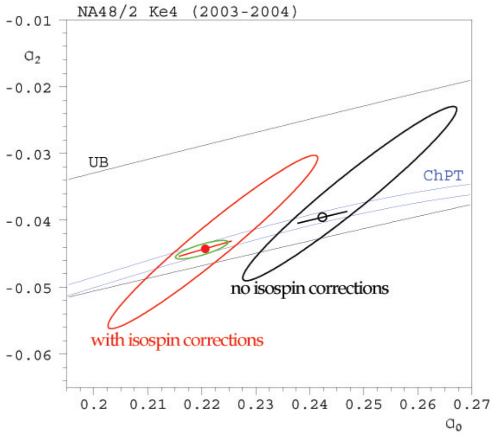

Figure 2: Fit results of the NA48/2 Ke4 data in the (a_0, a_2) plane without (black) and with (red) isospin mass effects). Circles are the result of a one-parameter fit imposing the ChPT constraint. The small (green) ellipse corresponds to the best prediction for ChPT.

difference. A ChPT calculation gives a shift of 12-15 mrad for $m_{\pi\pi} > 300\ MeV/c^2$ [12]. Note, that these corrections had also to be applied on the data of the previous Ke4 measurements.

The NA48/2 phase measurements fit results from Ke4 data are shown in Fig.1 together with the earlier measurements of the Geneva-Saclay [3] and BNL E865 [4] experiments. Fig.2 shows the fit of NA48/2 Ke4 data in the (a_0, a_2) plane with and without isospin mass corrections. Errors are statistical only. The ellipses correspond to 68% CL of the statistical uncertainties.

After subtracting isospin effects, numerical solution of Roy equations are used to extract scattering lengths from the phase measurements in a 2-parameter fit: $a_0 m_{\pi+} = 0.2220 \pm 0.0128_{stat} \pm 0.0050_{syst} \pm 0.0037_{theor}$; $a_2 m_{\pi+} = -0.0432 \pm 0.0086_{stat} \pm 0.0034_{syst} \pm 0.0028_{theor}$; with a correlation of 96.7%.

Using the ChPT constraint (the so-called Universal Band) we obtain, from a one-parameter fit: $a_0 m_{\pi+} = 0.2206 \pm 0.0049_{stat} \pm 0.0018_{syst} \pm 0.0064_{theor}$, in excellent agreement with the ChPT prediction [13]. The present world average on the pionic scattering length from Ke4, dominated by the NA48/2 results presented here, is: $a_0 m_{\pi+} = 0.2199 \pm 0.0125_{exp} \pm 0.0037_{th}$; $a_2 m_{\pi+} = -0.0430 \pm 0.0083_{exp} \pm 0.0028_{th}$.

3.2 Measurement of $K^\pm \to \pi^0\pi^0\pi^\pm$ (K3π) Decays and the Cusp Analysis

As first reported from an analysis of a partial data set [14], the $\pi^0\pi^0$ invariant mass ($M_{\pi^0\pi^0}$ or M_{00}) spectrum of $K^\pm \to \pi^0\pi^0\pi^\pm$ decays shows a cusp-like behaviour in the region around $M_{00} = 2m_{\pi+}$, as shown in Fig.3.

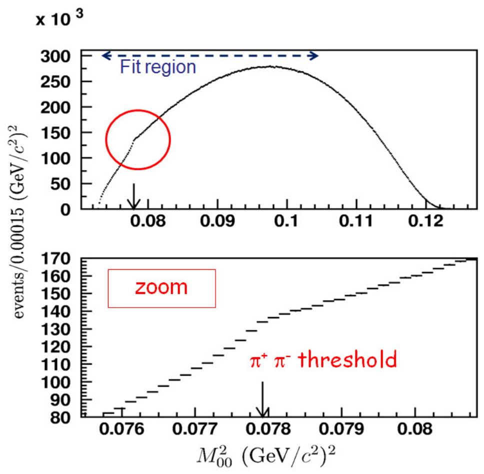

Figure 3: M^2_{00} spectrum of $K^\pm \to \pi^0\pi^0\pi^\pm$ events. The arrow indicates the cusp point at $4m^2_{\pi+}$.

