$B_s$ Decays and Mixing

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Theoretical remarks are offered regarding recent hadron collider results on the mixing and decays of $B_s$ mesons. Topics covered include: (1) CP-violating mixing in $B_s(\bar{B}_s) \rightarrow J/\psi \phi$, (2) the $D_0$ dimuon charge asymmetry, (3) information from triple products, (4) $B_s \rightarrow J/\psi f_0$, (5) new physics constraints, (6) some illustrative new physics scenarios.
1. Introduction

Recent results on $B_s$ decays and mixing have been presented by the CDF and D0 Collaborations at the Fermilab Tevatron and the LHCb Collaboration at CERN. We begin by discussing CP-violating mixing in $B_s (\bar{B}_s) \to J/\psi \phi$. Experiments at CDF and D0 suggested a mixing phase $\beta_s$ much larger than that in the Standard Model (SM). With such a large phase, we pointed out that time-dependent decays should display explicit time-dependence [1]. We update that analysis in Section 2.

The D0 Collaboration has presented evidence for a charge asymmetry in same-sign dimuons produced in $\bar{p}p$ collisions at $\sqrt{s} = 1.96$ TeV [2]. We suggest in Section 3 a test of whether this asymmetry is due to decays of $b$ quarks, as claimed, or background sources such as kaons [3].

In Section 4 we discuss what triple products in $B_{(i)} \to V_1 V_2$ actually measure. The answer [4] is CP violation, but only under certain conditions. The study of $B_s \to J/\psi f_0$, mentioned in Section 5, avoids the angular analysis needed to interpret $B_s \to J/\psi \phi$. In Section 6, we note constraints on new physics, and comment in Section 7 on a couple of scenarios for consideration should any hints for physics beyond the SM be borne out by further tests. We conclude in Section 8.

2. CP violation in interference between $B_s-\bar{B}_s$ mixing and $B_s \to J/\psi \phi$ decay

For formalism we refer to [5]. $B_s-\bar{B}_s$ mixing is expected to be dominated by the top quark in box graphs. The observed values $\Delta m_s = (17.77 \pm 0.10 \pm 0.07)$ ps$^{-1}$ (CDF [6]) and $(17.63 \pm 0.11 \pm 0.04)$ ps$^{-1}$ (LHCb [7]) agree with SM predictions. Denoting

$$|B_{sL}| = p|B_s\rangle + q|\bar{B}_s\rangle ; \quad |B_{sH}| = p|B_s\rangle - q|\bar{B}_s\rangle ,$$  \hspace{1cm} (2.1)

we expect for $\Delta \Gamma \ll \Delta m$, $q/p \approx \exp(2i\beta_s)\beta_s^{\text{SM}} = -\text{Arg}(-V_{ts}^\ast V_{tb}/V_{cs}^\ast V_{cb}) = (1.04 \pm 0.05)^\circ$ [5]. The SM $B_s \to J/\psi \phi$ CP asymmetry then should be governed by the small mixing phase $\phi_M = -44^\circ$ [8]) would imply detectable time-dependence of angular distribution coefficients, differing for tagged $B_s$ and $\bar{B}_s$ [1].

We review the discussion briefly. For a CP test, one tags the flavor at $t = 0$, denoting $\eta = \pm 1$ for a tagged $(B_s, \bar{B}_s)$. The coefficients of helicity amplitudes $|A_\parallel|\,^2,$ $|A_\perp|\,^2$ describing different angular dependences are denoted by $\mathcal{T}_+,$ $\mathcal{T}_-$, where

$$\mathcal{T}_\pm \equiv e^{-i\Gamma/2} \left[ \text{cosh}(\Delta \Gamma t)/2 \mp \text{cos}(\phi_M) \text{sinh}(\Delta \Gamma t)/2 \pm \eta \text{sin}(\phi_M) \text{sin}(\Delta m t) \right] .$$  \hspace{1cm} (2.2)

