PROCEEDINGS OF SCIENCE

Semi-analytical computation of the non-linear matter power spectrum

Benjamin AUDREN[*] **EPFL**
E-mail: benjamin.audren@epfl.ch

Julien Lesgourgues
EPFL, CERN
E-mail: Julien.Lesgourgues@cern.ch

We address the issue of computing the non-linear matter power spectrum on mildly non-linear scales with efficient semi-analytic methods. We implemented M. Pietroni's Time Renormalization Group (TRG) method and its Dynamical 1-Loop (D1L) limit in a numerical module for the new Boltzmann code CLASS. A careful comparison of the D1L, TRG and Standard 1-Loop approaches reveals that results depend crucially on the assumed initial bispectrum at high redshift. When starting from a common assumption, the three methods give roughly the same results, showing that the partial resumation of diagrams beyond one loop in the TRG method improves one-loop results by a negligible amount. A comparison with highly accurate simulations by M. Sato & T. Matsubara shows that all three methods tend to over-predict non-linear corrections by the same amount on small wavelengths. Percent precision is achieved until $k \sim 0.2\,h\mathrm{Mpc}^{-1}$ for $z \geq 2$, or until $k \sim 0.14\,h\mathrm{Mpc}^{-1}$ at $z = 1$.

XXIst International Europhysics Conference on High Energy Physics
21-27 July 2011
Grenoble, Rhônes Alpes France

[*]Speaker.

PoS(EPS-HEP2011)075

1. Introduction

Large scale structures in our universe have formed during the matter dominated era, starting from a very homogeneous state. In order to explain this mechanism, one must compute the evolution of the dominating species during this period: Cold Dark Matter (CDM) and baryons in the standard ΛCDM Model. It is generally agreed that structures formed from the gravitational collapse of small density perturbations. Depending on their wavelength, they entered at different times within the Hubble scale, which plays the role of a causal horizon for this process. In practice, one can apply the linear perturbation theory for wave numbers smaller than $0.1h\mathrm{Mpc}^{-1}$. In the opposite limit, for scales under ~ 10 Mpc today, the presence of highly non-linear structures such as galaxies points out the need for completely non-linear computations.

Current and upcoming surveys such as the Sloan Digital Sky Survey [1], the Large Synoptic Survey Telescope [2] and other Large Scale Structure (LSS) experiments probe this evolution with increasingly high precision. Through these new observations, cosmology opens a new window to confirm, constrain or infirm different high energy physics scenarios, related for instance to: neutrino masses, inflationary and dark energy models, or modifications of gravity.

The discriminating power of LSS observations depends crucially on the maximum wavenumber used in the comparison with the theory. By limiting the analysis to linear scales with $k < k_{max} = 0.1h\mathrm{Mpc}^{-1}$, one loses a lot of sensitivity, since the total amount of information scales like k_{max}^3. For instance, the strong dependence of neutrino mass error bars on k_{max} is illustrated in [1]. To further enhance the sensitivity, one would be tempted to use highly detailed N-body simulations. Unfortunately, in order to find both the best-fitting values and the error bars of the free cosmological parameters of a given scenario, one needs to compute a huge number of theoretical spectra corresponding to different points in parameter space. With the most efficient techniques (Monte Carlo Markov Chains), a minimum of 10'000 to 100'000 points is necessary, depending on the complexity of the model. N-body simulations are far too slow for being carried out in each of these points, or even in sizable fraction of them. This raises the need for semi-analytical tools to make accurate predictions on interesting scales. Even if such tools remain accurate only in a small range of mildly non-linear scales (just above $0.1h\mathrm{Mpc}^{-1}$), they can play a crucial role in measuring quantities like neutrino masses.

Semi-analytical methods have been proposed to actually calculate the non-linear power spectrum in Fourier space, taking into account the effects of mode coupling to some extent (for reviews and comparisons, see e.g. [2, 4]). Of course, all these approaches fail when dealing with the highly non-linear regime. However, their formulation stays consistent within the mildly non-linear regime, up to some k_{max} which depends on the method. Any tool implementing a good compromise between computing time on the one hand, and accuracy (i.e. large k_{max}) on the other hand, can be extremely useful for being employed at each point in parameter space when fitting cosmological models to the data.

[1] `http://www.sdss.org`
[2] `http://www.lsst.org`

I will present in this talk our implementation of the Time Renormalization Group (TRG) method proposed by Massimo Pietroni [3]. This method consists in integrating over time a coupled system of differential equations for the density and velocity power spectra and bispectra. Since the non-linearity computed at a given time step affect the time-derivative of the spectra at this step, the TRG method continuously includes higher-order corrections to the linear power spectra, and can be seen as a simple way to resum a sub-class of diagrams beyond one loop.

