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Applicability of Redundancy Calibration on Dense Phased Arrays

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Abstract. We aim to have an efficient and computationally cheap calibration method for dense phased arrays or any array which has enough redundant baselines. The most recently developed calibration method, multisource calibration requires a sky model. This takes computational capacity of the array's processor, especially when an extended structure like the galactic plane is present. This occurs due to short baselines. Redundancy calibration is independant of sky model. This in addition to having sufficient redundant baselines in dense phased arrays are the reasons why we study this method throughly. In the following paper, initial results of the redundancy calibration on dense phased arrays will be presented. The results are significantly promising that keeps up motivated to pursue this method in more details later on.

Keywords: dense phased array, calibration, redundancy, multisource, mutual coupling.

1. Introduction

In new generation of radio telescopes, we have a hierarchy of calibration schemes explained in Wijnholds et al. 2010. Calibration of phased arrays or station calibration is an important chain of this hierarchy. Its goal is to track the variations of complex electronic gain of recievers over time and frequency. A robust calibration together with beamforming should guarantee a stable beam pattern of the station for the central correlator. This is crucial for high fidelity imaging after the central correlator.

In phased arrays, correlation between all elements can be calculated. These correlations include many short baselines on which a non-resolved sky is captured. The most recent calibration method for the phased arrays is multisource calibration introduced by Wijnholds & van der Veen 2009. Multisource calibration method needs the presence of some relatively resolved point sources like Cas A and a model of the extended structures. Firstly, at a given time, detection of known sources is not guaranteed. Secondly, modeling the extended structures is hard and computationally expensive.

Dense phased arrays operating above ~ 100 MHz are often implemented as tiles, like HBAs (High Band Antenna) at LOFAR stations or EMBRACE (Electronic Multi-Beam Radio Astronomy ConcEpt). Having a regular antenna arrangement gives the possibility of having many redundant baselines i.e. with the same physical length and orientation (see Figure 2). This motivated us to try redundancy calibration instead. This method is independent of the sources in the sky. Its basic idea is that we should capture the same visibilities on the redundant baselines. It uses the data of all redundant baselines to obtain a convergent calibration solution.

In the following, we will build up a data model on the basis of which the two calibration methods can briefly be introduced. Afterwards the initial results of redundancy calibration on HBA data will be presented and discussed. The redundancy calibration method is not new but its application for dense phased array is novel. Therefore, there are some further steps left to take.

2. Methods

Multisource and reduandancy calibraion methods will be explained soon after data modeling.

2.1. data model

Lets assume that we have a phased array of p elements. Then we can express array signal vector, $x(t) = [x_1(t), x_2(t), ..., x_p(t)]^T$ like:

$$x(t) = \mathbf{\Gamma\Phi}\left(\sum_{k=1}^{q} \mathbf{a}_k s_k(t)\right) + \mathbf{n}(t) = \mathbf{\Gamma\Phi A s}(t) + \mathbf{n}(t) \qquad (1)$$

Where $\mathbf{s}(t)$ is $q \times 1$ vector containing q mutually independent i.i.d[a] Gaussian signals impinging on the array. They are also assumed to be narrow band, so we can define the q spatial signature vectors $\mathbf{a}_k$ which includes the phase delays due to the geometry and the directional response of the array. The receiver noise signals $n_i(t)$ are assumed to be mutually independent i.i.d Gaussian signals in a $p \times 1$ vector $\mathbf{n}(t)$ and uncorrelated. Thus $\mathbf{\Sigma}_n = \text{diag}(\sigma_n)$. Direction-independent complex gains; amplitudes and phases of gains which have to be calibrated are $\gamma = [\gamma_1, \gamma_2, ..., \gamma_p]^T$ and $\phi = [e^{j\phi_1}, e^{j\phi_2}, ..., e^{j\phi_p}]^T$

[a] independant (over time) and identically distributed.

correspondingly $\mathbf{\Gamma} = \text{diag}(\gamma)$ and $\mathbf{\Phi} = \text{diag}(\phi)$. $\mathbf{A} = [\mathbf{a}_2, ..., \mathbf{a}_q]$ (size $p \times q$). Then the model for the visibility matrix describing the correlations between all elements can be written:

$$\mathbf{R} = \mathbf{\Gamma\Phi A\Sigma}_s\mathbf{A}^H\mathbf{\Phi}^H\mathbf{\Gamma}^H + \mathbf{\Sigma}_n \quad (2)$$

where $\mathbf{\Sigma}_s$ and $\mathbf{A}$ are assumed to be known. We can calculate them by having time of observation, the telescope geometry and known astronomical catalouges.

