

Some Theoretical Aspects of Semi-Leptonic Decays

Thomas Mannel*

*Theoretische Elementarteilchenphysik,
Naturwiss. Techn. Fakultät,
Universität Siegen
D-57068 Siegen*

I discuss two recent aspects of semi-leptonic decays. First I will consider a few novel developments in the calculation of higher orders in the Heavy Quark Expansion for the inclusive width $B \rightarrow X_c \ell \bar{\nu}$. Secondly I shall report on an updated calculation of $B \rightarrow \pi \ell \bar{\nu}$ relevant for V_{ub} and comment on the role of $B \rightarrow \tau \bar{\nu}$ in the determination of V_{ub} .

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*Speaker.

1. Introduction

The theoretical description of semi-leptonic decays of heavy hadrons is in a very mature state. The major tool for reliable calculations is the Heavy Quark Expansion (HQE) and Heavy Quark Effective Theory (HQET) together with the Heavy Quark Symmetries (HQS) appearing in the heavy mass limit.

The determination of V_{cb} can be performed from exclusive as well as from inclusive decays. While the inclusive determination makes use of the HQE, the exclusive determination uses HQS, which constrain the form factors at the non-recoil point, where the four-velocities of the initial and final state hadrons are the same.

The theory for the inclusive determination based on HQE has reached the status of a precision calculation. It is based on the computation of the total rate, which in HQE is given as a combined series in $\alpha_s(m_b)^n$, $(\Lambda_{\text{QCD}}/m_b)^m$, and $(\Lambda_{\text{QCD}}/m_c)^k(\Lambda_{\text{QCD}}/m_b)^{l+3}$ [1]. Currently the leading term $m = k = 0$ is known to order α_s^2 [2, 3], the first sub-leading corrections are partially known to order $\alpha_s^2 \times (\Lambda_{\text{QCD}}/m_b)^2$ [4], while the tree level terms are known to order $(\Lambda_{\text{QCD}}/m_b)^m$ [5], and the term involving inverse powers of m_c $(\Lambda_{\text{QCD}}/m_c)^2(\Lambda_{\text{QCD}}/m_b)^3$ [6]. Overall, this calculation has reached a theoretical uncertainty at the level of one percent.

Exclusive determinations of V_{cb} rely on the decays $B \rightarrow D^{(*)}\ell\bar{\nu}$. The normalization of the form factors is known from HQS, but a precise determination of V_{cb} requires to take into account corrections to the HQS normalizations. This can be done either by QCD lattice calculations [7] or by exploiting QCD sum rules at the non-recoil point [8]. While the intrinsic uncertainties of the QCD sum rule calculations limit this method to uncertainties at the level of five percent, QCD lattice calculations can in principle achieve a better accuracy. However, the current uncertainty is comparable and at the level of four percent. Overall the resulting value for V_{cb} is compatible with the one extracted from inclusive decays, although the exclusive value extracted from the lattice calculations is close to two standard deviations smaller.

The determination of V_{ub} can also be performed from exclusive as well as from inclusive decays. However, in the application of HQE as well as the determination of the form factors is more complicated. Over the past year there has not been any significant progress in the inclusive method, which requires in general the input of non-perturbative functions, the so-called shape functions. This renders the discussion of sub-leading terms in the HQE as well as the calculation of QCD radiative corrections complicated.

Exclusive determinations of V_{ub} require the knowledge of form factors. Focussing on $B \rightarrow \pi\ell\bar{\nu}$, the relevant form factor can be estimated either from lattice QCD or from QCD light cone sum rules. Both methods are complementary, since lattice QCD is restricted to large values of the leptonic momentum transfer q^2 , while QCD sum rules work best at low q^2 . Extrapolating both methods yields a consistent picture for the form factor, giving us some confidence that we know the form factor at a level of five to ten percent.

