PROCEEDINGS OF SCIENCE

Towards exact field theory results for the Standard Model on fractional D6-branes

Gabriele Honecker[*†]
Institut für Physik (WA THEP)
Johannes-Gutenberg-Universität
D- 55099 Mainz, Germany
E-mail: Gabriele.Honecker@uni-mainz.de

Fractional D6-branes on toroidal orbifold backgrounds are known to be able to accommodate the particle spectrum and gauge group of the Standard Model, but up to now exact results for their low-energy effective action are missing. Here we discuss how the conceptual ansatz of matching the string theoretic gauge couplings at one-loop with the supergravity expressions is generalised from the six-torus to orbifold backgrounds on which the Standard Model spectrum can be realised on fractional D6-branes. The Kähler metrics and perturbatively exact holomorphic gauge kinetic functions can be classified in terms of the vanishing of some intersection angle and the related beta function coefficients, which potentially opens the possibility to extrapolate to smooth Calabi-Yau backgrounds.

The XXIst Europhysics Conference on High Energy Physics-HEP 2011
July 21-27 2011
Grenoble, Rhône-Alpes, France

MZ-TH/11-31

*Speaker.

†The work of G.H. is partially supported by the *Research Center Elementary Forces and Mathematical Foundations (EMG)* at JGU Mainz.

PoS(EPS-HEP2011)129

1. Introduction

Fractional D6-branes on $T^6/\mathbb{Z}_{2N}$ orientifold backgrounds of type IIA string theory have shown to be able to accommodate the Standard Model (SM) spectrum [1, 2], and improved models are expected on rigid D6-branes on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ with discrete torsion [3]. While extensive searches for SM spectra have been performed on a variety of toroidal orbifold backgrounds, the derivation of exact field theoretic results has to date focussed on the most simple case, the six-torus. We present here first results on the perturbatively exact holomorphic gauge kinetic functions and on the leading order Kähler metrics on type IIA $T^6/\mathbb{Z}_{2N}$ orientifold backgrounds by matching conformal field theory (CFT) results on gauge thresholds at one string-loop with standard supergravity expressions.

2. Kähler metrics and holomorphic gauge kinetic functions at one-loop

To study the low-energy effective field theory, the gauge couplings at one-loop can be computed using the magnetic background field method (see [4] for intersecting D6-branes on the six-torus and [5] on $T^6/\mathbb{Z}_{2N}$),

$$\left(\frac{M_{\text{Planck}}}{M_{\text{string}}}\right)^2 \text{Vol}(\text{D6}_a) + \int_0^\infty dl\, l^\varepsilon \frac{\partial^2}{\partial B_{\text{mag}}{}^2}\left[\langle \text{D6}_a(B_{\text{mag}})|e^{-l\pi H_{\text{closed}}}|\left(\sum_b |\text{D6}_b\rangle + |\text{O6}\rangle\right)\right]_{B_{\text{mag}}=0}$$

$$\overset{\frac{1}{\varepsilon}=\ln\left(\frac{M_{\text{string}}}{\mu}\right)^2}{=} \frac{1}{g^2_{a,\text{tree}}} + \frac{b_a}{16\pi^2}\ln\left(\frac{M_{\text{string}}}{\mu}\right)^2 + \frac{\Delta_a}{16\pi^2} = \frac{1}{g_a^2(\mu)} \quad \text{with} \quad \begin{array}{l} b_a = \sum_b b^{\mathscr{A}}_{ab} + b^{\mathscr{M}}_{aa'}, \\ \Delta_a = \sum_b \Delta^{\mathscr{A}}_{ab} + \Delta^{\mathscr{M}}_{a,\Omega\mathscr{R}}, \end{array} \tag{2.1}$$

with the beta function coefficients b_a of the gauge group G_a and gauge thresholds Δ_a due to massive strings decomposed into contributions from individual open string sectors with annulus ($\mathscr{A}$) and Möbius strip ($\mathscr{M}$) topology and endpoints on D6-branes a and b, the latter also including orbifold and orientifold images $(\theta^m a)_{m=0\ldots N-1}$ and $(\theta^n a')_{n=0\ldots N-1}$ on $T^6/\mathbb{Z}_{2N}$.