Taking $\phi_M = -44^\circ$, $\Delta \Gamma/\Gamma = 0.228$, and assuming the tagging $\eta$ to be diluted by a factor of 0.11, we concluded that wiggles should be distinguishable between the $B_s$-tagged and $\bar{B}_s$-tagged $\mathcal{T}_\pm$ distributions. We advocated making such a plot as evidence for CP violation in $B_s \to J/\psi \phi$ at a level beyond the SM. Here we update our estimate of $t$-dependence, finding the oscillations a bit smaller, but still visible. We take $\phi_M = (-39 \pm 17)^\circ$ based on an average between CDF [10, 11] and D0 [12] values, choose $\Delta \Gamma/\Gamma = 0.143$ based on an average between CDF $(0.075 \pm 0.035 \pm 0.010)$ and D0 $(0.15 \pm 0.06 \pm 0.01)$, and continue to assume a dilution factor of 11%. The resulting plot is shown in Fig. 1.
Figure 1: Relative intensities of $T_k$ signals as functions of $\Gamma_t$, for $B_s$ tags (solid) and $\bar{B}_s$ tags (dashed). This figure represents an update of a similar one in Ref. [1].

At this Conference, LHCb presented data restricting $\phi_M$ to the range $[-2.7, -0.5]$ [13] (68% c.l.), $1.2\sigma$ from the SM. We are eagerly awaiting data from ATLAS and CMS.

3. D0 dimuon asymmetry – Is it due to $b$’s? $K$’s?

The SM predicts a small asymmetry in the yield of same-sign muon pairs due to $b\bar{b}$ production followed by meson $\leftrightarrow$ antimeson oscillation: $A_{sl}^b \equiv \frac{N^{++} - N^{-+}}{N^{++} + N^{-+}} = (-2.0 \pm 0.3) \times 10^{-4}$ [14]. The D0 Collaboration reports a much larger value, $A_{sl}^b = (-9.57 \pm 2.51 \pm 1.46) \times 10^{-3}$, nearly 50 times the SM value [2]. (CDF is not ready to report such a measurement but has quoted a new average mixing parameter $\tilde{\chi}$ [15].)

D0 has interpreted its result as $3.2\sigma$ evidence for CP violation in neutral $B$ mixing. They have performed 16 systematic checks for which their results are found consistent with their nominal ones. Estimating the correct kaon decay backgrounds is crucial.

We have suggested a test [3] to see if a smaller asymmetry is obtained in a sample depleted in $b\bar{b}$ pairs. If one reduces the maximum allowed impact parameter of muon tracks, the signal should vanish more rapidly than background. The effect of our suggestion, an impact parameter cut of $b < 100\mu m$, is not yet known to us.

We denote quantities in the $B$ rest frame with an asterisk (*) and those in the lab frame with none. The lab energy of the $B$ is $E_B = \gamma m_B = m_B / \sqrt{1 - \beta^2}$. Muon angles with respect to the $B$
The dependence of \( \langle b \rangle \) on \( \gamma \beta \) is shown in Fig. 2.

An eyeball fit to the CDF \( b \) distribution [16] gives \( \langle b \rangle = 350 \) \( \mu \text{m} \). Table 1 denotes the effect of discarding events with \( b \) exceeding various values of \( b_0 \).

The D0 Collaboration defines a transverse impact parameter \( b_\perp \) relative to the closest primary vertex and a longitudinal distance \( b_\parallel \) from the point of closest approach to this vertex. They choose \( b_\perp < 3000 \) \( \mu \text{m} \) and \( b_\parallel < 5000 \) \( \mu \text{m} \). These are related to \( b \) as follows. The transverse and longitudinal components of muon momentum in the lab are \( p_\perp = p \sin \psi \), \( p_\parallel = p \cos \psi \). The distance \( d \) of a point along the \( \mu \) trajectory from the vertex is \( d^2 = b_\perp^2 + (s \sin \psi)^2 + (s \cos \psi - b_\parallel)^2 \), where \( s \) is the distance along the \( \mu \) trajectory from the transverse point of closest approach. The minimum of \( d \) is \( b = d_{\min} = [b_\perp^2 + (b_\parallel \sin \psi)^2]^{1/2} \). Little signal reduction is seen with \( b_\perp < 500 \) \( \mu \text{m} \), \( b_\parallel < 500 \) \( \mu \text{m} \) [2], but we advocate a tighter cut. The key question remains with regard to D0 muons:

**Table 1:** Fraction of events remaining for a given \( \langle b \rangle \) when events with \( b > b_0 \) are discarded [3].