Extracting V_{ub} independently from inclusive and exclusive ($B \rightarrow \pi\ell\bar{\nu}$) decays yields values which are only marginally consistent, the tension between the two values is at the level of 2.5σ 's. Including the recently measured value for the purely leptonic decay $B \rightarrow \tau\bar{\nu}$ using the lattice value for f_B confuses the situation further, and hence we cannot claim to know V_{ub} better than at a level of ten to fifteen percent, although the individual methods claim to be more precise than that.

In the following I shall focus on two recent subjects related to the determination of V_{cb} and V_{ub} . I will first discuss new methods for the estimation of the matrix elements appearing in the HQE at higher orders, which eventually may lead to a further reduction of the uncertainty in the inclusive determination of V_{cb} . As a second topic I shall present an update of the QCD light cone sum rule calculation of $B \rightarrow \pi \ell \bar{\nu}$ and comment on the role of $B \rightarrow \tau \bar{\nu}$ in the determination on V_{ub} .

2. New methods in the calculation of hadronic matrix elements in the HQE

Within HQE, the total semileptonic rate as well as the differential distributions are represented in terms of an expansion of the form

$$d\Gamma = d\Gamma_0 + \left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^2 d\Gamma_2 + \left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^3 d\Gamma_3 + \left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^4 d\Gamma_4 + d\Gamma_5 \left(a_0 \left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^5 + a_2 \left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^3 \left(\frac{\Lambda_{\text{QCD}}}{m_c}\right)^2 \right) + \dots + d\Gamma_7 \left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^3 \left(\frac{\Lambda_{\text{QCD}}}{m_c}\right)^4 \quad (2.1)$$

The coefficients $d\Gamma_i$ are themselves functions of m_c/m_b which are – up to logarithms of m_c – regular in the limit $m_c \rightarrow 0$, and which have an expansion in $\alpha_s(m_b)$. Furthermore, the $d\Gamma_i$ depend on non-perturbative parameters corresponding to matrix elements of increasing dimension.

The relevant hadronic matrix elements are

$$2M_H \mu_\pi^2 = -\langle H(v) | \bar{Q}_v (iD)^2 Q_v | H(v) \rangle : \quad \text{Kinetic Energy} \quad (2.2)$$

$$2M_H \mu_G^2 = \langle H(v) | \bar{Q}_v \sigma_{\mu\nu} (iD^\mu) (iD^\nu) Q_v | H(v) \rangle : \quad \text{Chromomagnetic Moment} \quad (2.3)$$

for $d\Gamma_2$ and

$$2M_H \rho_D^3 = -\langle H(v) | \bar{Q}_v (iD_\mu) (i\nu D) (iD^\mu) Q_v | H(v) \rangle : \quad \text{Darwin Term} \quad (2.4)$$

$$2M_H \rho_{LS}^3 = \langle H(v) | \bar{Q}_v \sigma_{\mu\nu} (iD^\mu) (i\nu D) (iD^\nu) Q_v | H(v) \rangle : \quad \text{Spin-Orbit Term} \quad (2.5)$$

for $d\Gamma_3$.

Going to higher orders one faces a proliferation of the number of independent matrix elements. At order $1/m_b^4$ we have already nine matrix elements, at order $1/m_b^5$ this increases to 18, and at $1/m_b^6$ there will be already 72 independent non-perturbative parameters.

Obviously these parameters cannot be determined from experiment any more, and hence we have to find a way to estimate them theoretically. To this end, we define a simple way for such an estimate based on a simple assumption which, however, can be systematically refined.

The higher order matrix elements can all be expressed in the form

$$\langle B | \bar{b} iD_{\mu_1} iD_{\mu_2} \cdots iD_{\mu_n} \Gamma b(0) | B \rangle, \quad (2.6)$$

where Γ denotes an arbitrary Dirac matrix. The representation is obtained by splitting the full chain $iD_{\mu_1} iD_{\mu_2} \cdots iD_{\mu_n}$ into $A = iD_{\mu_1} iD_{\mu_2} \cdots iD_{\mu_k}$ and $C = iD_{\mu_{k+1}} iD_{\mu_{k+2}} \cdots iD_{\mu_n}$.