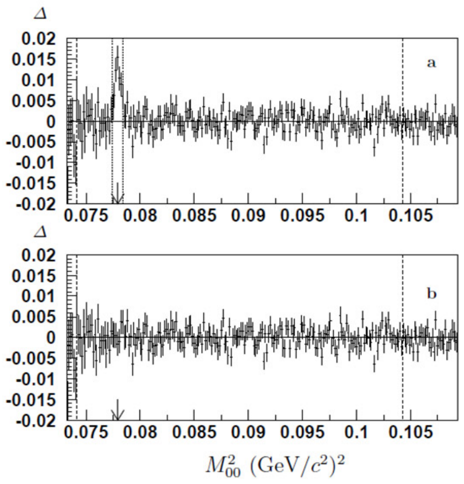

Figure 4: Fit residuals without (a) and with (b) pionium contribution included.

The interpretation of this structure was given by Cabibbo [15], expressing the $K^\pm \to \pi^0\pi^0\pi^\pm$ decay amplitude in terms of the $\pi^+\pi^- \to \pi^0\pi^0$ amplitude at threshold.

The existence of this threshold anomaly had been first predicted in 1961 [16] as due to $\pi^+\pi^- \to \pi^0\pi^0$ strong rescattering, having different real and imaginary behaviour below and above the $2\pi^+$ production threshold.

More recently several different theoretical approaches, using different formalisms and hypothesis, have been developed in order to exploit the experimental precision obtained by NA48/2 thanks to the sizeable statistics collected (60.31×10^6 events) and the excellent mass resolution $M_{\pi^0\pi^0}$.

Two theoretical approaches were used in the analysis of data: the Cabibbo-Isidori (CI) formulation [17], and the more recent Bern-Bonn (BB) formulation [18]. The BB approach uses a non-relativistic Lagrangian framework, which automatically satisfies unitarity and analiticity constraints, and allows to include electromagnetic contributions in a standard way. In the CI approach, the structure of the cusp singularity is treated using unitarity, analiticity and cluster decomposition properties of the S-matrix. The decay amplitude is expanded in powers of scattering lengths up to the second order and electromagnetic effects are omitted. To take them into account, the BB electromagnetic effect is used to correct the CI amplitude.

The $K3\pi$ analysis is essentially based on the liquid krypton calorimeter to identify the γs from the π^0 decay. The scattering lengths a_0 and a_2 were determined from fits of the full NA48/2 data set to the M_{00} spectrum. The free fit parameters were the scattering lengths a_0 and $a_0 - a_2$, the Dalitz plot parameters, the normalization, and the fraction f_{atom} of pionium formation at the cusp point. The fit region is chosen to reach a minimum total error. The fit residuals are shown in Fig.4 (a) and (b) without and with taking pionium formation into account.

In both approaches, the M^2_{00} distribution is fitted using the detector response matrix obtained from a large MonteCarlo simulation, to extract the scattering lengths $a_0 - a_2$, a_2 and the Dalitz slopes. The results are in good agreement in spite of the fact that they are based on different hypothesis. The final results presented here [19] are obtained from the fits to the most complete description of rescattering effects [20] (BB approach) with radiative corrections and the pionium fraction left floating in the fit:

$$(a_0 - a_2)m_{\pi+} = 0.2571 \pm 0.0048_{stat} \pm 0.0025_{syst} \pm 0.0014_{ext}$$

$$a_2 m_{\pi+} = -0.024 \pm 0.013_{stat} \pm 0.009_{syst} \pm 0.002_{ext}$$

with a correlation coefficient of -0.839. Imposing the ChPT constraint mentioned earlier results in the very precise value of:

$$(a_0 - a_2)m_{\pi+} = 0.2633 \pm 0.0024_{stat} \pm 0.0014_{syst} \pm 0.0019_{ext}$$

which is in perfect agreement with the prediction.

4. Conclusions

With the analyses of $K^\pm \to \pi^+\pi^-e^\pm\nu$ (Ke4) and $K^\pm \to \pi^0\pi^0\pi^\pm$ (K3π), NA48/2 dominates the measurements of S-wave scattering lengths. The two channels Ke4 and K3π provide complementary information on the scattering lengths a_0 and a_2. The two process are collected by different sub-detectors with different contribution to the experimental systematic uncertainty and using very different theoretical inputs. Therefore they can be easily combined, as illustrated in Fig.5, to:

$$a_0 m_{\pi+} = 0.2210(47)_{stat}(40)_{syst}$$

$$a_2 m_{\pi+} = -0.0429(44)_{stat}(28)_{syst}$$

and, assuming ChPT constraint:

$$(a_0 - a_2)m_{\pi+} = 0.2639(20)_{stat}(15)_{syst}.$$

With these new results, the experimental precision finally matches the one of the theoretical prediction [2]. The agreement between measurement and theory, which predicts $(a_0 - a_2)m_{\pi+} = 0.265 \pm 0.004$, is outstanding and provides one of the most stringent tests of ChPT to date.

