

Unitarity Triangle Measurements

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Recent results of measurements by the B-factory experiments BaBar and Belle related to the angles ϕ_1 and ϕ_3 of the unitary triangle are presented.

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1. Introduction

In the standard model (SM) the charged-current couplings to quarks are described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1]. The CKM matrix contains an irreducible phase, which is the source of CP violation in the SM. Unitarity of the matrix imposes six vanishing relations which can be represented as triangles in the complex plane.

The unitary triangle arises from the relation $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ by dividing by $V_{cd}V_{cb}^*$. Its angles are given by

$$\begin{aligned}\phi_1 = \beta &= \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \\ \phi_2 = \alpha &= \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \\ \phi_3 = \gamma &= \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right).\end{aligned}$$

Measurements of the sides and angles of the unitarity triangle provide a test for the SM and allow to constrain non-SM physics. Experimental determination of the angles is closely related to measurements of CP asymmetries, which has been a key objective of the B-factory experiments BaBar and Belle. Recent results of the B-factories on the angles ϕ_1 and ϕ_3 are summarised in this report.

2. Measurements of ϕ_1

The angle ϕ_1 can be obtained by measurements of time-dependent CP asymmetries. $A_{CP}(t)$ in B^0 and \bar{B}^0 decays to a common final state f is given by

$$A_{CP}(t) = \frac{\Gamma(\bar{B}^0(t) \rightarrow f) - \Gamma(B^0(t) \rightarrow f)}{\Gamma(\bar{B}^0(t) \rightarrow f) + \Gamma(B^0(t) \rightarrow f)} = \mathcal{S}_f \sin(\Delta mt) + \mathcal{A}_f \cos(\Delta mt)$$

with

$$\mathcal{S}_f = \frac{2\text{Im}(\lambda_f)}{|\lambda_f|^2 + 1}, \quad \mathcal{A}_f = \frac{|\lambda_f|^2 - 1}{|\lambda_f|^2 + 1}, \quad \lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}.$$

$A_f(\bar{A}_f)$ is the amplitude of the corresponding $B^0(\bar{B}^0) \rightarrow f$ decay, $\frac{q}{p}$ is a factor originating from $B^0 - \bar{B}^0$ mixing, \mathcal{S}_f is related to CP violation occurring in the interference of mixing and decay and \mathcal{A}_f is related to direct CP violation.

$b \rightarrow c\bar{c}s$ transitions to CP eigenstates such as $B^0 \rightarrow J/\psi K_{S,L}^0$ provide clean access to ϕ_1 [2][3]. These decays are dominated by only one tree amplitude with real CKM elements. $B^0 - \bar{B}^0$ mixing introduce a phase such that $\mathcal{S}_{J/\psi K_{S,L}^0} = -\xi_{CP} \sin(2\phi_1)$ and $\mathcal{A}_{J/\psi K_{S,L}^0} = 0$.

At the asymmetric-energy B-factories time-dependent CP asymmetries can be measured by the lifetime-difference of two neutral B mesons created in the decay of $\Upsilon(4S)$ mesons. Experimental difficulties are the finite vertex reconstruction resolution and imperfect knowledge about the flavor of the accompanying B meson.

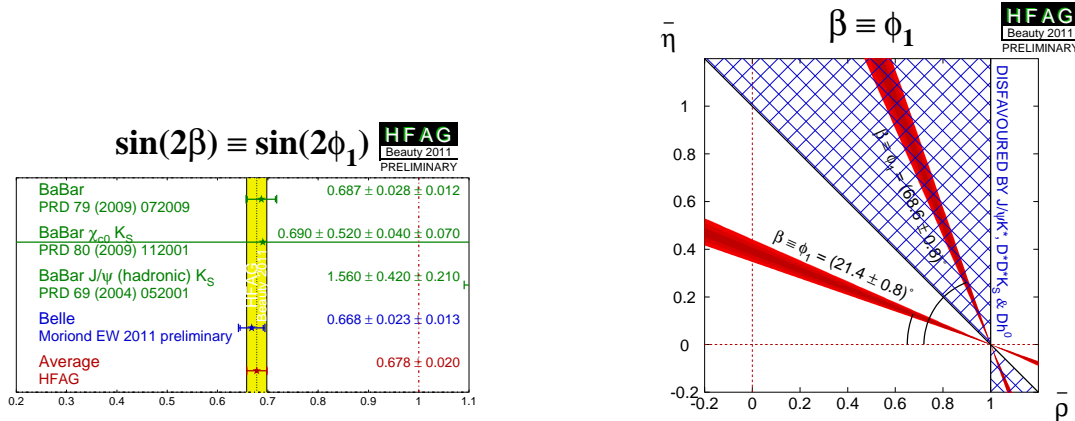


Figure 1: Average of $\sin(2\phi_1)$ measurements by the B-factory experiments BaBar and Belle and the corresponding angle ϕ_1 as provided by the Heavy Flavor Averaging Group (HFAG).