In order to discuss the idea of the method we shall first assume that the derivatives in A and C are all spatial derivatives. In the following we show the intermediate state representation

$$\langle B | \bar{b} A C \Gamma b(0) | B \rangle = \frac{1}{2M_B} \sum_n \langle B | \bar{b} A b(0) | n \rangle \cdot \langle n | \bar{b} C \Gamma b(0) | B \rangle, \quad (2.7)$$

where we have assumed the B mesons to be static and at rest, $|B\rangle = |B(p = (M_B, \vec{0}))\rangle$, and $|n\rangle$ are the single- b hadronic states with vanishing spatial momentum.

Eq. (2.7) can be proven based on the operator product expansion. We introduce a fictitious heavy quark Q which will be treated as static, and consider a correlator at vanishing spatial momentum transfer \vec{q}

$$T_{AC}(q_0) = \int d^4x e^{iq_0x_0} \langle B | iT \{ \bar{b} A Q(x) \bar{Q} C \Gamma b(0) \} | B \rangle. \quad (2.8)$$

We shall use the static limit for both b and Q , and hence introduce the ‘rephased’ fields $\tilde{Q}(x) = e^{im_Q q_0 x_0} Q(x)$ and likewise for b , and omit tilde in them in what follows. The form of the resulting exponent suggests to define $\omega = q_0 - m_b + m_Q$ as the natural variable for T_{AC} , and $\frac{1}{2M_B} T_{AC}(\omega)$ is assumed to have a heavy mass limit.

With large m_Q we can perform the OPE for $T_{AC}(\omega)$ at $|\omega| \gg \Lambda_{\text{QCD}}$ still assuming that $|\omega| \ll m_Q$ and neglecting thereby all powers of $1/m_Q$. In this case the propagator of Q becomes static,

$$iT \{ Q(x) \bar{Q}(0) \} = \frac{1 + \gamma_0}{2} \delta^3(\vec{x}) \theta(x_0) P \exp \left(i \int_0^{x_0} A_0 dx_0 \right), \quad (2.9)$$

and yields

$$T_{AC}(\omega) = \langle B | \bar{b} A \frac{1}{-\omega - \pi_0 - i0} C \frac{1 + \gamma_0}{2} \Gamma b | B \rangle, \quad (2.10)$$

where $\pi_0 = iD_0$ is the time component of the covariant derivative. This representation allows immediate expansion of $T_{AC}(\omega)$ in a series in $1/\omega$ at large $|\omega|$:

$$T_{AC}(\omega) = - \sum_{k=0}^{\infty} \langle B | \bar{b} A \frac{(-\pi_0)^k}{\omega^{k+1}} C \frac{1 + \gamma_0}{2} \Gamma b | B \rangle. \quad (2.11)$$

Alternatively, the scattering amplitude can be written through its dispersion relation

$$T_{AC}(\omega) = \frac{1}{2\pi i} \int_0^{\infty} d\varepsilon \frac{1}{\varepsilon - \omega + i0} \text{disc } T_{AC}(\varepsilon), \quad (2.12)$$

where we have used the fact that in the static theory the scattering amplitude has only one, ‘physical’ cut corresponding to positive ω . The discontinuity is given by

$$i \int d^4x e^{i\varepsilon x_0} \langle B | \bar{b} A Q(x) \bar{Q} C \Gamma b(0) | B \rangle \quad (2.13)$$

and amounts to

$$\text{disc } T_{AC}(\varepsilon) = \sum_{n_Q} i \int d^4x e^{-i\vec{p}_n \vec{x}} e^{i(\varepsilon - E_n)x_0} \langle B | \bar{b} A Q(0) | n_Q \rangle \langle n_Q | \bar{Q} C \Gamma b(0) | B \rangle, \quad (2.14)$$

where the sum runs over the complete set of the intermediate states $|n_Q\rangle$; their overall spatial momentum is denoted by \vec{p}_n and energy by E_n .

