

Slepton NLG (Non-Linear Gauge) in GRACE/SUSY-loop

M. Jimbo^{*,a} T. Ishikawa^b and M. Kuroda^c

^a Chiba University of Commerce
Ichikawa, Chiba 272-8512, Japan

^b KEK (Tsukuba)
Tsukuba, Ibaraki 305-0801, Japan

^c Meiji Gakuin University
Yokohama, Kanagawa 244-8539, Japan
E-mail: jimbo@cuc.ac.jp

We have been developing a program package called GRACE/SUSY-loop which is for the automatic calculations of the MSSM amplitudes in one-loop order. The non-linear gauge (NLG) fixing conditions play the crucial role in the calculations in one-loop order which contain a large number of Feynman diagrams. We present the recent progress in GRACE/SUSY-loop which is obtained by extending the non-linear gauge formalism to the slepton sector.

3rd Computational Particle Physics Workshop
September 23-25, 2010
KEK Tsukuba Japan

*Speaker.

1. Introduction

Supersymmetry (SUSY) between bosons and fermions at the unification-energy scale is one of the most promising hypotheses in the theory beyond the standard model (BSM), which is expected to resolve the remaining problems in the standard model (SM). The minimal supersymmetric extension of the SM (MSSM) is consistent with all the known high-precision experiments at a level comparable to the SM.

Since it is a broken symmetry at the electroweak-energy scale, the relic of SUSY is expected to remain as a rich spectrum of heavy SUSY particles, i.e. partners of usual matter fermions (leptons and quarks), gauge bosons and Higgs bosons, which are named sfermions (sleptons and squarks), gauginos and higgsinos, respectively. The quest of these new particles is one of the most important aims of the high-energy physics at present and future colliders of sub-TeV-energy or TeV-energy region.

In particular, experiments at the ILC offer high-precision determination of SUSY parameters via e^-e^+ -annihilation processes. Since the theoretical predictions with the high accuracy comparable to that of experiments is required to extract important physical results from the data, we have to include at least one-loop contributions in perturbative calculations of amplitudes.

Recently, we have calculated the radiative corrections to production processes and decay processes of SUSY particles in the framework of the MSSM using GRACE/SUSY-loop [1, 2, 3] which is a program package for automated computations of the MSSM in one-loop order. For the test of numerical results, we have used the non-linear gauge (NLG) formalism [4, 5, 6, 7, 8, 9] applied to GRACE/SUSY-loop [10].

In this paper, we show the recent progress in GRACE/SUSY-loop which is obtained by extending the non-linear gauge formalism to the slepton sector.

2. Features of GRACE/SUSY-loop

For many-body final states, each production process or decay process is described by a large number of Feynman diagrams even in tree-level order. There are still more Feynman diagrams in one-loop order even for two-body final states. For this reason, we have developed the GRACE system [11], which enables us to calculate amplitudes automatically.

A program package called GRACE/SUSY-loop is the version of the GRACE system for the calculation of the MSSM amplitudes in one-loop order, which includes the model files of the MSSM and can produce corresponding 2-point functions and counter terms. For the automatic calculation of the MSSM amplitudes in one-loop order, there exist other program packages independently developed by other groups, SloopS [12] and FeynArt/Calc [13].

As explained in [1], the renormalization scheme adopted for the electroweak (EW) interactions in GRACE/SUSY-loop is a variation of the on-mass-shell scheme [14, 15, 16, 17, 18], which is an MSSM extension of the scheme in the SM used in GRACE-loop [11]. There are some degrees of freedom in the renormalization conditions of the sfermion sector. We can choose different sets of residue conditions, decoupling conditions on the transition terms between the lighter and the heavier sfermions.

In GRACE / SUSY-loop, we use the technique of the NLG formalism [4, 5, 6, 7, 8, 9] in order to confirm the validity of calculations by imposing the NLG invariance on physical results. The NLG formalism is an extension of the linear R_ξ -gauge. The gauge fixing lagrangian for the EW interactions in the NLG [10, 12] is given as follows:

$$\mathcal{L}_{\text{gf}} = -\frac{1}{\xi_W} |F_{W^\pm}|^2 - \frac{1}{2\xi_Z} (F_Z)^2 - \frac{1}{2\xi_\gamma} (F_\gamma)^2, \quad (2.1)$$

$$F_{W^\pm} = (\partial_\mu \pm ie\tilde{\alpha}A_\mu \pm igc_W\tilde{\beta}Z_\mu)W^{\pm\mu} \pm i\xi_W \frac{g}{2}(\nu + \tilde{\delta}_H H^0 + \tilde{\delta}_h h^0 \pm i\tilde{\kappa}G^0)G^\pm, \quad (2.2)$$