Matching 1-loop in string theory and supergravity. To obtain the canonical supergravity formulation, the string theoretic expression (2.1) needs to be matched with the field theoretic one,

$$\frac{1}{g_a^2(\mu)} = \Re(\text{f}_a) + \frac{b_a}{16\pi^2}\left[\ln\left(\frac{M_{\text{Planck}}}{\mu}\right)^2 + \mathscr{K}\right] + \frac{C_2(\mathbf{Adj}_a)}{8\pi^2}\left[\mathscr{K} - \ln g_a^2(\mu^2)\right] - \sum_a \frac{C_2(\mathbf{R}_a)}{8\pi^2}\ln K_{\mathbf{R}_a}(\mu^2),$$

containing the holomorphic gauge kinetic function f_a, the Kähler potential $\mathscr{K}$ and Kähler metrics $K_{\mathbf{R}_a}$ of the matter representations $\mathbf{R}_a \in \{(\mathbf{N}_a, \overline{\mathbf{N}}_b), (\mathbf{N}_a, \mathbf{N}_b), \mathbf{Anti}_a, \mathbf{Sym}_a, \mathbf{Adj}_a\}$ of $U(N_a) \times U(N_b)$ with quadratic Casimirs $C_2(\mathbf{R}_a)$. Using an iterative procedure (see [6] for $h_{21}^{\text{bulk}} = 3$ on the six-torus and partial results on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ with discrete torsion), the Kähler potential for the field theoretical dilaton S and bulk complex structure moduli U_l and Kähler moduli v_i on $T^6/\mathbb{Z}_{2N}$ and $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ without and with discrete torsion takes the form [7],

$$\mathscr{K}_{\text{bulk}} = -\alpha \ln S - \alpha \sum_{l=1}^{h_{21}^{\text{bulk}}} \ln U_l - \sum_{i=1}^{3} \ln v_i \qquad \text{with} \quad \alpha = 1,2,4, \quad \text{for} \quad h_{21}^{\text{bulk}} = 3,1,0,$$

for various numbers h_{21}^{bulk} of bulk complex structures U_l. At tree level, the holomorphic gauge kinetic function can be brought to the form $\text{f}_a^{\text{tree}} = S\tilde{X}_a^0 - \sum_{i=1}^{h_{21}^{\text{bulk}}} U_i \tilde{X}_a^i$, where $\tilde{X}_a^i$ are the (suitably normalised) bulk wrapping numbers of orientifold even three-cycles, $\Pi_a = \sum_{i=0}^{h_{21}} \left(\tilde{X}_a^i \Pi_i^{\text{even}} + \tilde{Y}_a^i \Pi_i^{\text{odd}}\right)$,