The spatial integration over d^3x and integration over time dx_0 in Eq. (2.14) yield $(2\pi)^3 \delta^3(\vec{p}_n)$ and $2\pi \delta(E_n - \varepsilon)$, respectively. Therefore only the states with vanishing spatial momentum are projected out, and we denote them as $|n\rangle$:

$$\text{disc } T_{AC}(\varepsilon) = \sum_n 2\pi i \delta(\varepsilon - E_n) \langle B | \bar{b} A Q(0) | n \rangle \langle n | \bar{Q} C \Gamma b(0) | B \rangle. \quad (2.15)$$

Inserting the optical theorem relation (2.15) into the dispersion integral (2.12) we get

$$T_{AC}(\omega) = \sum_n \frac{\langle B|\bar{b}AQ(0)|n\rangle \langle n|\bar{Q}C\Gamma b(0)|B\rangle}{E_n - \omega + i0}, \quad (2.16)$$

and the large- ω expansion takes the form

$$T_{AC}(\omega) = - \sum_{k=0}^{\infty} \frac{1}{\omega^{k+1}} \sum_n E_n^k \langle B|\bar{b}AQ(0)|n\rangle \langle n|\bar{Q}C\Gamma b(0)|B\rangle. \quad (2.17)$$

Equating the leading terms in $1/\omega$ of $T_{AC}(\omega)$ in Eq. (2.11) and in Eq. (2.17) we arrive at the relation

$$\langle B|\bar{b}AC \frac{1+\gamma_0}{2} \Gamma b(0)|B\rangle = \sum_n \langle B|\bar{b}AQ(0)|n\rangle \cdot \langle n|\bar{Q}C\Gamma b(0)|B\rangle \quad (2.18)$$

which is the intermediate state representation (2.7). Note that the projector $(1+\gamma_0)/2$ in the left hand side can be omitted since the \bar{b} field satisfies $\bar{b} = \bar{b}(1+\gamma_0)/2$ in the static limit.

Operators involving time derivatives can be obtained by considering higher values of k in Eqs. (2.11) and (2.17) which describe the subleading in $1/\omega$ terms in the asymptotics of $T_{AC}(\omega)$. We readily generalize the saturation relation (2.7):

$$\langle B|\bar{b}A\pi_0^k C \frac{1+\gamma_0}{2} \Gamma \frac{1+\gamma_0}{2} b(0)|B\rangle = \sum_n (E_B - E_n)^k \langle B|\bar{b}AQ(0)|n\rangle \cdot \langle n|\bar{Q}C \frac{1+\gamma_0}{2} \Gamma \frac{1+\gamma_0}{2} b(0)|B\rangle. \quad (2.19)$$

Thus, each insertion of operator $(-\pi_0)$ inside a composite operator acts as a factor of the intermediate state excitation energy. This is expected, for equation of motion of the static quark field Q allows to equate

$$i\partial_0 \bar{Q}Cb(x) = \bar{Q}\pi_0 Cb(x)$$

for any color-singlet operator $\bar{Q}Cb(x)$. At the same time we have

$$i\partial_0 \langle n|\bar{Q}Cb(x)|B\rangle = -(E_n - M_B) \langle n|\bar{Q}Cb(x)|B\rangle.$$

The intermediate state representation (2.7) still does not assume any approximation aside from the static limit for the b quark, yet it may be used to apply a dynamic QCD approximation. The one we employ here uses as an input the B -meson heavy quark expectation values (2.6) of dimension 5 and 6, which are expressed through μ_π^2 , μ_G^2 , ρ_D^3 and ρ_{LS}^3 .

All operators with four and more derivatives must have an even number of spatial derivatives due to rotational invariance. Thus the operators with four derivatives have either four spatial derivatives, or two time and two spatial derivatives.

We shall discuss the $D=7$ operators with four spatial derivatives, and apply (2.18):

$$\langle B|\bar{b}iD_j iD_k iD_l iD_m \Gamma b|B\rangle = \sum_n \langle B|\bar{b}iD_j iD_k b|n\rangle \langle n|\bar{b}iD_l iD_m \Gamma b|B\rangle. \quad (2.20)$$

The intermediate states $|n\rangle$ in the sum are either the ground-state multiplet B, B^* , or excited states with the suitable parity of light degrees of freedom. The ground-state factorization approximation assumes that the sum in (2.20) is to a large extent saturated by the ground state spin-symmetry doublet. Hence we retain only the contribution of the ground state and discard the contribution of

higher excitations. In the case of dimension seven operators the result is expressed in terms of the expectation values with two derivatives, i.e. μ_π^2 and μ_G^2 ; matrix elements involving B^* are related to them by spin symmetry.