$$F_Z = \partial_\mu Z^\mu + \xi_Z \frac{g_Z}{2}(\nu + \tilde{\varepsilon}_H H^0 + \tilde{\varepsilon}_h h^0)G^0, \quad (2.3)$$

$$F_\gamma = \partial_\mu A^\mu, \quad (2.4)$$

where $\nu = \sqrt{\nu_1^2 + \nu_2^2}$, $M_W = \frac{gv}{2}$, $M_Z = \frac{g_Z v}{2}$, h^0 and H^0 stands for the lighter and heavier CP even Higgs boson, respectively, G^\pm and G^0 stands for the Goldstone boson which corresponds to gauge boson W^\pm and Z , respectively. The gauge fixing lagrangian (2.1) contains seven independent NLG-parameters, $(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}_H, \tilde{\delta}_h, \tilde{\kappa}, \tilde{\varepsilon}_H, \tilde{\varepsilon}_h)$. The numerical tests are performed by varying these parameters.

3. Renormalization conditions in the slepton sector

The bare mass term in the slepton sector of MSSM lagrangian is given by

$$\mathcal{L}_0^{\text{mass}} = -\left(\tilde{\ell}_L^* \tilde{\ell}_R^*\right)_0 \begin{pmatrix} m_{\tilde{\ell}_L}^2 & m_{\tilde{\ell}_{LR}}^2 \\ m_{\tilde{\ell}_{LR}}^{2*} & m_{\tilde{\ell}_R}^2 \end{pmatrix}_0 \begin{pmatrix} \tilde{\ell}_L \\ \tilde{\ell}_R \end{pmatrix}_0, \quad \ell = e, \mu, \tau, \quad (3.1)$$

$$\begin{aligned} m_{\tilde{\ell}_L}^2 &= \tilde{m}_{\tilde{\ell}_L}^2 + m_\ell^2 + M_Z^2 \cos 2\beta (T_{3\ell} - Q_\ell \sin^2 \theta_W), \\ m_{\tilde{\ell}_R}^2 &= \tilde{m}_{\tilde{\ell}_R}^2 + m_\ell^2 + M_Z^2 \cos 2\beta Q_\ell \sin^2 \theta_W, \\ m_{\tilde{\ell}_{LR}}^2 &= -m_\ell (A_\ell + \mu \tan \beta). \end{aligned} \quad (3.2)$$

It contains three parameters $\tilde{m}_{\tilde{\ell}_L}$, $\tilde{m}_{\tilde{\ell}_R}$ and A_ℓ . Diagonalizing the mass matrix, we determine the mixing angle θ_ℓ and the mass eigenvalues $m_{\tilde{\ell}_1}, m_{\tilde{\ell}_2}$ ¹,

$$\begin{pmatrix} \cos \theta_\ell & \sin \theta_\ell \\ -\sin \theta_\ell & \cos \theta_\ell \end{pmatrix}_0 \begin{pmatrix} m_{\tilde{\ell}_L}^2 & m_{\tilde{\ell}_{LR}}^2 \\ m_{\tilde{\ell}_{LR}}^{2*} & m_{\tilde{\ell}_R}^2 \end{pmatrix}_0 \begin{pmatrix} \cos \theta_\ell & -\sin \theta_\ell \\ \sin \theta_\ell & \cos \theta_\ell \end{pmatrix}_0 = \begin{pmatrix} m_{\tilde{\ell}_1}^2 & 0 \\ 0 & m_{\tilde{\ell}_2}^2 \end{pmatrix}_0, \quad (3.3)$$

while the mass of the $\tilde{\nu}$ is given by $(m_{\tilde{\nu}_\ell}^2)_0 = (\tilde{m}_{\tilde{\nu}_{\ell L}}^2 + \frac{1}{2}M_Z^2 \cos 2\beta)_0$. We assume the SU(2)_L conditions on their left-handed soft SUSY-breaking mass terms, $\tilde{m}_{\tilde{e}_L}^2 = \tilde{m}_{\tilde{\nu}_{e L}}^2$, $\tilde{m}_{\tilde{\mu}_L}^2 = \tilde{m}_{\tilde{\nu}_{\mu L}}^2$ and $\tilde{m}_{\tilde{\tau}_L}^2 = \tilde{m}_{\tilde{\nu}_{\tau L}}^2$, which lead to the following condition,

$$m_{\tilde{\nu}_\ell}^2 = \cos^2 \theta_\ell m_{\tilde{\ell}_1}^2 + \sin^2 \theta_\ell m_{\tilde{\ell}_2}^2 - m_\ell^2 + M_W^2 \cos 2\beta. \quad (3.4)$$

This relation is valid among the bare quantities as well as among the renormalized quantities.

