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Scattering Amplitudes in String/Gauge Theory Correspondence

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We review some generalizations of the Alday-Maldacena conjecture in the AdS/CFT correspondence. We discuss several scattering processes in different theories, but we mainly focus on the theories with fundamental matter and on finite temperature theories. Namely, the dual theory of the D3(N_c)-D7(N_f) model at the probe limit, the $\mathcal{N}=4$ super Yang-Mills at finite temperature with nonzero R-charge densities, and the Non-Commutative gauge theory at finite temperature. In these theories we review the analysis and the corresponding scattering amplitude results. We also discuss briefly the possibility of existence of different minimal surface solutions corresponding to a scattering amplitude, that become dominant at different regions due to different saddle points becoming dominant at high energies when the string theory is expanded around them. Finally, we mention some future directions for the scattering amplitudes in the above theories.

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PoS(Kruger 2010)059

1. Introduction

Recently there was important progress on the gluon scattering amplitudes processes using the Alday-Maldacena conjecture [1]. The problem of scattering amplitudes in high energy physics is a significant one. In the weak coupling regime one can solve the quantum field theory by completely specifying its scattering matrix. If the theory under examination is highly symmetric, like the $\mathcal{N} = 4$ Yang-Mills, then higher order perturbative calculations of its scattering matrix can be done. On the other hand, the strong coupling regime of any field theory should be accessible through the AdS/CFT duality [2]. For the case of the $\mathcal{N} = 4$ SYM theory, the gravity dual is the well known $AdS_5 \times S^5$ background. By now it is well understood how to construct the finite temperature dual backgrounds as well as backgrounds for less supersymmetric field theories, eg. the Sasaki-Einstein ones which are dual to $\mathcal{N} = 1$ SYM theories, or even for some confining theories, like the Sakai-Sugimoto background which is conjectured to be dual to $3+1$ dimensional large N_c QCD and captures the chiral symmetry breaking due to the shape of the flavor branes. Moreover, the process of adding flavors in these theories is also well understood and is equivalent to inserting fundamental matter in the theory by adding flavor branes. Hence it is clear that the string/gauge theory duality in principle can be used for several calculations in completely different theories.

However, the scattering on-shell states of a gauge theory carry color charge, and the closed string theory need to be extended by an open string sector. The integrated correlation function of open string vertex operators then correspond to gauge theory scattering amplitudes. Nevertheless, one can extract the partial amplitudes, which carry the kinematic dependence by separating the terms in the correlation functions of vertex operators, to terms depending on color charges and terms depending on the particle momenta. The partial amplitudes are the ones that can be described by the closed string theory.

In this note we focus at the strong coupling, gravity dual calculations. It has been noted by Alday and Maldacena [1] that the partial amplitudes at strong coupling are related to a polygonal light-light Wilson loops. Hence the scattering amplitude calculation is reduced to a minimal surface calculation with its boundaries determined by the momenta of the scattered particles.

More specifically, the gluon scattering amplitude is defined as the scattering of open strings whose ends are located on an IR-brane sitting in the bulk of the corresponding geometry. The associated calculation is hard, but the problem is simplified by performing a ‘disk’ T-duality on the dual background geometry [1]. After T-duality the problem reduces to finding the expectation value of a light-like Wilson loop on the UV of the T-dual geometry. The T-dual of the AdS_5 background is again an AdS_5, a fact that makes the calculations in $\mathcal{N} = 4$ SYM relatively simple. Generally, the T-duals of other backgrounds differ from the original ones. For example, in the case of the dual background of the finite temperature SYM, the T-dual metric is different than the initial one, while in the non-commutative dual background, the T-duality changes the B-fields too, eg.[6].

The scattering amplitudes/Wilson loop conjecture is by now well studied in the case of the maximally supersymmetric 4-dimensional Yang-Mills. However, it is of equal importance to understand the duality in other field theories with less supersymmetry, richer field content and even broken conformal invariance.