Applying this we obtain for spin-singlet and spin-triplet B expectation values of $D=7$ involving spatial derivatives only:

$$\frac{1}{2M_B} \langle B | \bar{b} i D_j i D_k i D_l i D_m b | B \rangle = \frac{(\mu_\pi^2)^2}{9} \delta_{jk} \delta_{lm} + \frac{(\mu_G^2)^2}{36} (\delta_{jm} \delta_{kl} - \delta_{jl} \delta_{km}) \quad (2.21)$$

$$\begin{aligned} \frac{1}{2M_B} \langle B | \bar{b} i D_j i D_k i D_l i D_m \sigma_{ab} b | B \rangle = & -\frac{\mu_\pi^2 \mu_G^2}{18} (\delta_{jk} \delta_{la} \delta_{mb} - \delta_{jk} \delta_{lb} \delta_{ma} + \delta_{lm} \delta_{ja} \delta_{kb} - \delta_{lm} \delta_{jb} \delta_{ka}) + \\ & \frac{(\mu_G^2)^2}{36} [\delta_{jm} (\delta_{lb} \delta_{ka} - \delta_{la} \delta_{kb}) - \delta_{jl} (\delta_{ka} \delta_{mb} - \delta_{kb} \delta_{ma}) + \\ & \delta_{kl} (\delta_{ja} \delta_{mb} - \delta_{jb} \delta_{ma}) - \delta_{km} (\delta_{ja} \delta_{lb} - \delta_{jb} \delta_{la})]. \quad (2.22) \end{aligned}$$

Finally, we need to consider the expectation values of the form $\langle B | \bar{b} i D_j i D_0^k i D_l [\sigma] b | B \rangle$ for $k=2,3$ which evidently belong to the tower of $\mu_{\pi,G}^2$ and $\rho_{D,LS}^3$. Likewise, their values could be considered as the input describing strong dynamics, along with the latter; yet they have not been constrained experimentally. The intermediate states saturating such expectation values have opposite parity to the ground state (P -wave states) regardless of number of time derivatives. The counterpart of the ground-state saturation approximation here is retaining the contribution of the lowest P -wave resonance in the sum; then each power of time derivative amounts to the extra power of $-\bar{\epsilon}$, where $\bar{\epsilon} = M_P - M_B \approx 0.4 \text{ GeV}$.

In fact, there are two families of the P -wave excitations of B mesons corresponding to spin of light degrees of freedom $\frac{3}{2}$ or $\frac{1}{2}$. The combinations $\mu_\pi^2 - \mu_G^2$, $\rho_D^3 + \rho_{LS}^3$, ... receive contributions only from the $\frac{1}{2}$ -family, whereas the $\frac{3}{2}$ -family gives rise to $\frac{\mu_\pi^2 + 2\mu_G^2}{3}$, $\frac{\rho_D^3 - 2\rho_{LS}^3}{3}$, etc. [9] (the transition amplitude into the lowest $\frac{1}{2}$ P -state appears to be suppressed). Therefore, it makes sense to consider these two structures separately and approximate

$$\langle B | \bar{b} i D_j (-i D_0)^{k+1} i D_l b | B \rangle = \left(\bar{\epsilon}_{3/2}^k \frac{2\rho_D^3 - \rho_{LS}^3}{9} + \bar{\epsilon}_{1/2}^k \frac{\rho_D^3 + \rho_{LS}^3}{9} \right) \delta_{jl} \quad (2.23)$$

$$\langle B | \bar{b} i D_j (-i D_0)^{k+1} i D_l \sigma_{jl} b | B \rangle = -\bar{\epsilon}_{3/2}^k \frac{2\rho_D^3 - \rho_{LS}^3}{3} + \bar{\epsilon}_{1/2}^k \frac{2\rho_D^3 + 2\rho_{LS}^3}{3}. \quad (2.24)$$