$(\phi_{ab}^{(1)}, \phi_{ab}^{(2)}, \phi_{ab}^{(3)})$	$K_{\mathbf{R}_a}$	$\delta_b^{\text{loop},\mathscr{A}} \, \mathrm{f}_{SU(N_a)}$
(0,0,0)	$K_{\mathbf{Adj}_a} = \frac{\sqrt{2\pi}}{c_a} \frac{f(S,U)}{v_i} \sqrt{\frac{V_{aa}^{(j)} V_{aa}^{(k)}}{V_{aa}^{(i)}}}$ $K_{\mathbf{R}_a \neq \mathbf{Adj}_a} = f(S,U) \sqrt{\frac{2\pi V_{ab}^{(i)}}{v_j v_k}}$ $(ijk) \simeq (1,2,3)$ cyclic	$-\sum_{i=1}^3 \frac{b_{ab}^{\mathscr{A},(i)}}{4\pi^2} \ln \eta(iv_i)$ $-\sum_{i=1}^3 \frac{\tilde{b}_{ab}^{\mathscr{A},(i)}(1-\delta_{\sigma_{ab}^i,0}\delta_{\tau_{ab}^i,0})}{8\pi^2} \times$ $\times \ln\left(e^{-\frac{\pi(\sigma_{ab}^i)^2 v_i}{4}} \frac{\vartheta_1(\frac{\tau_{ab}^i - i\sigma_{ab}^i v_i}{2}, iv_i)}{\eta(iv_i)}\right)$
$(0^{(i)}, \phi^{(j)}, \phi^{(k)})$ $\phi^{(j)} = -\phi^{(k)}$	$f(S,U) \sqrt{\frac{2\pi V_{ab}^{(i)}}{v_j v_k}}$	$-\frac{b_{ab}^{\mathscr{A}}}{4\pi^2} \ln \eta(iv_i) - \frac{\tilde{b}_{ab}^{\mathscr{A}}(1-\delta_{\sigma_{ab}^i,0}\delta_{\tau_{ab}^i,0})}{8\pi^2} \times$ $\ln\left(e^{-\frac{\pi(\sigma_{ab}^i)^2 v_i}{4}} \frac{\vartheta_1(\frac{\tau_{ab}^i - i\sigma_{ab}^i v_i}{2}, iv_i)}{\eta(iv_i)}\right)$ $+\sum_{l=j,k} \frac{N_b I_{ab}^{\mathbb{Z}_2^{(l)}}}{8\pi^2 c_a} \left(\frac{\text{sgn}(\phi_{ab}^{(l)})}{2} - \phi_{ab}^{(l)}\right))$
$(\phi^{(1)}, \phi^{(2)}, \phi^{(3)})$ $\sum_{i=1}^3 \phi^{(i)} = 0$	$f(S,U) \sqrt{\prod_{i=1}^3 \frac{1}{v_i} \left(\frac{\Gamma(\lvert\phi_{ab}^{(i)}\rvert)}{\Gamma(1-\lvert\phi_{ab}^{(i)}\rvert)}\right)^{-\frac{\text{sgn}(\phi_{ab}^{(i)})}{\text{sgn}(I_{ab})}}}$	$\sum_{i=1}^3 \frac{N_b I_{ab}^{\mathbb{Z}_2^{(i)}}}{8\pi^2 c_a} \left(\frac{\text{sgn}(\phi_{ab}^{(i)}) + \text{sgn}(I_{ab})}{2} - \phi_{ab}^{(i)}\right)$

Table 1: Kähler metrics $K_{\mathbf{R}_a}$ of open string matter states on D6-branes on T^6 and $T^6/\mathbb{Z}_{2N}$ and $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ without and with discrete torsion in dependence of the supersymmetric intersection angles $(\vec{\phi}_{ab})$ containing the universal factor $f(S,U) = (S \prod_{i=1}^{h_{21}^{\text{bulk}}} U_i)^{-\alpha/4}$ as well as the corresponding annulus contributions $\delta_b^{\text{loop},\mathscr{A}} \, \mathrm{f}_{SU(N_a)}$ to the holomorphic gauge kinetic functions as classified in [7]. The Möbius strip contributions to the gauge kinetic functions from aa' sectors on $T^6/\mathbb{Z}_{2N}$ are given in table 2.

specified in [7] for all orbifold backgrounds. The term proportional to $C_2(\mathbf{Adj}_a)$ in the supergravity expression contributes only to the matching of the aa sector, i.e. strings with both endpoints on the same stack of D6-branes. Analogously to the beta function coefficients and gauge thresholds, the one-loop contributions to the holomorphic gauge kinetic function form a sum over contributions from different open string sectors, $\delta_{\text{total}}^{\text{loop}} \mathrm{f}_a = \sum_b \delta_b^{\text{loop},\mathscr{A}} \mathrm{f}_a + \delta_{a'}^{\text{loop},\mathscr{M}} \mathrm{f}_a$. The open string Kähler metrics take an universal shape for all factorisable six-torus and toroidal orbifold backgrounds [7] as displayed in table 1, which fits with the alternative derivation on the six-torus by means of disc scattering amplitudes [8]. While the Kähler metrics and annulus contributions to the holomorphic gauge kinetic functions are fully classified by the vanishing of some intersection angle $(\phi_{ab}^{(i)})$, and the non-trivial orbifold background dependence is absorbed in the beta function coefficients, the Möbius strip contributions depend also on the relative angles $(\phi_{a,\Omega\mathscr{R}}^{(i)})$ with respect to the O6-planes as displayed in table 2 for $T^6/\mathbb{Z}_{2N}$.

