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Bigravity as a Tool for Massive Gravity

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The formulation of massive gravity as bigravity is discussed. We argue that bigravity is more than a tool to tackle massive deformation of gravity.

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Recently, there has been a renewed interest in the search of a modified theory of gravity at large distances through a massive deformation of GR (see for a recent review [1]). A great deal of effort was devoted to extend at the nonlinear level [2] the seminal work of Fierz and Pauli (FP) [3].

In order to construct a massive deformation of GR we need to build non trivial scalar function of the metric field. To do that an extra tensor field field g_2 is needed. It is useful to introduce a "fictitious" space $\mathscr{M}_2$ where g_2 is a metric together with spacetime manifold $\mathscr{M}_1$ and a map $\Phi : \mathscr{M}_1 \to \mathscr{M}_2$. Given a metric g_2 in $\mathscr{M}_2$ one cal pull it back to $\mathscr{M}_1$

$$G = \Phi^*(g_2) , \qquad G_{\mu\nu}(x) = \frac{\partial \Phi^A}{\partial x^\mu} \frac{\partial \Phi^B}{\partial x^\nu} g_{2AB}(\Phi(x)) . \tag{1}$$

Under a diff we have $\Phi \to \Phi_{f_1 f_2} = f_2^{-1} \cdot \Phi \cdot f_1$. The building blocks of diff invariant modified theory of gravity can be formed starting from the following fundamental geometrical object

$$X^\mu_\nu = g_1^{\mu\alpha} G_{\alpha\nu} \quad (1,1) \text{ tensor in} \mathscr{M}_1 . \tag{2}$$

For instance, X^μ_μ is scalar in $\mathscr{M}_1$. In general, it is convenient to define [4]

$$\tau_n = \mathrm{Tr}(X^n) = (X^n)^\mu_\mu \equiv \mathrm{Tr}(Z^n) . \tag{3}$$

Sometimes, see [10], a related object is used to build invariant actions:

$$H_{\mu\nu} = g_{1\,\mu\nu} - G_{\mu\nu} = g_{1\,\mu\nu} - g_{1\,\mu\alpha} X^\alpha_\nu , \qquad \tau_n = \mathrm{Tr}[(1 - g_1^{-1} H)^n] . \tag{4}$$

The class of theories where both g_1 and g_2 are dynamical and with a non-trivial Φ, are invariant under the largest set of diffs: $\mathrm{Diff}_1 \times \mathrm{Diff}_2$. Sometimes, the second metric is taken non dynamical, then the theory exhibits invariance under Diff_1 only. For instance when g_2 is a flat and coincides with Minkowski metric $\tilde{\eta}_{AB}$, expanding the map Y as

$$Y^A(x) = \delta^A_\mu x^\mu + \pi^A(x) ; \tag{5}$$

one gets

$$G_{\mu\nu} = \delta^A_\mu \delta^B_\nu \tilde{\eta}_{AB} + \tilde{\eta}_{\mu B} \partial_\nu \pi^A + \tilde{\eta}_{\nu B} \partial_\mu \pi^A + \partial_\mu \pi^A \partial_\nu \pi^B \, \tilde{\eta}_{AB} . \tag{6}$$

When g_1 and g_2 are dynamical (bigravity theories), but the identification map Φ is chosen to be the identity (unitary gauge), only invariance under diagonal diffs for which $f_1 = f_2 = f$ is present. Such a choice of Φ is preserved by a diagonal diffs. In the unitary gauge $\mathscr{M}_1$ and $\mathscr{M}_2$ are identified and the gauge group $\mathrm{Diff}_1 \times \mathrm{Diff}_2$ is broken down to Diff_D, the group of diagonal diffs. This class of theories is usually called bigravity. When in bigravity the dynamics of g_2 is switched off, taking for simplicity g_2 flat, it can be written (at least locally) as

$$g_{2\,\mu\nu} = \partial_\mu \phi^a \partial_\nu \phi^b \tilde{\eta}_{ab} , \qquad \tilde{\eta} = \omega^2 \, \mathrm{Diag}\left(c^2, 1, 1, 1\right) . \tag{7}$$

The "flat" tetrads 1-form $e^a = d\phi^a$ are written in terms of the scalar fields ϕ^a that can be interpreted as local "flat" coordinates in $\mathscr{M}_1$. The inverse metric can written in terms of a dual basis of vector fields

$$E_a = \frac{\partial}{\partial \phi^a} = \frac{\partial x^\mu}{\partial \phi^a} \frac{\partial}{\partial x^\mu} \tag{8}$$

as

$$g_2{}^{\mu\nu} = E^\mu_a E^\nu_b \,\tilde{\eta}^{ab} = \frac{\partial x^\mu}{\partial \phi^a}\frac{\partial x^\nu}{\partial \phi^b}\,\tilde{\eta}^{ab} \,. \tag{9}$$