In this note we discuss briefly processes which, except gluons, involve quarks in the fundamental representation. We also examine processes which extend the gluon scattering correspondence to finite temperature. The scattering amplitude in finite temperature is a challenging problem in the dual field theory, especially due to difficulties to define asymptotic states in the strong coupling scattering.

There are several other generalizations of the Alday-Maldacena conjecture which we do not have the space to discuss here analytically. An almost straightforward generalization can be done for the β-deformed theories with a real deformation parameter. It has been found that the scattering amplitudes are the same as in $\mathcal{N}=4$ super Yang-Mills theory [3]. This result is also naturally expected from the form of the Wilson loops in these deformed theories, studied in [4], which at least in the large N limit appear independent of the deformation parameter. The situation is similar for the orbifolds of $\mathcal{N}=4$ super Yang-Mills where, with a rescaling of the coupling constant, the amplitudes remain the same as the initial theory. Moreover, a first try to extend the gluon scattering correspondence to the finite temperature was made in [10, 11]. Until now the only configuration that has been considered is the one interpreted as the forward scattering of a low energy gluon off a high energy gluon, which turns out to give only a phase contribution to the amplitude.

Furthermore, one can consider processes that involve asymptotics gluons and local operators, where the corresponding world-sheet is "zig-zag" periodic, or variations of it where the processes involve mesonic operators and final quarks-antiquarks, where the corresponding worldsheet can be extended again to a "zig-zag" one.

The plan of this short review is that in section 2 we briefly discuss processes that involve gluons and quarks [7]. In section 3 we discuss the gluon scattering amplitude at strong coupling in finite temperature and present some results that have been obtained in [11]. An analytic review for the scattering amplitudes can be found in [5].

2. Scattering amplitudes involving gluons and quarks

It has been known for some time that we can add fundamental matter in these theories by adding D-branes in the bulk. In the case of $AdS_5 \times S^5$ we can do that by adding D7-branes that wrap an S^3 of the S^5 and the AdS_5, where the resulting configuration preserves $\mathcal{N}=2$ supersymmetry. Usually it is enough to consider the probe limit where the $N_f \ll N_c$, and hence the backreaction of the flavor branes to the background is negligible. This corresponds to the field theory of saying that the dynamics of the glueballs affect the dynamics of the mesons but not vice-versa, and that the field theory looks conformal up to a very small scale.

In this configuration the gluons are open strings between $D3$-branes while the quarks are open strings between the D3 and D7 branes. If the flavor branes wrap the whole AdS_5 then the quarks are massless. The scattering amplitudes can be computed in a world-sheet with a topology of the disk with vertex operator insertions as in the no flavored case, with the difference now that the vertex operators can additionally correspond to open string states belonging to the $(3,7)$ sector.

Following the same procedure as in the undeformed background, we T-dualize along the longitudinal directions of the D3 brane and obtain the usual polygonic light-like Wilson loop, specified by the momenta of the scattering particles. However, if the boundary ends on D7 branes it still satisfies the Neumann boundary conditions in the radial direction and therefore can be extended in the bulk. Hence one expects that the worldsheet of the light-like Wilson loop ends on a folded string, since in the boundary the world-sheet satisfies Dirichlet boundary conditions [7]. In the probe

PoS(Kruger 2010)059

limit, it is possible by using symmetry arguments to relate the minimal surfaces corresponding to gluon scattering amplitudes, to the minimal surfaces corresponding to the scattering where quarks and gluons are involved. More specifically the minimal surface corresponding to the amplitude ($\bar{q}$ggq) with momenta $k_{1,2,3,4}$ respectively, is the half of the minimal surface corresponding to the amplitude (gggggg) with momenta $2k_1,\ k_2,\ k_3,\ 2k_4,\ k_3$ and k_2. It would be very interesting to see how this relation is modified away from the probe limit where the background is modified due to the backreaction of the flavor branes.