Note that assuming $\bar{\epsilon}_{1/2} = \bar{\epsilon}_{3/2} = \bar{\epsilon}$ implies $\rho_D^3 \simeq \bar{\epsilon} \mu_\pi^2$ and $-\rho_{LS}^3 \simeq \bar{\epsilon} \mu_G^2$; the first relation seems to be satisfied by the preliminary values of μ_π^2 and ρ_D^3 extracted from experiment.

This ground state saturation method can be extended also to higher dimensional operators in an obvious way. Furthermore, there is also the possibility for a refinement of the method by including more states aside from the ground state. In this way a systematic approach can be constructed to obtain reliable estimates for the higher order matrix elements.

As expected, the effect on the total rate and hence on V_{cb} is small. However, the moments of differential distributions are more strongly affected; this is currently under consideration.

3. Update on the exclusive determination of V_{ub}

Currently, $B \rightarrow \pi l \nu_l$ is the most reliable exclusive channel to extract $|V_{ub}|$ [10]. There is a steady progress in measuring the branching fraction and q^2 -distribution for $l = \mu, e$ (see [11, 12, 13])

for the latest results). The hadronic vector form factor $f_{B\pi}^+(q^2)$ and its scalar counterpart $f_{B\pi}^0(q^2)$ relevant for this decay are defined as

$$\langle \pi^+(p) | \bar{u} \gamma_\mu b | \bar{B}^0(p+q) \rangle = f_{B\pi}^+(q^2) \left[2p_\mu + \left(1 - \frac{m_B^2 - m_\pi^2}{q^2} \right) q_\mu \right] + f_{B\pi}^0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q_\mu, \quad (3.1)$$

where $f_{B\pi}^+(0) = f_{B\pi}^0(0)$. The most recent lattice QCD computations with three dynamical flavours [14, 15] predict these form factors at $q^2 \geq 16 \text{ GeV}^2$, in the upper part of the semileptonic region $0 \leq q^2 \leq (m_B - m_\pi)^2 \simeq 26.4 \text{ GeV}^2$, with an accuracy reaching 10%. There are also recent results available [16] in the quenched approximation on a fine lattice. QCD light-cone sum rules (LCSR) with pion distribution amplitudes (DA's) allow one to calculate the $B \rightarrow \pi$ form factors [17, 18, 19, 20, 21] at small and intermediate momentum transfers, $0 \leq q^2 \leq q_{max}^2$, where the choice of q_{max}^2 varies between 12 and 16 GeV^2 .

The QCD light cone sum rule calculation has been updated for the prediction the integral:

$$\Delta\zeta(0, q_{max}^2) \equiv \frac{G_F^2}{24\pi^3} \int_0^{q_{max}^2} dq^2 p_\pi^3 |f_{B\pi}^+(q^2)|^2 = \frac{1}{|V_{ub}|^2 \tau_{B^0}} \int_0^{q_{max}^2} dq^2 \frac{d\mathcal{B}(B \rightarrow \pi \ell \nu_\ell)}{dq^2}, \quad (3.2)$$

where $p_\pi = \sqrt{(m_B^2 + m_\pi^2 - q^2)/4m_B^2 - m_\pi^2}$ is the pion 3-momentum in the B -meson rest frame, and the above equation is valid for $\ell = e, \mu$ in the limit $m_l = 0$. As in [21], the value $q_{max}^2 = 12.0 \text{ GeV}^2$ is adopted. The predicted $\Delta\zeta(0, 12 \text{ GeV}^2)$ is used to extract $|V_{ub}|$ from the most recent BABAR-collaboration results [11, 12] for the measured partial branching fraction integrated over the same q^2 -region. Furthermore, we predict the form factors in the whole semileptonic region by fitting the LCSR results at $q^2 \leq q_{max}^2$ to the z -series parameterization in the form suggested in [22].