¹We use the convention, $m_{\tilde{\ell}_1} < m_{\tilde{\ell}_2}$

In the slepton sector, there are three mass renormalization constants, five wavefunction renormalization constants, and one mixing angle renormalization constants for each generation. They are $\delta m_{\tilde{\ell}_1}$, $\delta m_{\tilde{\ell}_2}$, $\delta m_{\tilde{v}_\ell}$, $\delta Z_{\tilde{\ell}_i \tilde{\ell}_j}$ ($i, j = 1, 2$), $\delta Z_{\tilde{v}_\ell}$, $\delta \theta_\ell$. We introduce the wavefunction renormalization constants in the unmixed fields $\tilde{\ell}_L$ and $\tilde{\ell}_R$ for each charged slepton.

$$\begin{pmatrix} \tilde{\ell}_L \\ \tilde{\ell}_R \end{pmatrix}_0 = \begin{pmatrix} Z_L^{1/2} & 0 \\ 0 & Z_R^{1/2} \end{pmatrix} \begin{pmatrix} \tilde{\ell}_L \\ \tilde{\ell}_R \end{pmatrix}, \quad (3.5)$$

which can be also written as

$$\begin{aligned} \begin{pmatrix} \tilde{\ell}_L \\ \tilde{\ell}_R \end{pmatrix}_0 &= \begin{pmatrix} \cos \theta_\ell & -\sin \theta_\ell \\ \sin \theta_\ell & \cos \theta_\ell \end{pmatrix}_0 \begin{pmatrix} \tilde{\ell}_1 \\ \tilde{\ell}_2 \end{pmatrix}_0 \\ &= \begin{pmatrix} \cos \theta_\ell & -\sin \theta_\ell \\ \sin \theta_\ell & \cos \theta_\ell \end{pmatrix}_0 \begin{pmatrix} Z_{11}^{1/2} & Z_{11}^{1/2} \\ Z_{21}^{1/2} & Z_{22}^{1/2} \end{pmatrix} \begin{pmatrix} \tilde{\ell}_1 \\ \tilde{\ell}_2 \end{pmatrix}. \end{aligned} \quad (3.6)$$

The first equation of (3.6) means, in particular,

$$\begin{pmatrix} \tilde{\ell}_L \\ \tilde{\ell}_R \end{pmatrix} = \begin{pmatrix} \cos \theta_\ell & -\sin \theta_\ell \\ \sin \theta_\ell & \cos \theta_\ell \end{pmatrix} \begin{pmatrix} \tilde{\ell}_1 \\ \tilde{\ell}_2 \end{pmatrix}, \quad (3.7)$$

and inserting this expression in (3.5), we find

$$\begin{pmatrix} \tilde{\ell}_L \\ \tilde{\ell}_R \end{pmatrix}_0 = \begin{pmatrix} Z_L^{1/2} & 0 \\ 0 & Z_R^{1/2} \end{pmatrix} \begin{pmatrix} \cos \theta_\ell & -\sin \theta_\ell \\ \sin \theta_\ell & \cos \theta_\ell \end{pmatrix} \begin{pmatrix} \tilde{\ell}_1 \\ \tilde{\ell}_2 \end{pmatrix}. \quad (3.8)$$

Therefore, four wavefunction renormalization constants, $Z_{11}^{1/2}, Z_{12}^{1/2}, Z_{21}^{1/2}, Z_{22}^{1/2}$ can be expressed in terms of three independent renormalization constants, $Z_L^{1/2}, Z_R^{1/2}$ and $\delta \theta_\ell$, for each charged slepton.

We have adopted the following renormalization conditions in our paper on the chargino pair-production and decays [1].

- the on mass-shell conditions for all the three sleptons in each generation
- the residue conditions for all the three sleptons in each generation
- the decoupling conditions for the on-shell $\tilde{\ell}_i$ with $\tilde{\ell}_j$, ($j \neq i, \tilde{\ell} = \tilde{e}, \tilde{\mu}, \tilde{\tau}$)
- SU(2) relation for $\delta \theta_\ell$

which lead to the following expressions for the renormalization constants.