In dense phased arrays like HBA and EMBRACE, the tiles are tightly packed. This may cause mutual coupling between the tiles. It is an important and frequency dependant effect on amplitudes and phases of the observed visibilities. It can be shown that the mutual coupling effect can be included as a matrix $\mathbf{M}$ multiplication in our data model. Referring to Warnick et al. (2006) and Warnick et al. (2005), a first order approximation for $\mathbf{M}$ can be assumed as Eq. 3, to define the off diagonal elements and $M_{ii} = 1$ as the diagonal elements.

$$M_{ij} = -m_a\lambda/r_{ij}(1 - m_b|\cos(\phi_{ij})|)\exp(2\pi j r_{ij}/\lambda) \quad (3)$$

Where m_a and m_d are constant values, r_{ij} is the distance between two elements, λ represents the frequency, ϕ_{ij} shows the orientation of the baseline between the two elements. Having a model of mutual coupling, we can simply disentangle it from the observed data.

2.2. Multisource calibration method

The multisource calibration problem can be formulated as a least squares minimization problem:

$$\{\hat{\mathbf{g}}, \hat{\sigma}_n\} = argmin_{\mathbf{g},\sigma_n} ||\mathbf{\Gamma\Phi A\Sigma}_s\mathbf{A}^H\mathbf{\Phi}^H\mathbf{\Gamma}^H + \mathbf{\Sigma}_n - \hat{\mathbf{R}}||_F^2 \quad (4)$$

This estimates the noise and complex gain of each reciever element using the measured visibility, $\hat{\mathbf{R}}$ and the modeled visibility, $\mathbf{\Gamma\Phi A\Sigma}_s\mathbf{A}^H\mathbf{\Phi}^H\mathbf{\Gamma}^H + \mathbf{\Sigma}_n$. In Wijnholds & van der Veen (2009), the mathematical solutions to this problem are comprehensively discussed. Here we just emphasize that the initial and important assumptions that let the method lead to a straightforward solution are:

- The receivers' noise are uncorrelated and accordingly $\mathbf{\Sigma_n}$ is diagonal.
- Complex gains are direction-independent (they are basically direction-dependant but assumed to be known. This direction dependency can be absorbed in the known sky $\mathbf{\Sigma_s}$).
- The impinging signals are narrow-band. This means we can represent time delays by phase shifts.

Figure 1 shows a sky map scanned by HBA tiles. One can see the galactic plane due to the many short baselines and the Sun as the dominant radio source. To apply the multisource calibration on HBA data, we need to have the corresponding data model; model of the extended structure and an unstable source like Sun.

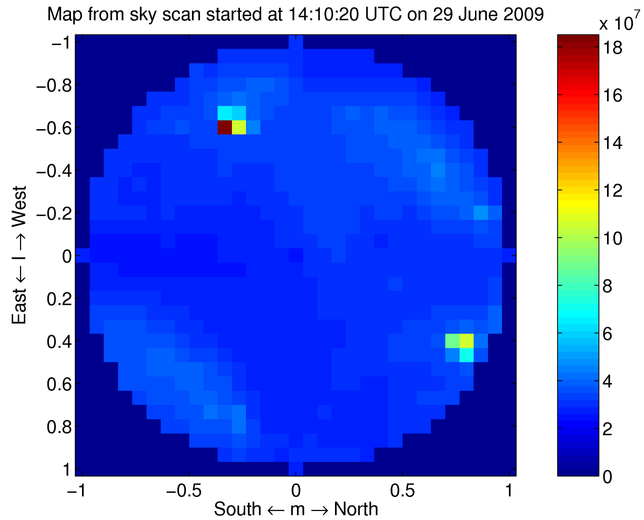

Fig. 1: The sky map scanned by HBA tiles at 14:10:20 UTC on 29 June 2009. The galactic plane appears at north-west, Sun appears at south-west. One can also see their corresponding grating response in the map.