The details of the calculation can be found in [10]; our predictions for $f_{B\pi}^+$ are, within errors, in a reasonable agreement with the lattice QCD results obtained by HPQCD [14] and Fermilab/MILC [15] collaborations. We also observe an agreement with the normalization and shape of the form factors obtained by the QCDSF collaboration [16], in particular, they predict $f_{B\pi}^+(0) = 0.27 \pm 0.07 \pm 0.05$.

Furthermore, we estimate the total width of $B \rightarrow \pi \ell \nu_\ell$ in units of $1/|V_{ub}|^2$ and the integral (3.2) for the large q^2 -region:

$$\begin{aligned} \frac{1}{|V_{ub}|^2} \Gamma(B \rightarrow \pi \ell \nu_\ell) &= \Delta\zeta(0, 26.4 \text{ GeV}^2) = 7.71_{-1.61}^{+1.71} \text{ ps}^{-1}, \\ \Delta\zeta(16 \text{ GeV}^2, 26.4 \text{ GeV}^2) &= 1.88_{-0.59}^{+0.53} \text{ ps}^{-1}. \end{aligned} \quad (3.3)$$

Using the most recent data from BaBar we update our value for V_{ub} from the LCSR calculation to be

$$|V_{ub}| = (3.50_{-0.33}^{+0.38} |_{th.} \pm 0.11 |_{exp.}) \times 10^{-3}. \quad (3.4)$$

4. The role of $B \rightarrow \tau \bar{\nu}$ in the determination of V_{ub} .

The measurement of the leptonic width $B \rightarrow \tau \nu_\tau$ can also be used to extract V_{ub} , once the value of the B meson decay constant f_B is known. Currently, the central value for the leptonic width

$B \rightarrow \tau \nu_\tau$ measured by both BABAR and Belle collaborations is larger than the SM prediction:

$$\mathcal{B}(B^- \rightarrow \tau \bar{\nu}_\tau) = \frac{G_F^2}{8\pi} |V_{ub}|^2 m_\tau^2 m_B \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 \tau_{B^-}, \quad (4.1)$$

if one employs f_B predicted from lattice QCD or QCD sum rules, together with $|V_{ub}|$ extracted from $B \rightarrow \pi \ell \nu_\ell$.

The recent discussions on this situation are mostly concentrated on the value of $|V_{ub}|$. Indeed, the tension decreases, if one uses in (4.1) the somewhat larger value of $|V_{ub}|$ extracted from the inclusive $b \rightarrow u$ decays. On the other hand, the CKM fits [23, 24] yield a smaller $|V_{ub}|$, consistent with the determinations from $B \rightarrow \pi \ell \nu_\ell$.

Let us emphasize that, independent of the actual $|V_{ub}|$ value, there exists a tension between the ratio of semileptonic and leptonic B widths and the QCD predictions for the two relevant hadronic matrix elements $f_{B\pi}^+(q^2)$ and f_B . To demonstrate that, we define the following observable:

$$R_{s/l}(q_1^2, q_2^2) \equiv \frac{\Delta \mathcal{B}_{B \rightarrow \pi \ell \nu_\ell}(q_1^2, q_2^2)}{\mathcal{B}(B \rightarrow \tau \nu_\tau)} \left(\frac{\tau_{B^-}}{\tau_{B^0}} \right) = \frac{\Delta \zeta(q_1^2, q_2^2)}{(G_F^2/8\pi) m_\tau^2 m_B (1 - m_\tau^2/m_B^2)^2 f_B^2}, \quad (4.2)$$

where the partial branching fraction $\Delta \mathcal{B}$ and the integral $\Delta \zeta$ defined as in (3.2), are taken over the same region $q_1^2 \leq q^2 \leq q_2^2$ of the momentum transfer.