$$\delta m_{\tilde{\ell}}^2 = -Re\Sigma_{\tilde{\ell}\tilde{\ell}}(m_{\tilde{\ell}}^2), \quad \tilde{\ell} = \tilde{e}_1, \tilde{e}_2, \tilde{v}_e, \tilde{\mu}_1, \tilde{\mu}_2, \tilde{v}_\mu, \tilde{\tau}_1, \tilde{\tau}_2, \tilde{v}_\tau, \quad (3.9)$$

$$\delta Z_{\tilde{\ell}\tilde{\ell}} = \Sigma'(m_{\tilde{\ell}}^2), \quad \tilde{\ell} = \tilde{e}_1, \tilde{e}_2, \tilde{\mu}_1, \tilde{\mu}_2, \tilde{\tau}_1, \tilde{\tau}_2, \quad (3.10)$$

$$\delta Z_{\tilde{v}_\ell} = \Sigma'(m_{\tilde{v}_\ell}^2), \quad \ell = e, \mu, \tau, \quad (3.11)$$

$$\frac{1}{2} \delta Z_{\tilde{\ell}_i \tilde{\ell}_j} = -\frac{\Sigma_{\tilde{\ell}_i \tilde{\ell}_j}(m_{\tilde{\ell}_j}^2)}{m_{\tilde{\ell}_i}^2 - m_{\tilde{\ell}_j}^2}, \quad i \neq j, \quad \tilde{\ell} = \tilde{e}, \tilde{\mu}, \tilde{\tau}, \quad (3.12)$$

$$\delta \theta_\ell = \frac{\delta m_{\tilde{v}_\ell} - \delta(M_W^2 \cos 2\beta - m_\ell^2) - \cos^2 \theta_\ell \delta m_{\tilde{\ell}_1}^2 - \sin^2 \theta_\ell \delta m_{\tilde{\ell}_2}^2}{\sin 2\theta_\ell (m_{\tilde{\ell}_2}^2 - m_{\tilde{\ell}_1}^2)}, \quad \ell = e, \mu, \tau. \quad (3.13)$$

4. Extension of non-linear gauge formalism

We can extend the NLG functions (2.2) and (2.3) by including bilinear forms of sleptons with new NLG parameters \tilde{c} 's [3] as follows:

$$\begin{aligned} F_{W^+} = & (\partial_\mu + ie\tilde{\alpha}A_\mu + igc_W\tilde{\beta}Z_\mu)W^{+\mu} + i\xi_W \frac{g}{2}(v + \tilde{\delta}_H H^0 + \tilde{\delta}_h h^0 + i\tilde{\kappa}G^0)G^+ \\ & + i\xi_W g \left[\sum_{i=1,2} \left\{ \tilde{c}_i^e(\tilde{e}_i^*\tilde{v}_e) + \tilde{c}_i^\mu(\tilde{\mu}_i^*\tilde{v}_\mu) + \tilde{c}_i^\tau(\tilde{\tau}_i^*\tilde{v}_\tau) \right\} \right], \end{aligned} \quad (4.1)$$

$$\begin{aligned} F_Z = & \partial_\mu Z^\mu + \xi_Z \frac{g_Z}{2}(v + \tilde{\epsilon}_H H^0 + \tilde{\epsilon}_h h^0)G^0 \\ & + \xi_Z g_Z \left[\tilde{c}^{v_e v_e}(\tilde{v}_e^*\tilde{v}_e) + \tilde{c}^{v_\mu v_\mu}(\tilde{v}_\mu^*\tilde{v}_\mu) + \tilde{c}^{v_\tau v_\tau}(\tilde{v}_\tau^*\tilde{v}_\tau) \right. \\ & \left. + \sum_{i,j=1,2} \left\{ \tilde{c}_{ij}^{ee}(\tilde{e}_i^*\tilde{e}_j) + \tilde{c}_{ij}^{\mu\mu}(\tilde{\mu}_i^*\tilde{\mu}_j) + \tilde{c}_{ij}^{\tau\tau}(\tilde{\tau}_i^*\tilde{\tau}_j) \right\} \right], \end{aligned} \quad (4.2)$$

while F_{W^-} is hermitian conjugate to F_{W^+} .

In this paper, we focus on the NLG parameter \tilde{c}_1^τ in the NLG function F_{W^\pm} , and set $\xi_W = 1$ in order to avoid the instability in the one-loop calculations. The Feynman rules of vertices in the linear gauge are modified by introducing the NLG parameter \tilde{c}_1^τ as shown in Figure 1.