2.3. Redundancy calibration method

The redundancy calibration method was introduced by Noordam and De Bruyn in 1982 Noordam et al. (1982). It has successfully been applied for the WSRT (Westerbork Synthesis Radio Telescope) since then. Its basic idea is very simple but clever; theoretically, the visibilities on redundant baselines are the same. We call those true visibilities. What we actually measure are the true visibilities multiplied by antennas and baseline dependant gains and summed by some errors Wieringa et al. (1991):

$$R_{ij}^{obs} = R_{ij}^{true}G_iG_j^*G_{ij} + c_{ij} + e_{ij} \quad (5)$$

Where G_{ij} is baseline dependant complex gain, c_{ij} is additive error due to e.g. correlator offset and e_{ij} is the thermal noise plus possible interference. If we assume that errors in G_{ij} as well as c_{ij} are negligible, we obtain:

$$R_{ij}^{obs} = R_{ij}^{true}G_iG_j^* + e_{ij} \quad (6)$$

By taking the natural logarithm from both sides of the Eq. 6, we obtain individual linear equations for the amplitude and phases of complex values:

$$r_{ij}^{obs} = r_{ij}^{true} + g_i + g_j + a_{ij} \quad (7)$$

$$\psi_{ij}^{obs} = \psi_{ij}^{true} + \phi_i - \phi_j + b_{ij} \quad (8)$$

Note that even for a Gaussian noise e_{ij}, the error terms a_{ij} and b_{ij} will have complicated distributions that depend on the SNR. For the rest of this paper, we assume a high SNR. So we can ignore the error terms. The resulting set of linear equations written for all the redundant baselines can be solved using a single step least squares method. This estimator will converge, if we add some constraints based on the actual situation of the array. We set those constraints to the best of our knowledge. Since we have to specify the absolute flux level, we set:

$$\Sigma g_i = 0 \tag{9}$$

We also have to constrain the absolute element phase. We can enforce this constraint by specifying that the average phase for all elements is zero:

$$\Sigma \phi_i = 0 \tag{10}$$

There might also be an arbitrary linear phase slope over the array. This phase slope corresponds to a position shift of the field. This arises because the redundancy can not detemine an absolute position. This can either be absorbed in the true visibilities or in the element phases. It can be shown, this is the null space of the matrix we have built up by Eq. 8. Since we have a two dimentional array unlike WSRT, we constraint x and y in the same manner:

$$\Sigma_{i=1}^{p} \phi_i x_i = 0 \tag{11}$$

$$\Sigma_{j=1}^{p} \phi_j y_j = 0 \tag{12}$$

This method is independant of sky model. Instead it requires: a) enough SNR to do a meaningful comparision between the redundant visibilities, b) enough redundancy in the array to get all the elements involved in the system of equations.

3. Discussion

Independancy of redundancy calibration from a sky model is a strong reason why we investigate this method. In dense phased arrays like HBA and EMBRACE, the tiles are set in a regular arrangement. This provides a significant number of redundant baselines. We studied the performance of reduncany calibration using different data sets. The presented results are on data captured on 26th May 2009 at 13:12:40 UTC using RCU mode 5 (frequency 110-190 MHz). The visisbility in each subband was integrated over one second.

Figure 2 shows 36 different types of redundant baselines that are available in a 24 tile HBA station. We present the result of method on the type indexed 10. Referring to Eq. 7 and Eq. 8, the parameters to be estimated are the true visibilities and the amplitues and phases of the complex reciever gains. In Figure 3 the left panels show the amplitudes and phases of the observed visibilities. One can clearly see that the visibilities (in both amplitude and phase) are redundant. Regarding the mutual coupling, we can certainly say that its effect is very small in the case of HBA data. Based on the model given in Eq. 3, its effcet on redundant baselines of the same type but by different elements are different. Therefore if it were a large effcet, we wouldn't see what we see now in the left panels. The right panels show how the method estimates amplitude and phase of the true visibility using the data of left panels. Deviation from the observed values is about a few percent. Phase wrapping is seen in lower right panel. This is a standard problem that we still have to solve.

Figure 4 shows how the method estimates amplitudes of complex reciever gain. The elements number 9, 10, 15 and 16 are chosen for this. Under normal circumstances, we expect this value to vary smoothly over frequency. One can see that its variance is about a few percent.

Figure 5 shows how the method estimates phases of complex reciever gain for the same elements. This value is also supposed to vary smoothly over frequency. But we can not justify the way these phases vary. We may have to reconstrain the phases in different way. This in addition to the phase wrapping problem require further work which will be suggested later in this paper.

4. Conclusion and further work

Herein we presented the initial results of redundancy calibration method on real data of a dense phased array. The results are sufficiently promising to make us believe this method is not only computationally cheap but also statistically efficient. Therefore it is potentially the method of choice for calibrating dense phased arrays like HBA at LOFAR stations, EMBRACE and hopefully dense phased arrays in SKA. This may be also a message for decision makers that we may consider redundancy in SKA final layout.