<table>
<thead>
<tr>
<th>( b_0 ) (( \mu \text{m} ))</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle b \rangle ) (( \mu \text{m} ))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>0.237</td>
<td>0.542</td>
<td>0.748</td>
<td>0.866</td>
<td>0.930</td>
</tr>
<tr>
<td>300</td>
<td>0.080</td>
<td>0.237</td>
<td>0.400</td>
<td>0.542</td>
<td>0.658</td>
</tr>
<tr>
<td>450</td>
<td>0.040</td>
<td>0.129</td>
<td>0.237</td>
<td>0.347</td>
<td>0.450</td>
</tr>
</tbody>
</table>

boost are denoted by \( \theta^* \) in the \( B \) rest frame and \( \theta \) in the lab. The transformation between them is \( \sin \theta = \sin \theta^*/[\gamma(1 + \beta \cos \theta^*)] \). The isotropy of muon emission in \( \cos \theta^* \) can be used to calculate the average values of \( \sin \theta \) and \( \langle b \rangle = \gamma \beta \sin \theta c \tau \), where \( c \tau = 450 \mu \text{m} \) and

\[
\langle \sin \theta \rangle = \frac{1}{2} \int_0^{\pi} \frac{\sin^2 \theta^* d\theta^*}{\gamma(1 + \beta \cos \theta^*)} = \frac{\pi}{2} \frac{1}{1 + \gamma}.
\] (3.1)

The dependence of \( \langle b \rangle \) on \( \gamma \beta \) is shown in Fig. 2.
are they really from $b$ decays? This question should be answered by imposing an upper bound of $b_0 < 100 \mu m$ on the impact parameter $b_0$.

4. What do triple products in $B(s) \to V_1V_2$ measure?

A spinless particle decaying to four spinless particles gives rise to three independent momenta in its rest frame. One can form a T-odd expectation value out of (e.g.) $a$ in its rest frame. The transversity amplitudes depend on angular analyses:

\[
A_T = \frac{\Gamma(TP > 0) - \Gamma(TP < 0)}{\Gamma(TP > 0) + \Gamma(TP < 0)}; \quad TP \equiv p_1 \cdot (p_2 \times p_3);
\]

they are tiny in the SM. A true T-violation is signified by

\[
\mathcal{A}_T^{\text{true}} = \frac{\Gamma(TP > 0) + \bar{\Gamma}(TP > 0) - \Gamma(TP < 0) - \bar{\Gamma}(TP < 0)}{\Gamma(TP > 0) + \bar{\Gamma}(TP > 0) + \Gamma(TP < 0) + \bar{\Gamma}(TP < 0)}.
\]

The matrix element for $B(p) \to V_1(k_1, \epsilon_1) + V_2(k_2, \epsilon_2)$ can be written

\[
M = a\epsilon_1^* \cdot \epsilon_2^* \frac{b}{m_B} (p \cdot \epsilon_1)(p \cdot \epsilon_2) + i\frac{c}{m_B} \epsilon_{\mu \nu \rho \sigma} p^\mu q^\nu e^\rho e^\sigma; \quad q \equiv k_1 - k_2
\]