When $c \neq 1$, the flat background will break Lorentz invariance. In this class of theories we still have invariance under Diff_D. Notice that, by a suitable choice of coordinates one can set $\phi^a = \delta^a_\mu x^\mu$, using up all the gauge freedom. Decomposing the scalars as

$$\phi^a(x) = \delta^a_\mu x^\mu + \theta^a(x) \,; \tag{10}$$

one gets that expression for $H_{\mu\nu}$ is basically the same of one obtained when $g_{2AB} = \eta_{AB}$ but with $Y \neq \mathrm{id}$. For all class of theories the action can be written as

$$\begin{aligned} S = &\int_{\mathscr{M}_1} d^4x\sqrt{g_1}\left(M_{pl}\,R_1 + L_{\text{matter}}\right) + \kappa M_{pl}\int_{\mathscr{M}_2} d^4y\sqrt{g_2}\,R_2 \\ &-4\int_{\mathscr{M}_1} d^4x\,(g_1 G)^{1/4}\,V(X)\,; \end{aligned} \tag{11}$$

where V is a scalar function built out of X and it encodes the IR modifications of GR. The general properties of the massive gravity for both the Lorentz invariant (LI) and Lorentz breaking (LB) phases were studied in [5]. In the linearized LI phase, a combination of a massless and massive spin 2 modes mediate gravitational interactions, however the vDVZ discontinuity is present and in the zero mass ($m^2 \to 0$) there is an anomalous correction (25%) to the light deflection from the sun that is experimentally excluded [6]. In the LI phase, the standard weak field expansion is not viable. The LI phase, at the linearized level, is very similar to Fierz and Pauli (FP) [3] theory.

In the LB phase, together with the massive tensor mode there is always a massless one in the spectrum of metric perturbations. The corresponding phenomenology is quite rich [5, 7, 8]. The linearized theory can be interpreted as a diff-invariant realization of massive gravity, free of ghosts and phenomenologically viable (no vDVZ discontinuity is present). The only propagating degrees of freedom at linearized level are the spatial transverse traceless tensor modes (2 polarizations for each metric) physically representing a massless and a massive graviton (gravitational waves) oscillating one in the other and with different speeds, resulting in a nontrivial dispersion relation. The possibly superluminal speed c^2 in the second gravitational sector does not lead to causality violations, because the new metric has the character of 'æther'. The physical consequence is that gravitational wave experiments become frame-dependent.

A possible way to circumvent the physical consequences of the discontinuity IN LI case phase was proposed in [9]; the idea is that the linearized approximation breaks down near a massive object like the sun and an improved perturbative expansion must be used that leads to a continuous zero mass limit. In addition, FP is problematic as an effective theory. Regarding FP as a gauge theory where the gauge symmetry is broken by a explicit mass term m, one would expect a cutoff $\Lambda_2 \sim mg^{-1} = (mM_{pl})^{1/2}$, however the real cutoff is $\Lambda_5 = (m^4 M_{pl})^{1/5}$ or $\Lambda_3 = (m^2 M_{pl})^{1/3}$, much lower than Λ_2 [10]. A would-be Goldstone mode is responsible for the extreme UV sensitivity of the FP theory, that becomes totally unreliable in the absence of proper UV completion.

Recently it was shown that there exists a non linear completion of the FP theory [11] that is free of ghosts, avoiding the presence of the Boulware-Deser instability [12]. Then the propagation of only five degrees of freedom and the absence of instabilities was generalized in [13]; this was shown also in the Stuckelberg language in [14].

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The need for a second dynamical metric also follows from rather general grounds. Indeed, it was shown in [15] that in the case of non singular static spherically symmetric geometry with the additional property that the two metrics are diagonal in the same coordinate patch, a Killing horizon for g_1 must also be a Killing horizon for g_2. Thus, it seems that in order that the Vaishtein mechanism is effective and GR is recovered in the near horizon region of a black hole, g_2 has to be dynamical. Indeed, in a recent study of spherically symmetric solutions of a class of ghost free massive gravity theories in the bigravity formulation [16], generically there is room for the Vaishtein mechanism only when the two metrics are simultaneously diagonal in the same coordinate patch. The same conclusion is found looking at FRW solutions [17] which do not exist when the second metric is non dynamical.

Finally, it is also interesting to point out that in massive gravity, drastic modifications of gravity can take place with a non-analytic modification with respect to Schwarzschild [18]. In such cases the notion of total gravitational energy can be tricky and an ad hoc study is required [19]. Notice however that such a non-analytic deviation are not present in the class of ghost free massive gravity theories [16].

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