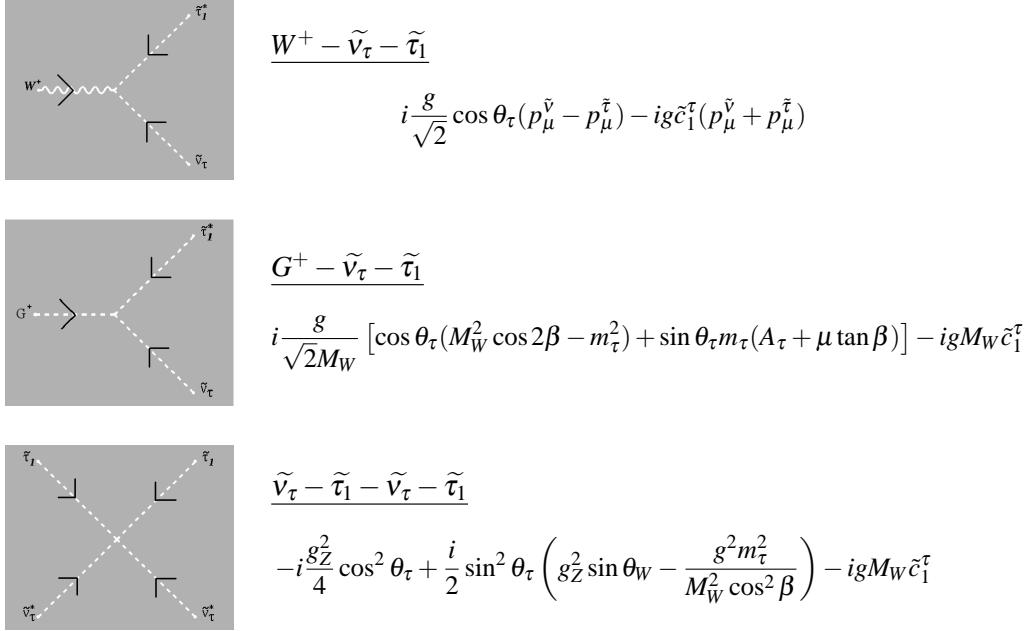


Figure 1: Feynman rules including the NLG couplings

The gauge invariance of NLG in the one-loop calculations is guaranteed by the BRST transformation, which leads to the introduction of the Faddeev-Poppov ghosts, ω_\pm , ω_Z and ω_γ , and anti-ghosts, $\bar{\omega}_\pm$, $\bar{\omega}_Z$ and $\bar{\omega}_\gamma$. The corresponding ghost lagrangian to the NLG parameter \tilde{c}_1^τ is given

as follows:

$$\begin{aligned} \mathcal{L}_{ghost} = & -i\xi_W g \tilde{c}_1^\tau \bar{\omega}_+ \left(\frac{i}{2} \right) \left[\sqrt{2}g \cos \theta_\tau \omega_+ \tilde{v}_\tau^* - \cos \theta_\tau g_Z \omega_Z (\cos \theta_\tau \tilde{v}_1^* - \sin \theta_\tau \tilde{v}_2^*) \right. \\ & \left. - 2e\omega_\gamma (\cos \theta_\tau \tilde{v}_1^* - \sin \theta_\tau \tilde{v}_2^*) \right] \tilde{v}_\tau \\ & -i\xi_W g \tilde{c}_1^\tau \bar{\omega}_+ \left(\frac{i}{2} \right) (2g_Z \sin \theta_W^2 \omega_z - 2e\omega_\gamma) (\tilde{v}_1^*) \tilde{v}_\tau \\ & -i\xi_W g \tilde{c}_1^\tau \bar{\omega}_+ \tilde{v}_1^* \left(\frac{-i}{2} \right) \left[\sqrt{2}g \omega_+ (\cos \theta_\tau \tilde{v}_1 - \sin \theta_\tau \tilde{v}_2) + g_z \omega_Z \tilde{v}_\tau \right] + (h.c.) . \quad (4.3) \end{aligned}$$

5. Numerical tests

We have calculated the slepton decay widths in one-loop order including the NLG gauge fixing functions (4.1). Here we present the results of the process, $\tilde{\tau}_2 \rightarrow \tilde{\tau}_1 + h^0$. Typical Feynman diagrams of this process concerned in the NLG coupling \tilde{c}_1^τ is shown in Figure 2. We have investigated coefficients of the zeroth power to fourth power of \tilde{c}_1^τ in the UV part and UV finite part. Table 1 shows numerical results, in which we have used SUSY parameters as $M_2 = 400$ GeV, $\mu = -100$ GeV, $\tan\beta = 30$, $m_{\tilde{\tau}_1} = 495.84$ GeV, $m_{\tilde{\tau}_2} = 608.23$ GeV and $\theta_\tau = 0.74\pi$. Then we have confirmed the NLG invariance of vertices for the two-body decays in one-loop order.