The transversity amplitudes depend on $a, b, c$ as $A_\parallel(a), A_0(a, b),$ and $A_\perp(c).$ Under CP conjugation, $a \to \bar{a}, b \to \bar{b},$ $ic \to -i\bar{c}.$ Angular distributions depend on the angle $\phi$ and polar angles $\theta_1, \theta_2,$ each in the rest frame of the decaying $V_1$ or $V_2$:

\[
\frac{d\Gamma}{d \cos \theta_1 d \cos \theta_2 d \phi} \sim |A_0|^2 \cos^2 \theta_1 \cos^2 \theta_2 + (1/2)|A_\parallel|^2 \sin^2 \theta_1 \sin^2 \theta_2 \sin^2 \phi
\]

\[
+ (1/2)|A_\parallel|^2 \sin^2 \theta_1 \cos^2 \theta_2 \cos^2 \phi + (1/2 \sqrt{2}) \text{Re}(A_0 A_\parallel^*) \sin 2\theta_1 \sin 2\theta_2 \cos \phi
\]

\[
- (1/2 \sqrt{2}) \text{Im}(A_\parallel A_0^*) \sin 2\theta_1 \sin 2\theta_2 \sin \phi - (1/2) \text{Im}(A_\perp A_0^*) \sin^2 \theta_1 \sin^2 \theta_2 \sin 2\phi.
\]

The last two terms are T-odd and of two distinct types. The interfering amplitudes are characterized by a weak phase difference $\phi_w$ and a strong phase difference $\delta$. In addition to the “true” TP $\mathcal{A}_T^{\text{true}}$ defined above, one can define [4] a “fake” TP:

\[
\mathcal{A}_T^{\text{fake}} = \frac{\Gamma(TP > 0) - \bar{\Gamma}(TP > 0) - \Gamma(TP < 0) + \bar{\Gamma}(TP < 0)}{\Gamma(TP > 0) + \bar{\Gamma}(TP > 0) + \Gamma(TP < 0) + \bar{\Gamma}(TP < 0)},
\]
Table 2: Longitudinal and transverse fractions $f_L$ and $f_T$ for some $b \to s$-penguin $B \to V V$ processes.

<table>
<thead>
<tr>
<th></th>
<th>$B_s \to \phi \phi$</th>
<th>$B^+ \to \phi K^{*-}$</th>
<th>$B^+ \to \rho^0 K^{*-}$</th>
<th>$B^0 \to \rho^0 K^{*-}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[22]</td>
<td>$0.348 \pm 0.041 \pm 0.021$</td>
<td>$0.49 \pm 0.05 \pm 0.03$</td>
<td>$0.52 \pm 0.10 \pm 0.04$</td>
<td>$0.57 \pm 0.09 \pm 0.08$</td>
</tr>
<tr>
<td>$f_L$</td>
<td>$0.49 \pm 0.05 \pm 0.03$</td>
<td>$0.52 \pm 0.10 \pm 0.04$</td>
<td>$0.57 \pm 0.09 \pm 0.08$</td>
<td>\n</td>
</tr>
</tbody>
</table>

where $TP_{\text{true}} \propto \sin \phi_v \cos \delta$, $TP_{\text{fake}} \propto \cos \phi_v \sin \delta$. The two T-odd observables are

$$ A_T^{(1)} = \frac{\text{Im}(A_0 \bar{A}_0^*)}{|A_0|^2 + |A_0|^2 + |A_0|^2} , \quad A_T^{(2)} = \frac{\text{Im}(A_0 A_0^*)}{|A_0|^2 + |A_0|^2 + |A_0|^2} . $$

(4.6)

For CP conjugates, one has similar definitions with barred amplitudes and a minus sign from complex conjugation of the imaginary coefficient of $c$. The TP asymmetries $A_T^{\pm}$ then satisfy

$$ A_T^{\pm} = \text{Im}(A_0 A_0^* - \bar{A}_0 \bar{A}_0^*), \quad (i = 0, 1) . $$

(4.7)

The observables $A_T^{(1,2)}$ are related to those in Dorigo’s talk [21] by “$u$” $\leftrightarrow$ $A_T^{(2)}$; “$v$” $\leftrightarrow$ $A_T^{(1)}$; he reports on their measurement in $B_s \to \phi \phi$.