Extension with the inclusion of massive external quark states is done in [8] where has been found the worldsheet which corresponds to the scattering of two massive quarks and two massless gluons and the one of the four massive quarks, and their corresponding scattering amplitudes.

3. Gluon Scattering Amplitudes in Finite Temperature Gauge/Gravity Dualities

The results of this section were mainly obtained in [11]. We are looking at a particular kinematic configuration corresponding to a forward scattering of a soft gluon off an energetic one, in finite temperature theories. These are the:

•Finite Temperature $\mathcal{N}=4$ super Yang-Mills.

To introduce temperature in the AdS/CFT we need to deform the AdS_5 background by placing a big black hole in the center of the AdS space, or equivalently, to calculate the near horizon geometry of a near extremal black 3-brane. The 'disk' T-duality on this space does not generate any new fields.We place our light-like Wilson loop at $r=r_{horizon}$, which makes sense to do so since the point $r=r_h$ is at the UV of the T-dual metric as also commented in [10].

• Finite Temperature $\mathcal{N}=4$ super Yang-Mills with chemical potential.

To introduce a chemical potential in the previous background we need to introduce non-zero R-charges, which correspond to rotating near extremal D3-branes. These backgrounds are characterized by a non extremality parameter and in general by three rotation parameters which can be thought as corresponding to chemical potentials. The background we consider here has two non zero angular momentum parameters which correspond to two generators of the Cartan subalgebra of $SO(6)$ and we set them equal to μ. It also has the parameter r_h which is a non-extremality parameter. We choose to work in the Grand Canonical Ensemble (GCE) where the thermodynamic quantities, which we keep constant, are the temperature T and the chemical potential $\hat{\mu}$, which are related to the non-extremality and rotation parameters.

• Noncommutative Yang-Mills theory at finite temperature.

To construct this dual background we introduce the B-field to the non-extremal D3 brane background using U-duality. After the T-duality the metric as well as the B-fields are modified. We have two parameters that appear here, one is the non-commutative parameter a and the position of the black hole horizon r_h. The non-commutativity in dual field theory has been introduced only between the (x_0,x_1) and (x_2,x_3), where x_i are the D3-brane directions where the field theory lives. A second remark concerns the signature of the metric which is chosen to be $(-2,2)$ and we need to take that in account for the ansatz of the world-sheet we consider.

Now we need to define our Wilson loop which corresponds to the kinematic configuration of the scattered particles. Due to the presence of a black hole, it is natural to place the light-like Wilson loop at its horizon, which in the T-dual background is located at the UV. However, it is very difficult to find the minimal surface with the most general boundary conditions even for the case of four gluons, due to the complicated T-dual metric and because through the introduction of temperature we have lost integrability, whose role was crucial in obtaining the area of the minimal surface in the case of pure AdS. Hence, we limit ourselves to a Wilson loop with particular boundary conditions. Namely, we consider a rectangular loop with one edge much bigger than the other. We call the long edge of this Wilson loop L and the short one L_2, where $L \gg L_2$, where both of them being in the light-like direction. This light-like Wilson loop corresponds to an amplitude at strong coupling for forward scattering of a low energy gluon off a high energy gluon. Since one edge of the loop is much larger than the other we make the following ansatz for the embedding of the world-sheet [1]

$$y_0 = \tau + f(\sigma) \, , \qquad y_1 = \tau \, , \qquad y_2 = \sigma \, , \qquad r = r(\sigma), \tag{3.1}$$

with appropriate boundary conditions. Here (σ, τ) are the world-sheet coordinates, y_i are the T-dual variables of the x_i respectively and r is the radial coordinate.

After some analysis we find the corresponding minimal surface which is directly related to the gluon scattering amplitude for each of these three backgrounds. We find that for increasing the chemical potential or the non-commutative parameter, the on-shell action corresponding to our Wilson loop in the T-dual space decreases. For all three theories the length of the short side of the Wilson loop is constrained by an upper bound which depends on the temperature, the R-charge density and the non-commutative parameter. Due to this constraint, in the limit of zeroth temperature our approach breaks down since the upper bound goes to zero, while by keeping the temperature finite and taking the chemical potential and the non-commutative parameter to approach the zero value, the limit is smooth.