We have also calculated cross sections of the scattering processes systematically to test the NLG invariance of up to four-point vertices in one-loop order. Here we present the results of the process, $\tilde{\tau}_1 + \tilde{\tau}_1^* \rightarrow \tilde{\tau}_1 + \tilde{\tau}_1^*$. Typical Feynman diagrams of this process concerned in the NLG coupling \tilde{c}_1^τ is shown in Figure 3. Table 2 shows numerical results.

graph	UV part			Finite part	
	$(\tilde{c}_1^\tau)^0$	$(\tilde{c}_1^\tau)^1$	$(\tilde{c}_1^\tau)^2$	$(\tilde{c}_1^\tau)^1$	$(\tilde{c}_1^\tau)^2$
Virtual					
1	-2.864048E+02	8.100752E+02	-5.728097E+02	-6.902655E+00	6.310783E+00
2	2.640140E+02	-3.733722E+02	0.000000E+00	4.340137E+00	0.000000E+00
3	1.281803E+03	-2.015262E+03	2.864048E+02	2.219785E+01	-3.032220E+00
4	-2.459420E+02	3.478145E+02	0.000000E+00	-4.165453E+00	0.000000E+00
6	0.000000E+00	0.000000E+00	0.000000E+00	2.651421E-01	0.000000E+00
7	-1.470710E+03	1.877379E+03	2.864048E+02	-2.268075E+01	-3.192840E+00
8	0.000000E+00	0.000000E+00	0.000000E+00	-3.626202E-01	0.000000E+00
9	0.000000E+00	0.000000E+00	0.000000E+00	-7.996177E-02	-8.572352E-02
10	1.839419E+02	-5.202662E+02	3.678837E+02	5.671871E+00	-4.659912E+00
50	0.000000E+00	0.000000E+00	0.000000E+00	1.638727E-02	1.756807E-02
82	1.467649E+03	2.020973E+02	0.000000E+00	-2.152933E+00	0.000000E+00
94	-9.034420E+01	0.000000E+00	-3.678837E+02	0.000000E+00	4.642344E+00
134	0.000000E+00	-2.025188E+02	0.000000E+00	1.726551E+00	0.000000E+00
135	0.000000E+00	-2.025188E+02	0.000000E+00	1.726551E+00	0.000000E+00
138	-1.279135E+03	2.020973E+02	0.000000E+00	-2.266272E+00	0.000000E+00
Counter Term					
145	5.330665E+02	-1.255253E+02	0.000000E+00	2.666161E+00	0.000000E+00
Total					
	-6.032040E-20	5.714153E-27	1.019217E-29	-3.995728E-23	-2.514610E-31

Table 1: Test for NLG invariance of $\tilde{\tau}_2 \rightarrow \tilde{\tau}_1 + h^0$

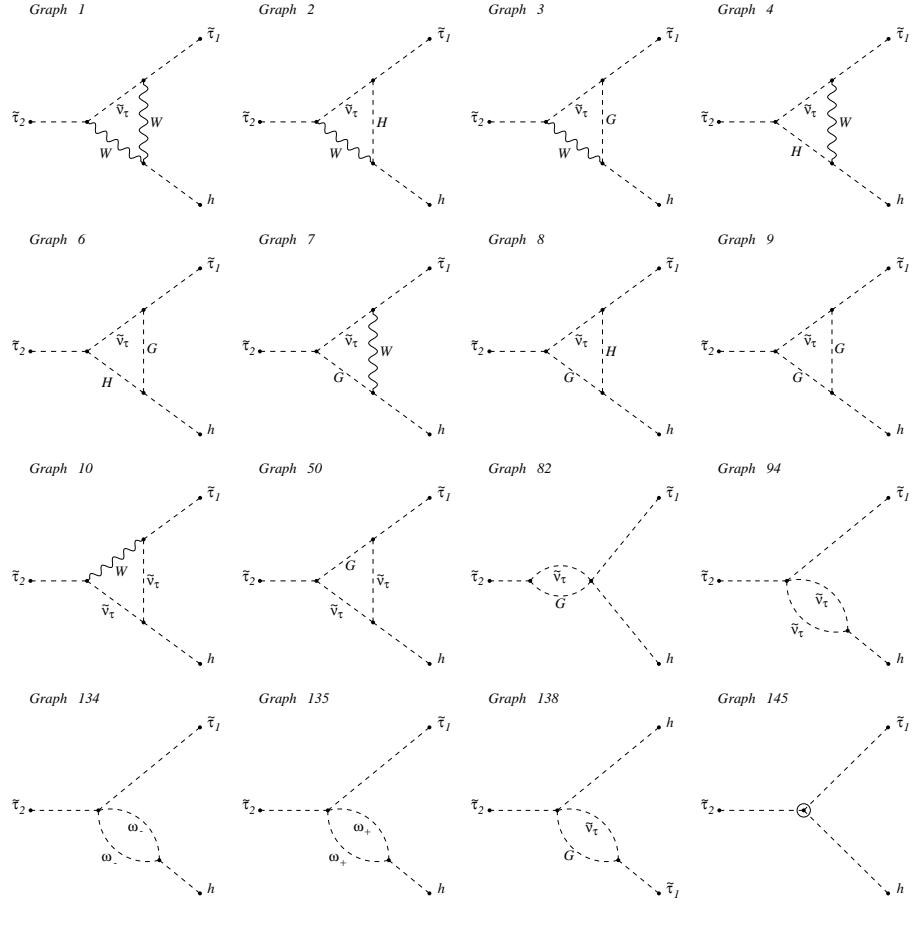


Figure 2: Typical Feynman diagrams of $\tilde{\tau}_2 \rightarrow \tilde{\tau}_1 + h^0$