The decays $B \to \phi K^*$ and $B_s \to \phi \phi$ are both dominated by the $b \to s$ penguin diagram. Factorization predicts dominant longitudinal polarization of the vector mesons, in contrast to observations [22, 23, 24] (Table 2). By contrast, the tree-dominated decay $B^0 \to \rho^+ \rho^-$ has $f_L = 0.992 \pm 0.024^{+0.026}_{-0.013}$ [25], or nearly 1 as predicted. There is no reason to trust factorization for the penguin amplitude, which may be due to rescattering from charm-anticharm intermediate states.

From $B^0 \to \phi K^{*0}$ amplitudes quoted by [4] we estimate

$$ A_T^{(1)} = -0.260 \pm 0.048; \quad A_T^{(2)} = 0.203 \pm 0.050; \quad A_T^{(3)} = 0.005 \pm 0.070; \quad A_T^{(4)} = 0.010 \pm 0.064 . $$

(4.8)

These values imply a large fake $A_T^{(1)}$ (since $A_T^{(1)} - A_T^{(1)} \neq 0$); no true $A_T^{(1)}$ (since $A_T^{(1)} + A_T^{(1)}$ is consistent with zero); and no fake or true $A_T^{(2)}$ (since both $A_T^{(2)}$ and $A_T^{(2)}$ are consistent with zero). The large fake $A_T^{(1)}$ simply reflects the importance of strong final-state phases.

5. $B_s \to J/\psi f_0$ vs. $B_s \to J/\psi f_0$

Helicity or transversity analysis for $B_s \to J/\psi f_0$ (S-, P-, D-wave) is avoided for $B_s \to J/\psi f_0$ (pure P-wave). As $CP(J/\psi) = CP(f_0) = +$, the overall final state is CP odd. An estimate of the rate for this process [26] is

$$ R_{f_0/\phi} \equiv \frac{\Gamma(B_s \to J/\psi f_0, f_0 \to \pi^+ \pi^-)}{\Gamma(B_s \to J/\psi f_0, f_0 \to K^+ K^-)} \simeq 20% , $$

(5.1)

to be compared with experimental values $0.252^{+0.046}_{-0.032} \pm 0.027$ [27], $\simeq 0.18 \sim 30\%$ (stat. error) [28], and $0.292 \pm 0.020 \pm 0.017$ [21]. The CKM structure for this process is the same as for $B_j \to J/\psi f_0$. Although $f_0$ decays mainly to $\pi \pi$, it seems to be “fed” mainly from $s\bar{s}$: Comparing $J/\psi \to \phi \pi \pi$ and $J/\psi \to \omega \pi \pi$ [29], one sees a $\pi \pi$ peak at $M(f_0) \simeq 980$ MeV in $\phi \pi \pi$, not $\omega \pi \pi$. 


6. New physics constraints

Two (of $\sim 100$) theoretical analyses [30, 31] emphasize the correlation between $a_{sl}^d$, $\Delta m_q$, $\Delta \Gamma_q$, and the mixing angle $\phi_q$, where $A_{sl}^b = (0.506 \pm 0.043)a_{sl}^d + (0.494 \pm 0.043)a_{sl}^s$. The questions of whether $\beta_s$ or $a_{sl}^d$ are nonstandard are separate; they are related by $a_{sl}^d = (|\Delta \Gamma_q|/\Delta m_s)\tan \phi_q$. If the $D0$ dimuon asymmetry is mainly from $a_{sl}^s$, Ref. [31] finds $a_{sl}^s = (-12.5 \pm 4.8) \times 10^{-3}$ by combining with the $D0$ measurement $(-1.7 \pm 9.1) \times 10^{-3}$. Using in this formula the (CDF, LHCb) average $\Delta m_s = (17.70 \pm 0.08)$ ps$^{-1}$ and the (CDF, D0) average $\Delta \Gamma_s = 0.094 \pm 0.031$ ps$^{-1}$, one expects $\phi_s = (-67^{+18}_{-7})^\circ$. Comparing with $\phi_M^s = (-39 \pm 17)^\circ$, this would favor slightly larger $\Delta \Gamma_s$ or a nonstandard value of $a_{sl}^d$. In Ref. [5] it is noted that one must respect the SM prediction of $\Delta m_q$. New physics must affect mainly phases of mixing amplitudes.