We just started exploring this method for the next generation of radio telescopes. We still have a long way to go. Thus some of the major further steps are suggested as:

1. The reduandancy calibration method should be applied on simulated data. This helps us to improve or change the constraints we set at the beginning.
2. The reduandancy calibration method should be evaluated mathematically as an estimator; Monte-Carlo simulation and CRLB (Cramer-Rao Lower Bound) evaluation are required. In this way, we can compare it with the existing calibration methods.
3. We should study its sensitivity toward the strength of RFI sources and SNR values.
4. We should also quantify the errors due to different assumptions that we make.

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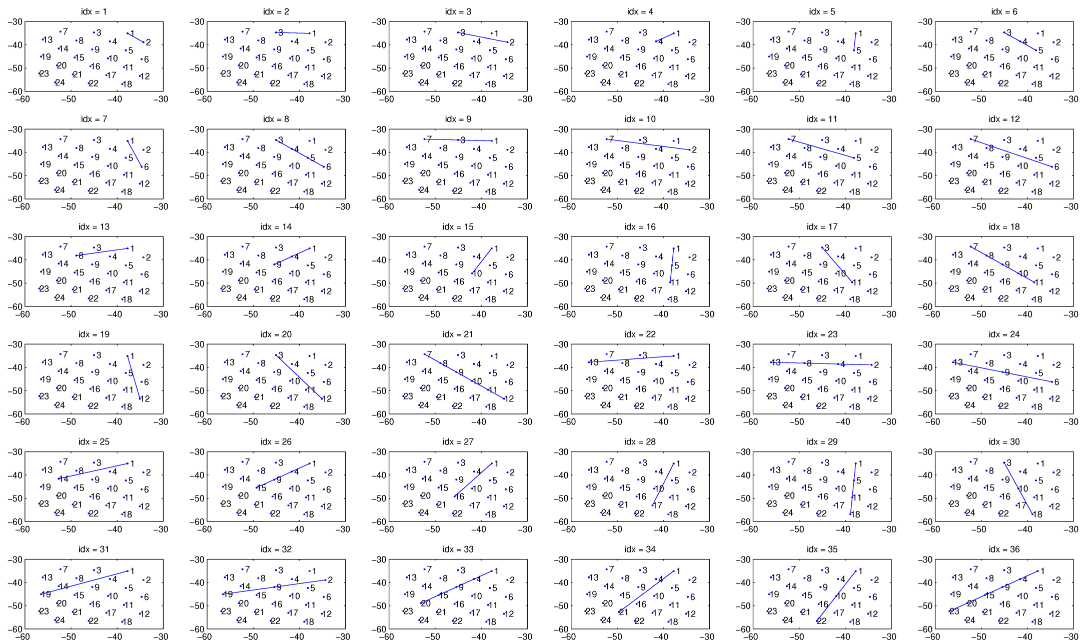

Fig. 2: 36 different redundand baselines indicated on layout of 24 tiles of HBA.

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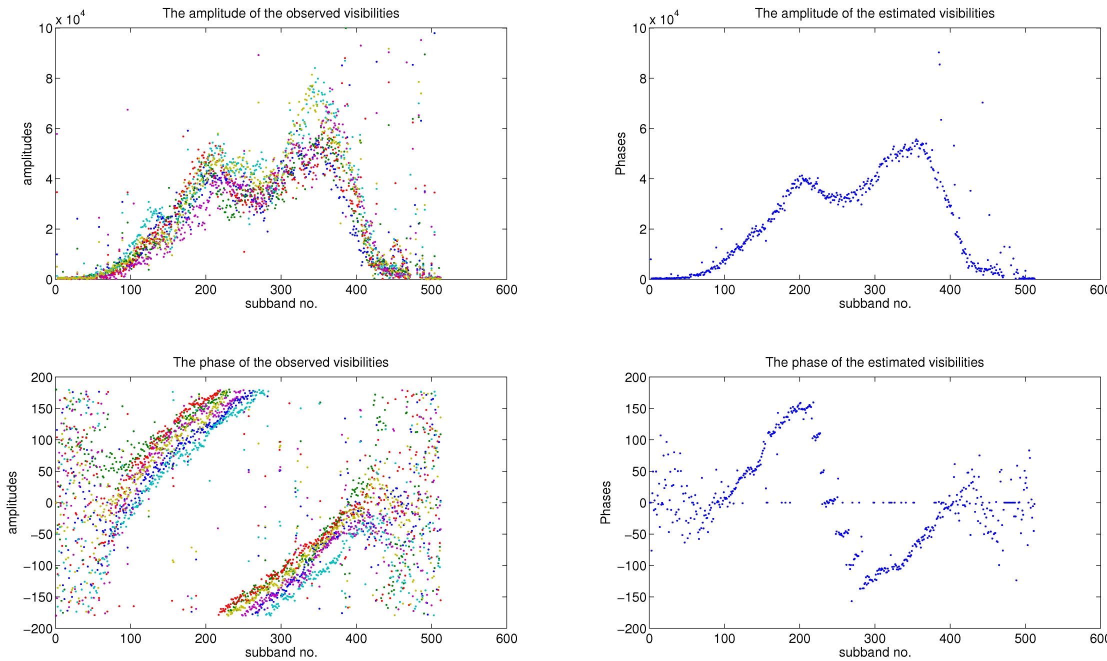