References

[1] S.Weinberg, Phys. Rev. Lett. **17** 616 (1966).

[2] G.Colangelo, J.Gasser, H.Leutwyler, Nucl. Phys. **B 603** 125 (2001); G.Colangelo, J.Gasser, H.Leutwyler, Phys. Rev. Lett. **B 86** 5008 (2001).

[3] L.Rosselet et al. [Geneva-Saclay Collaboration], Phys. Rev. **D 15** 574 (1977).

[4] S.Pislak et al. [E865 Collaboration], Phys. Rev. Lett.**87** 221801 (2001); Phys. Rev. **D 67** 072004 (2003); Erratum ibid. **81** 119903 (2010);

[5] B.Adeva et al. [DIRAC Collaboration], Phys. Lett. **B 619** 50 (2005).

[6] J.R.Batley et al., Phys. Lett. **B 638** 22 (2006).

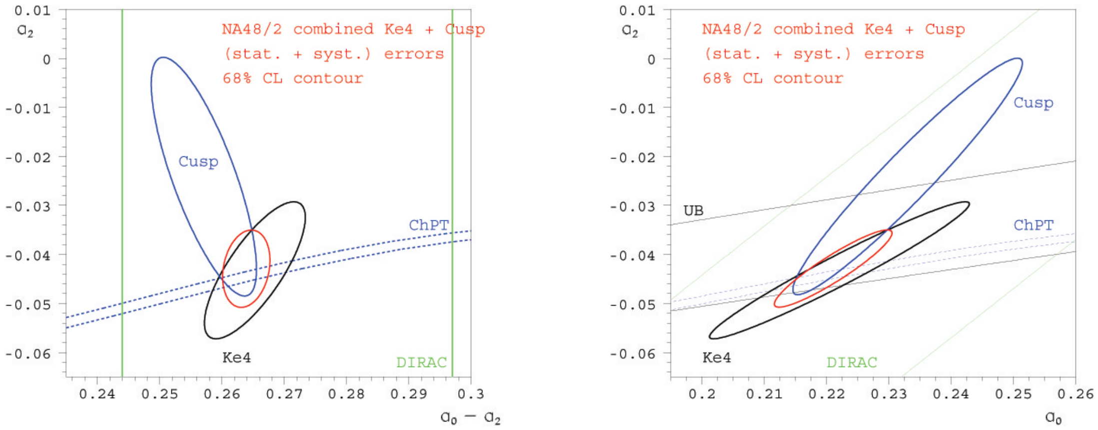

Figure 5: NA48/2 Ke4 (black) and K3π (blue) results as 2-parameter fits in the $((a_0 - a_2), a_2)$ (left figure) and (a_0, a_2) (right figure) plane. In each plane the red contour corresponds to the combination of results. The correlation coefficient is 0.28 (resp. 0.91). The green lines correspond to the DIRAC result band.

[7] V.Fanti et al. [NA48 Collaboration], Nucl. Instr. Meth. **A 574** 433 (2007).

[8] C.Nakamura et al. [Particle Data Group], J. Phys. **G 37** 075021 (2010).

[9] N.Cabibbo, A.Maksymowicz, Phys. Rev. **B 438** 137 (1965).

[10] R.Brun, R.Hagelberg, M.Hansroul and J.C.Lassalle, CERN-DD-78-2-REV (http://wwwasd.web.cern.ch/wwwasd/geant/).

[11] S.M.Roy, Phys. Lett. **B 36** 353 (1971).

[12] G.Colangelo, J.Gasser, A.Rusetsky, Eur. Phys. J. **C 59** 777 (2009).

[13] G.Colangelo et al., Nucl. Phys. **B 638** 187 (2006).

[14] J.R.Batley et al. [NA48/2 Collaboration], Phys. Lett. **B 633** 173 (2006).

[15] N.Cabibbo, Phys. Rev. Lett. **93** 12181 (2004).

[16] B.Budini, L.Fonda, Phys. Rev. Lett. **6** 419 (1961).

[17] N.Cabibbo, G.Isidori, JHEP **0503** 021 (2005).

[18] G.Colangelo, J.Gasser, B.Kubis, A.Rusetzky, Phys. Lett. **B 638** 187 (2006); M.Bissinger, A.Fuhrer, J.Gasser, B.Kubis, A.Rusetzky, Phys. Lett. **B 659** 576 (2008).

[19] J.R.Batley et al. [NA48/2 Collaboration], Eur. Phys. J. **C 64** 589 (2009).

[20] M.Bissegger (BFGKR) et al., Nucl. Phys. **B 806** 178 (2009).