The upper bound on the length L gives a hint of the existence of other string solutions that are valid for larger values of L_{2max}. This could be related to the conjectured existence of different saddle points that become dominant at high energies when the string theory is expanded around them. Hence one could observe a kind of "phase transition" in the gluon scattering amplitudes as a reflection of the change in the character of the amplitudes due to different saddle point becoming dominant.

To present some representative results from [11], we choose the ones obtained in the presence of the chemical potentials. We plot in Figure 1 the L_2/T vs the world-sheet turning points for different chemical potentials and a fixed temperature, and the dependence of the action with respect to L_2/T. The existence of an upper bound in L_2 can be seen from the plots in Figure 1. For each L_2 there are two solutions corresponding to different turning points for the world-sheet but the natural branch is the left one where the string world-sheet stays close to the boundary. This can also be seen for the right plot of the Figure 1, where we can also observe that the on-shell action decreases with increase of the chemical potential. The results for the other two theories are qualitatively similar and can be found in [11], together with a further detailed analysis.

For the minimal surfaces of all the four point amplitudes we calculated we have used an appropriate cut off which corresponds in the field theory of the IR cut-off regularization. This is

[1] Our ansatz differs from the one in [10]. We have allowed y_0 to depend on σ in order to satisfy our equations of motion for y_0 exactly.

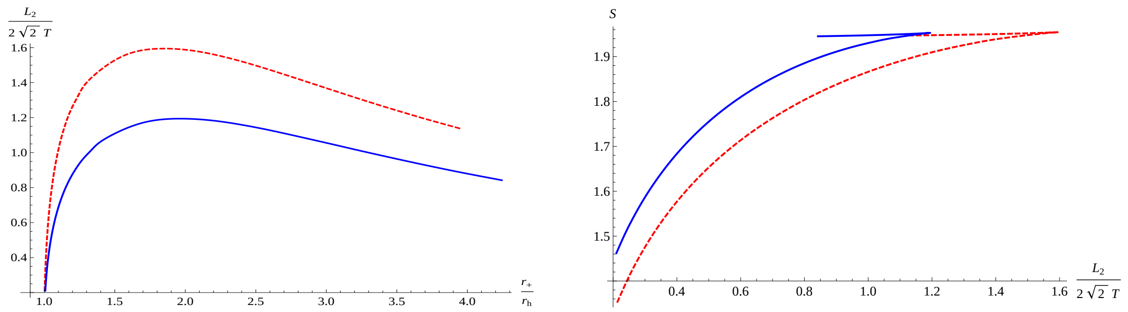

Figure 1: On the left, the dependence of L_2 in terms of r_+/r_h (where r_+ is the world-sheet turning point) for $T = \sqrt{2}/\pi$. Where the red-dashed are the results for $\xi_2 \simeq 2.66573$ and with blue being the results for $\xi_1 \simeq 1.11072$. Notice that for higher chemical potentials the $L_{2,max}$ takes higher values. We observe that for the same values of L_2 the world-sheet is less extended in higher chemical potentials than in lower ones. On the right, the dependence of the action on the L_2/T, for the same values of L_2 we get higher values of action for lower chemical potentials.

consistent and it is equivalent to using the Legendre transform of the action in a formal treatment, since the background satisfies the conditions of [9]. Although the amplitude for our kinematic configuration turns out to be a pure phase, it is interesting to notice that the minimal surfaces we found have the properties that one would expect for a real amplitude of the form $\exp(-S)$ in finite temperature gauge theories.

It would be very interesting to consider the most general planar four point gluon scattering amplitude using the corresponding general polygonic sequence, where we expect that the exponent in the amplitude should have a real part too. Moreover, by achieving that, we would be able to directly calculate the viscosity coefficient at finite temperature.

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PoS(Kruger 2010)059