Exp.	$\Delta \mathcal{B}(10^{-4})$ [Ref.]	$\mathcal{B}(B \rightarrow \tau \nu_\tau)(10^{-4})$ [Ref.]	$R_{s/l}$
BABAR	0.32 ± 0.03 [11] $0.33 \pm 0.03 \pm 0.03$ [12]	1.76 ± 0.49 [25, 26]	$0.20^{+0.08}_{-0.05}$
Belle	0.398 ± 0.03 [13]	$1.54^{+0.38+0.29}_{-0.37-0.31}$ [27]	$0.28^{+0.13}_{-0.07}$
QCD	$\Delta \zeta(\text{ps}^{-1})$ [Ref.]	$f_B(\text{MeV})$ [Ref.]	$R_{s/l}$
HPQCD	2.02 ± 0.55 [14]	190 ± 13 [28]	0.52 ± 0.16
FNAL/MILC	$2.21^{+0.47}_{-0.42}$ [15]	212 ± 9 [29]	0.46 ± 0.10

Table 1: The ratio $R_{s/l}$ for the region $16\text{GeV}^2 < q^2 < 26.4\text{GeV}^2$, measured and calculated from (4.2) using the lattice QCD results. The weighted average over the two BABAR measurements is taken and all errors are added in quadrature.

The above equation for the ratio $R_{s/l}$ follows solely from the $V - A$ structure of the weak currents in SM and V_{ub} cancels out in the ratio. The form factor $f_{B\pi}^+$ and decay constant f_B entering r.h.s. are obtained by one and the same QCD method: lattice QCD or the combination of LCSR and QCD sum rule. In Tables 1 and 2 we collect the inputs for this equation, obtained from different measurements and QCD calculations. The disagreement between the calculated and measured

Exp.	$\Delta\mathcal{B}(10^{-4})$ [Ref.]	$\mathcal{B}(B \rightarrow \tau\nu_\tau)(10^{-4})$ [Ref.]	$R_{s/l}$
BABAR	0.88 ± 0.06 [11] $0.84 \pm 0.03 \pm 0.04$ [12]	1.76 ± 0.49 [25, 26]	$0.52^{+0.20}_{-0.12}$
QCD	$\Delta\zeta$ [Ref.]	$f_B(\text{MeV})$ [Ref.]	$R_{s/l}$
LCSR/QCDSR	$4.59^{+1.00}_{-0.85}$ [10]	210 ± 19 [30]	$0.97^{+0.28}_{-0.24}$

Table 2: The same as in Table 1 for the region $0 \leq q^2 \leq 12.0\text{GeV}^2$ where the QCD sum rule results are used.

ratio $R_{s/l}$ goes beyond the theoretical and experimental errors, especially in the case of the lattice calculations which have smaller uncertainties.

Decreasing further the theoretical and experimental errors in (4.2), especially in the $B \rightarrow \tau\nu_\tau$ width, becomes therefore a very important task. Possible effects beyond the SM in $B \rightarrow \tau\nu_\tau$ are already being discussed in the literature, and, in particular, $B \rightarrow D\tau\nu_\tau$ is proposed as a channel which has common new physics contributions with the leptonic B decay (see e.g. [31] and references therein).

5. Summary

The current situation concerning the determination of V_{cb} from exclusive and inclusive decays is satisfactory. The inclusive determination has currently an uncertainty at the level of 1.5% and yields a value of V_{cb} that is compatible with the one determined from exclusive decays. With the 2010 update of the lattice value for the form factors the exclusive value for V_{cb} went up and became more compatible with the inclusive one.

The situation with V_{ub} is less satisfactory. The inclusive determinations yield values which tend to be higher than the exclusive ones. Due to the phase space cuts the HQE is much less precise than in the case of V_{cb} , and due to our ignorance of the shape functions it is very difficult to improve the precision. The theoretical control over the exclusive channels such as $B \rightarrow \pi\ell\bar{\nu}$ has improved due to more precise sum rule calculations and better lattice simulations. The uncertainties of both approaches are at the level of ten percent, where the inclusive method is believed to be slightly better. However, the exclusive value of V_{ub} is approximately 2.5σ 's smaller than the inclusive one. Both methods have been scrutinized in detail to find a possible source for the tension. However, until today no contributions have been found which were omitted and which could account for the tension.

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