BaBar reported [4] measurements of the time-dependent CP asymmetry parameters in the decays to $J/\psi K_S^0$, $J/\psi K_L^0$, $\psi(2S)K_S^0$, $\eta_c K_S^0$, $\chi_{c1} K_S^0$ and $J/\psi K^*(892)^0$ using its full data sample of $465 \times 10^6 B\bar{B}$ pairs collected on the $\Upsilon(4S)$:

$$\mathcal{A} = -0.024 \pm 0.020(\text{stat}) \pm 0.016(\text{syst})$$

$$\sin(2\phi_1) = 0.687 \pm 0.028 \quad \pm 0.012$$

Belle recently updated its $\sin(2\phi_1)$ measurement in decays to $J/\psi K_S^0$, $J/\psi K_L^0$, $\psi(2S)K_S^0$ and $\chi_{c1} K_S^0$ using the final data sample of $770 \times 10^6 B\bar{B}$ pairs. Compared to the previous analysis [5], this new measurement gains in statistics not only by increased luminosity but also by application of improved track finding algorithms. Also the systematic uncertainty could significantly be reduced due to improved vertex reconstruction and description of resolution effects. The preliminary result which represents current world's most precise $\sin(2\phi_1)$ measurement is:

$$\mathcal{A} = 0.007 \pm 0.016(\text{stat}) \pm 0.013(\text{syst})$$

$$\sin(2\phi_1) = 0.668 \pm 0.023 \quad \pm 0.013$$

Considering this new result the average of measurements by the B-factories is $\sin(2\phi_1) = 0.678 \pm 0.020$. This corresponds to an value of $\phi_1 = (21.4 \pm 0.8)^\circ$ (Figure 1).

3. Measurements of ϕ_3

The angle ϕ_3 enters as relative weak phase between $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$ transitions.

In decays such as $B^\pm \rightarrow DK^\pm$ these transitions interfere, when the D^0 and \bar{D}^0 decay to a common final state. The two interfering amplitudes differ in a factor of $r_B e^{i(\pm\phi_3 + \delta_B)}$, where r_B is the magnitude ratio of the amplitudes and δ_B the relative strong phase between them. Determination of ϕ_3 based on this interference could provide a SM anchor point, since only tree-level diagrams contribute. Experimentally the measurement of ϕ_3 is challenging due to Cabibbo- and color-suppressions of the decays and small values of r_B .

Several methods utilising this interference have been proposed: The GLW method [6][7] is applied to Cabibbo-suppressed D decays to CP eigenstates, the ADS method [8][10] to doubly Cabibbo-suppressed and Cabibbo-favored D decays and the GSSZ method [11] to multibody D decays.

The currently most sensitive method is the GGSZ method, which relies on multibody D decays into selfconjugated states such as $D \rightarrow K_S^0 \pi^+ \pi^-$. The sensitivity arises by the dependence of the interference on the position in the Dalitz plane. In some regions of the Dalitz plane Cabibbo-favored and doubly Cabibbo-suppressed amplitudes interfere or are enriched by CP eigenstates.

Previous ϕ_3 measurements using the GGSZ method were model-dependent. The amplitudes needed for description of the Dalitz structure were obtained by fits to tagged D^0 decays using model assumptions such as in the Isobar or in the K-matrix model [13]. The systematic uncertainty in ϕ_3 associated to these model assumptions can be as large as 10° .

3.1 Model-dependent GGSZ ϕ_3 measurement by BaBar

BaBar reported [14] a measurement of ϕ_3 in the decays of $B^\pm \rightarrow D^{(*)} K^{(*)\pm}$ using a data sample of $465 \times 10^6 B\bar{B}$ pairs. The result including magnitude ratios $r_{B/s}^{(*)}$, relative strong phases $\delta_{B/s}^{(*)}$ and hadronic parameter κ is:

$$\begin{aligned}\phi_3 &= (68 \quad +15_{-14}^{\text{(stat)}} \pm 4^{\text{(syst)}} \pm 3^{\text{(model)}})^\circ \\ r_B &= (9.6 \quad \pm 2.9 \quad \pm 0.5 \quad \pm 0.4)\% \\ r_B^* &= (13.3 \quad +4.2_{-3.9} \quad \pm 1.3 \quad \pm 0.3)\% \\ \kappa r_s &= (14.9 \quad +6.6_{-6.2} \quad \pm 2.6 \quad \pm 0.6)\% \\ \delta_B &= (119 \quad +19_{-20} \quad \pm 3 \quad \pm 3)^\circ \\ \delta_B^* &= (-82 \quad \pm 21 \quad \pm 5 \quad \pm 3)^\circ \\ \delta_s &= (111 \quad \pm 32 \quad \pm 11 \quad \pm 3)^\circ\end{aligned}$$

This measurement excludes the hypothesis of no direct CP violation with a confidence level equivalent to 3.5 standard deviations.