6. Summary

We have developed the program package GRACE / SUSY-loop for the MSSM amplitudes in one-loop order, and extended the non-linear gauge formalism applied to GRACE / SUSY-loop by introducing the gauge fixing terms of bilinear forms of sleptons. Then we have confirmed the NLG invariance of the MSSM amplitudes in one-loop order for decay processes and scattering processes using GRACE / SUSY-loop.

Acknowledgments

This work is partially supported by Grant-in-Aid for Scientific Research(B) (20340063) and Grant-in-Aid for Scientific Research on Innovative Areas (21105513).

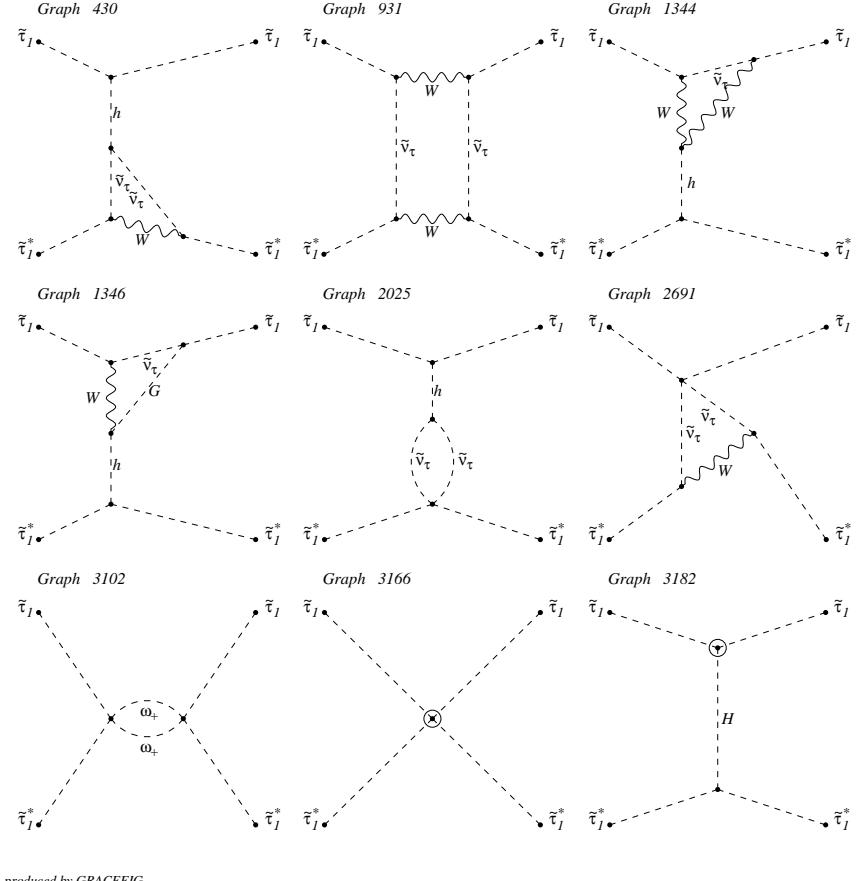


Figure 3: Typical Feynman diagrams of $\tilde{\tau}_1 + \tilde{\tau}_1^* \rightarrow \tilde{\tau}_1 + \tilde{\tau}_1^*$

References

- [1] J. Fujimoto, T. Ishikawa, Y. Kurihara, M. Jimbo, T. Kon and M. Kuroda, *Phys. Rev.* **D75** (2007) 113002.
- [2] K. Iizuka, T. Kon, K. Kato, T. Ishikawa, Y. Kurihara, M. Jimbo and M. Kuroda, PoS (RADCOR2009) 068 (2010), [[arXiv:1001.2800 \(hep-ph\)](https://arxiv.org/abs/1001.2800)].
- [3] M. Jimbo, K. Iizuka, T. Ishikawa, K. Kato, T. Kon, Y. Kurihara, M. Kuroda, To appear in Proceedings of International Linear Collider Workshop 2010 (LCWS10 & ILC10), Beijing, China, Mar 26-30 2010 (2010), [[arXiv:1006.3491 \(hep-ph\)](https://arxiv.org/abs/1006.3491)].
- [4] K. Fujikawa, *Phys. Rev* **D7** (1973) 393.
- [5] M.B. Gavela, G. Girardi, C. Malleville and P. Sorba, *Nucl. Phys.* **B193** (1981) 257.
- [6] H.E. Haber and D. Wyler, *SCIPIP-88/19* (1988).
- [7] M. Capdequi-Peyranère, H.E. Haber, P. Irulegui, *PM-90-06, SCIPIP-90-03*, in *Rigorous Methods in Particle Physics, Springer Tract. Phys.* **119** (1990).