7. A cursory look at new physics scenarios

Supersymmetry has generic flavor-changing (but controllable) effects [32]. Randall-Sundrum [33] scenarios in which different quarks lie at different points along a fifth dimension offer a language for understanding quark mixings; but there is no predictive scheme yet. Theories with an extra (flavor-changing) $Z$ can induce mixing as desired. In Ref. [31] a contribution to $\Delta \Gamma$ is introduced through a new light pseudoscalar (an on-shell state in $B_s \to \bar{B}_s$). These are just some examples of a wealth of models on the market. Some of them predict other observable consequences but there are too many to enumerate exhaustively. Two of my current favorites are (1) a fourth generation, and (2) a hidden sector.

Lunghi and Soni [34] note the tension between $\sin 2\beta = \sin 2\phi_3 = 0.668 \pm 0.023$ (measured in $B$ decays) and that $0.867 \pm 0.048$ in (their) CKM fit. They note effects of new physics on both $\Delta$Flavor = 1 (penguin) and $\Delta$Flavor = 2 (box) amplitudes but give no specifics on $\beta_s$ or $a_{sl}^s$.

In a “hidden sector” let an extended gauge sector $G$ describe dark matter, and let there be particles $Y$ with charges in both the SM and in $G$, and particles $X$ with charges only in $G$. A box diagram describing $B_s \to \bar{B}_s$ mixing in this scenario is shown in Fig. 3. Table 3 gives examples of ordinary, mixed, and “shadow” matter. There are clearly many opportunities in such a scenario for new contributions to penguin and box diagrams.

<table>
<thead>
<tr>
<th>Type of matter</th>
<th>Std. Model</th>
<th>G</th>
<th>Example(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordinary</td>
<td>Charged</td>
<td>Uncharged</td>
<td>Quarks, leptons</td>
</tr>
<tr>
<td>Mixed ($Y$)</td>
<td>Charged</td>
<td>Charged</td>
<td>Superpartners</td>
</tr>
<tr>
<td>Shadow ($X$)</td>
<td>Uncharged</td>
<td>Charged</td>
<td>$E'_8$ of $E_8 \otimes E'_8$</td>
</tr>
</tbody>
</table>

8. Summary

$B_s$ decays and mixing provide potential mirrors of new physics. While the phase $\beta_s$ has moved toward its Standard Model value, even the currently measured value of $\beta_s$ should be manifested in
time-dependent quantities.

The D0 collaboration [2] claims a dimuon charge asymmetry. At this conference [15] CDF has reported a remeasurement of $\chi$ and we look forward to their further progress on dimuons. The signal requires subtraction of a big kaon background. Is what’s left really due to $b$ quark decays? We have proposed an impact parameter cut of $b < 100 \mu m$ to find out [3].

Using triple products in four-body decays, one can construct T-odd observables providing strong and weak phase information. There is interest in what new physics one can learn from $B_s \to \phi \phi$ [21].

As for whether there is new physics in any of the above hints, I urge you to have your favorite model ready; there are enough to go around.

Acknowledgments

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References


[11] O. Leroy, presented at 2011 La Thuile Meeting on behalf of LHCb, estimates the CDF value as

\[ \phi_M = (-31 \pm 29)^\circ. \]

[12] D0 Collaboration, D0 6098-CONF, quotes a value of \( \phi_M = (-44^{+23}_{-21} \pm 1)^\circ. \)


[21] CDF Collaboration, presented by M. Dorigo, this Conference, arXiv:1105.4437 [hep-ex]. This value has been updated by the CDF Collaboration subsequent to the Conference.

[22] CDF Collaboration, “Measurement of the Polarization Amplitudes of the \( B_s \to \phi \phi \) Decay,” CDF Note 10120, March 31, 2011.