3.2 Model-independent GGSZ ϕ_3 measurement by Belle

Belle performed a ϕ_3 measurement in a model-independent approach. In this approach the $D \rightarrow K_S^0 \pi^+ \pi^-$ phase space was sub-divided and in each sub-space information on the relative strong phases could be applied, which have been obtained by CLEO in quantum-correlated $\psi(3770) \rightarrow D\bar{D}$ decays [15].

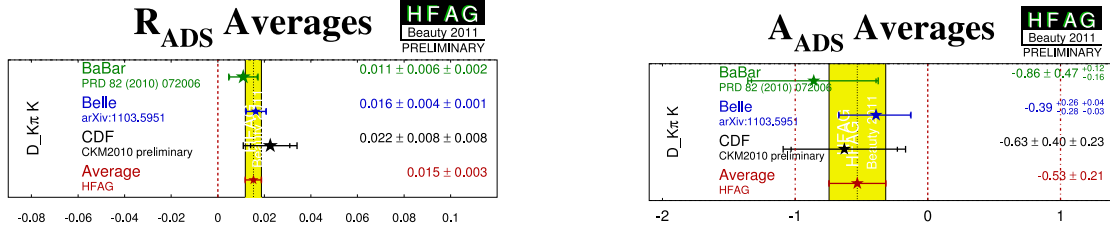


Figure 2: Measurements of ADS observables in $B^\pm \rightarrow DK^\pm$ and $D \rightarrow K^\mp \pi^\pm$ decays. \mathcal{R}_{ADS} (left) is given by the ratios of suppressed to favored decays and \mathcal{A}_{ADS} (right) is related to direct CP asymmetries.

It was shown [16][17] that this model-independent approach introduces no bias in the ϕ_3 measurement. The uncertainty due to modeling is replaced by experimental uncertainties on the $D \rightarrow K_S^0 \pi^+ \pi^-$ amplitude and is expected to reduce to approx. 1° by future measurements of BE-SIII. Drawback of the method is a loss in statistical sensitivity due to binning.

The preliminary Belle result of the model-independent ϕ_3 measurement in the decays $B^\pm \rightarrow DK^\pm$ and $D \rightarrow K_S^0 \pi^+ \pi^-$ using $770 \times 10^6 B\bar{B}$ pairs is:

$$\begin{aligned} \phi_3 &= (77.3 \quad +15.1 \text{(stat)} \pm 4.2 \text{(syst)} \pm 4.3 \text{(phase precision)})^\circ \\ r_B &= 0.145 \pm 0.030 \quad \pm 0.011 \quad \pm 0.011 \\ \delta_B &= (129.9 \pm 15.0 \quad \pm 3.9 \quad \pm 4.7)^\circ \end{aligned}$$

3.3 ADS ϕ_3 measurement by Belle

In $B^\pm \rightarrow DK^\pm$ the ratio \mathcal{R}_{ADS} of decay rates of suppressed to favored decays such as $D \rightarrow K^+ \pi^-$ and $D \rightarrow K^- \pi^+$ and the direct CP asymmetry \mathcal{A}_{ADS} are observables related to ϕ_3 according to the ADS method.

Recently Belle could measure \mathcal{R}_{ADS} and \mathcal{A}_{ADS} in these decays [18]:

$$\begin{aligned} \mathcal{R}_{ADS} &= (1.63 \quad +0.44 \text{(stat)} \quad +0.07 \text{(syst)} \pm 0.41 \pm 0.13) \times 10^{-2} \\ \mathcal{A}_{ADS} &= -0.39 \quad +0.26 \quad +0.04 \quad -0.28 \quad -0.03 \end{aligned}$$

In this measurement the significance of \mathcal{R}_{ADS} is 4.1σ . Thus this is the first evidence of a signal in an ADS measurement. By combination with other measurements ϕ_3 can be extracted from these observables in a model-independent way.

An overview of results of ADS measurements in $B^\pm \rightarrow DK^\pm$ and $D \rightarrow K^\mp \pi^\pm$ is given in Figure 2.

4. Summary

All angles of the unitary triangle have been objective of measurements by the B-factory experiments BaBar and Belle. Recent results related to the angles ϕ_1 and ϕ_3 have been summarised.

The determination of $\sin(2\phi_1)$ by time-dependent CP asymmetries became a precision measurement. The uncertainty of the corresponding average of ϕ_1 is less than 1° .

Present measurements of ϕ_3 are still statistically dominated. Prospective measurements by other experiments such as Super-B, Belle II or LHCb can contribute to a better knowledge of ϕ_3 .

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