Fig. 3: Upper left: the amplitude of the observed visibilities. Upper right: the estimated amplitudes for true visibilities. Lower left: the phase of the observed visibilities. Lower right: the estimated phases for true visibilities.

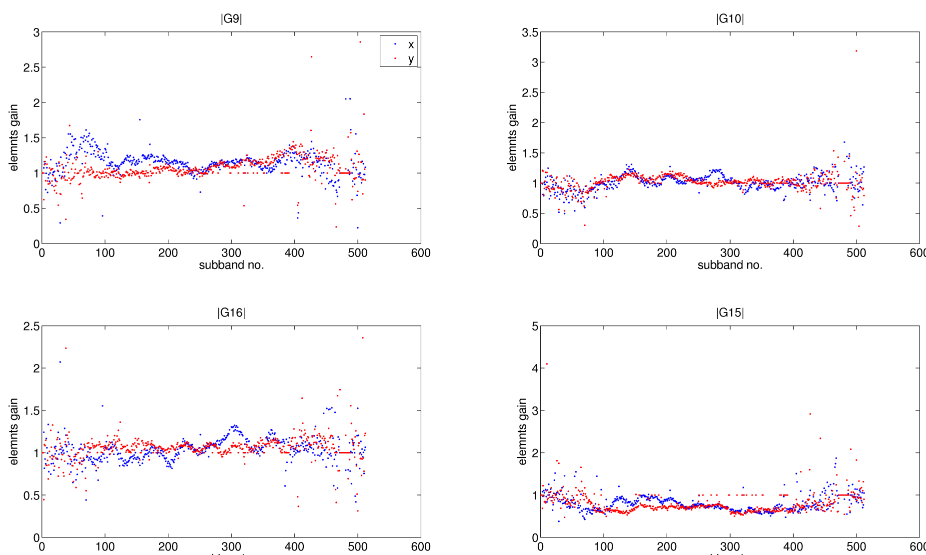

Fig. 4: Amplitude of complex recievers' gains. Out of 24 elements, the elements no. 9, 10, 15 and 16 are chosen.

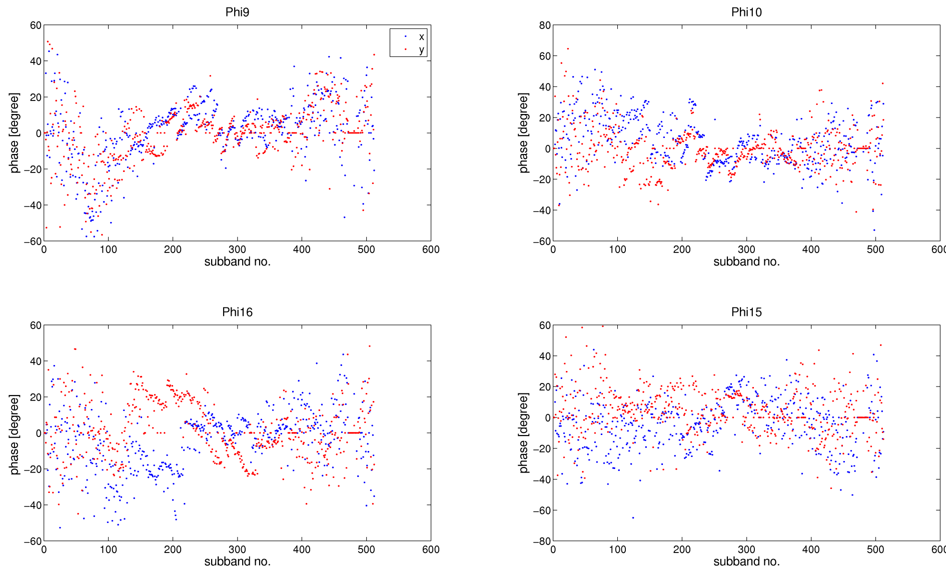

Fig. 5: Phase of complex recievers' gains. Out of 24 elements, the elements no. 9, 10, 15 and 16 are chosen.