A simple variant of the TRG equations, consisting in using products of the linear power spectra in the non-linear source term of the equations, provides strictly one-loop results (we called this limit D1L for Dynamical 1-Loop) The main focus of this paper consists in a detailed comparison between S1L (Standard 1-Loop), D1L and TRG results for realistic ΛCDM models. We will see that assumptions concerning the initial bispectrum at high redshift crucially affect the results at low redshift. This point had been overlooked in the past, and will lead us to the conclusion that when starting from the same assumptions, the three methods only differ by a negligible amount. Hence, using the TRG method (at the order discussed in the current literature) instead of the much faster D1L algorithm does not appear to be justified.

2. Results and discussion

We showed two important results. First, that the two 1-Loop methods match perfectly when compared within the same assumptions. Indeed, the S1L is usually computed in the Einstein-de-Sitter limit, and with an initial bispectrum due to the non-linear evolution before the starting redshift.

Then, we showed that the TRG method improves only marginally over the 1-Loop schemes when compared to recent accurate N-body simulations (by Sato et al. [5]) with the correct initial conditions. Differences between TRG and D1L are visible at low redshift, but they remain very small on scales of interest. This shows that the partial resumation of diagrams beyond one loop in the TRG method improves one-loop results by a negligible amount, for a much larger computing time. Hence, the one-loop limit of the TRG equations (namely, the D1L scheme) is preferable in practice. This is to contrast with previous works done on this method, where a good agreement was shown. According to our work, it is likely that this agreement was purely accidental, and due to a wrong setting in the initial condition for the TRG.

We released these methods in the form of a C module, named `trg.c` and integrated in the Boltzmann code `CLASS`, version 1.2 ([6]). We presented in the companion paper some convergence tests proving the reliability of this implementation.

The D1L algorithm is a fast and practical tool for obtaining one-loop results for the density/velocity power spectra (and tree-level results for the bispectra), even for non-trivial cosmological models or in presence of an arbitrary initial bispectrum. There are several ways to incorporate such assumptions in one-loop calculations, but in the D1L equations, this flexibility is present from the beginning.

PoS(EPS-HEP2011)075

In this paper, we used this opportunity for showing the importance of assumptions concerning the initial bispectrum. In our universe, the non-linearity of the gravitational equations is sufficient to induce a tiny bispectrum even at very high redshift on cosmological scales of interest in this paper. At any given initial redshift, this bispectrum is small enough to be well approximated by a tree-level calculation, but nevertheless large enough to impact results at small redshift by several percents. This observation is consistent with previous studies of initial conditions for N-body simulations. In particular, the 2LPT method has been developed in order to deal with transient effects in simulations [7], i.e. in order to remove spurious decaying modes by implementing initial conditions infered from second-order Lagrangian perturbation theory, including an initial tree-level bispectrum. However, this fact had been overlooked in the context of TRG calculations.

3. Summary

We have shown that previous claims that TRG results improve over one-loop predictions were the consequence of neglecting this initial bispectrum. When using the same initial bispectrum in all methods, the difference between S1L, D1L and TRG results almost disappears. All these results can be checked with the publicly released TRG module for `CLASS`. The D1L method implemented in it provides a convenient tool to test non linear models that induce different initial conditions easily, as opposed to the traditional standard one-loop method with built-in assumptions.

References

[1] S. Hannestad, "Can cosmology detect hierarchical neutrino masses?," *Phys.Rev.* **D67** (2003) 085017, astro-ph/0211106.

[2] F. Bernardeau, S. Colombi, E. Gaztanaga, and R. Scoccimarro, "Large-scale structure of the universe and cosmological perturbation theory," *Phys. Rept.* **367** (2002) 1–248, astro-ph/0112551.

[3] M. Pietroni, "Flowing with Time: a New Approach to Nonlinear Cosmological Perturbations," *JCAP* **0810** (2008) 036, 0806.0971.

[4] J. Carlson, M. White, and N. Padmanabhan, "A critical look at cosmological perturbation theory techniques," *Phys.Rev.* **D80** (2009) 043531, 0905.0479.

[5] M. Sato and T. Matsubara, "Nonlinear Biasing and Redshift-Space Distortions in Lagrangian Resummation Theory and N-body Simulations," 1105.5007.

[6] J. Lesgourgues, "The Cosmic Linear Anisotropy Solving System (CLASS) I: Overview," 1104.2932.

[7] M. Crocce, S. Pueblas, and R. Scoccimarro, "Transients from Initial Conditions in Cosmological Simulations," *Mon.Not.Roy.Astron.Soc.* **373** (2006) 369–381, astro-ph/0606505.

PoS(EPS-HEP2011)075