- [8] F. Boudjema and A. Chopin, *Z. Phys.* **C73** (1996) 85.
- [9] G. Bélanger, F. Boudjema, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato, Y. Shimizu, *Phys. Rept.* **430** (2006) 117.
- [10] J. Fujimoto, T. Ishikawa, M. Jimbo, T. Kaneko, T. Kon, Y. Kurihara, M. Kuroda and Y. Shimizu, *Nucl. Phys. Proc. Suppl.* **157** (2006) 157.
- [11] F. Yuasa, J. Fujimoto, T. Ishikawa, M. Jimbo, T. Kaneko, K. Kato, S. Kawabata, T. Kon, Y. Kurihara, M. Kuroda, N. Nakazawa, Y. Shimizu and H. Tanaka, *Prog. Theor. Phys. Suppl.* **138** (2000) 18; <http://minami-home.kek.jp/>.
- [12] N. Baro and F. Boudjema, *Phys. Rev.* **D80** (2009) 076010.
- [13] T. Hahn, *Nucl. Phys. Proc. Suppl.* **89** (2000) 231; *Comput. Phys. Commun.* **140** (2001) 418.
- [14] P.H. Chankowski, S. Pokorski and J. Rosiek, *Nucl. Phys.* **B423** (1994) 437.
- [15] A. Yamada, *Phys. Lett.* **B263** (1991) 233; *Z. Phys.* **C61** (1994) 247.
- [16] A. Dabelstein, *Z. Phys.* **C67** (1995) 495.
- [17] W. Hollik, E. Kraus, M. Roth, C. Rupp, K. Sibold and D. Stöckinger, *Nucl. Phys.* **B639** (2002) 3.
- [18] T. Fritzsche and W. Hollik, *Eur. Phys. J.* **C24** (2002) 619.

graph	UV part				
	$(\tilde{c}_1^\tau)^0$	$(\tilde{c}_1^\tau)^1$	$(\tilde{c}_1^\tau)^2$	$(\tilde{c}_1^\tau)^3$	$(\tilde{c}_1^\tau)^4$
Virtual					
430	-5.230496E-02	-2.130068E-01	-2.168624E-01	0.000000E+00	0.000000E+00
931	-9.672486E-03	-7.878050E-02	-2.406194E-01	-3.266330E-01	-1.662726E-01
1344	8.144091E-02	3.316601E-01	3.376633E-01	0.000000E+00	0.000000E+00
1346	-3.644882E-01	-8.250864E-01	-1.688317E-01	0.000000E+00	0.000000E+00
2025	4.225897E-02	0.000000E+00	2.168624E-01	0.000000E+00	0.000000E+00
2691	7.814734E-03	3.182474E-02	7.250402E-02	1.633165E-01	1.662726E-01
3102	0.000000E+00	0.000000E+00	4.010324E-02	0.000000E+00	0.000000E+00
Counter Term					
3166	6.074190E-02	-1.272990E-01	0.000000E+00	0.000000E+00	0.000000E+00
3182	1.777580E+02	1.914021E+02	0.000000E+00	0.000000E+00	0.000000E+00
Total					
	-1.108258E-21	-1.813692E-25	-9.099667E-29	2.117843E-31	4.801768E-38
Finite part					
Virtual					
430		2.452794E-03	2.745929E-03	0.000000E+00	0.000000E+00
931		9.074788E-04	3.123851E-03	3.760592E-03	2.111032E-03
1344		-3.295736E-03	-3.852588E-03	0.000000E+00	0.000000E+00
1346		9.344057E-03	1.939682E-03	0.000000E+00	0.000000E+00
2025		0.000000E+00	-2.738628E-03	0.000000E+00	0.000000E+00
2691		-3.664649E-04	-1.035375E-03	-1.880605E-03	-2.105357E-03
3102		0.000000E+00	-4.706958E-04	0.000000E+00	0.000000E+00
Counter Term					
3166		1.427501E-03	0.000000E+00	0.000000E+00	0.000000E+00
3182		-2.146339E+00	0.000000E+00	0.000000E+00	0.000000E+00
Total					
		3.341261E-27	-3.304929E-29	-4.964405E-29	-2.529046E-29

Table 2: Test for NLG invariance of $\tilde{\tau}_1 + \tilde{\tau}_1^* \rightarrow \tilde{\tau}_1 + \tilde{\tau}_1^*$