Holomorphic gauge kinetic functions for massless U(1) gauge factors. Besides the $SU(N_a)$ gauge groups discussed above, $U(1)_a$ factors are ubiquitous in D-brane models and required for model building. While the tree-level gauge coupling of a single (anomalous) $U(1)_a$ is simply related to the $SU(N_a) \subset U(N_a)$ part by the normalisation factor $2N_a$,

$$\mathrm{f}_{U(1)_a}^{\text{tree}} = 2N_a \, \mathrm{f}_{SU(N_a)}^{\text{tree}}, \qquad \delta_{\text{total}}^{\text{loop}} \, \mathrm{f}_{U(1)_a} = 2N_a \Big(2\, \delta_{a'}^{\text{loop},\mathscr{A}} \mathrm{f}_{SU(N_a)} + \delta_{a'}^{\text{loop},\mathscr{M}} \mathrm{f}_{SU(N_a)} + \sum_{b \neq a,a'} \delta_b^{\text{loop},\mathscr{A}} \mathrm{f}_{SU(N_a)} \Big),$$

the one-loop correction differs in the aa and aa' sectors due to their vanishing and doubled $U(1)_a$ charge assignments, respectively [7].

The holomorphic gauge kinetic function of an (anomaly-free) massless linear combination of U(1)s, $Q_X = \sum_i x_i Q_{U(1)_i}$, such as the SM hyper charge, consists of a superposition of terms from

$(\phi^{(1)}_{aa'},\phi^{(2)}_{aa'},\phi^{(3)}_{aa'})$	$\delta^{\text{loop},\mathscr{M}}_{a'}\,\mathrm{f}_{SU(N_a)}$	$(\phi^{(1)}_{aa'},\phi^{(2)}_{aa'},\phi^{(3)}_{aa'})$	$\delta^{\text{loop},\mathscr{M}}_{a'}\,\mathrm{f}_{SU(N_a)}$
$(0,0,0)$ or $(\phi,0,-\phi)$ $\uparrow\uparrow\Omega\mathscr{R}$ on $T^2_{(2)}$	$-\frac{b^{\mathscr{M}}_{aa'}}{4\pi^2}\ln\eta(i\tilde{v}_2)+\tilde{c}_\phi\,\frac{\ln(2)}{2\pi^2}$ $-\frac{\tilde{b}^{\mathscr{M}}_{aa'}\,(1-\delta_{\sigma^2_{aa'},0}\delta_{\tau^2_{aa'},0})}{8\pi^2}\times$ $\ln\left(e^{-\frac{\pi(\sigma^2_{aa'})^2\tilde{v}_2}{4}}\,\frac{\vartheta_1(\frac{\tau^2_{aa'}-i\sigma^2_{aa'}\tilde{v}_2}{2},i\tilde{v}_2)}{\eta(i\tilde{v}_2)}\right)$	$i=2$ and $\perp\Omega\mathscr{R}$ on $T^2_{(2)}$ $(0^{(i)},\phi^{(j)},\phi^{(k)})_{\phi^{(j)}=-\phi^{(k)}}$ $i=1$ or 3	$\frac{\ln(2)}{16\pi^2}\left(\lvert\tilde{I}^{\Omega\mathscr{R}}_a\rvert+\lvert\tilde{I}^{\Omega\mathscr{R}\mathbb{Z}_2^{(2)}}_a\rvert\right)$ $-\frac{b^{\mathscr{M},(i)}_{aa'}}{4\pi^2}\ln\eta(i\tilde{v}_i)$ $+\frac{\ln(2)}{16\pi^2}\left(\lvert\tilde{I}^{\Omega\mathscr{R}}_a\rvert+\lvert\tilde{I}^{\Omega\mathscr{R}\mathbb{Z}_2^{(2)}}_a\rvert\right)$
$(0,0,0)$ $\uparrow\uparrow\Omega\mathscr{R}\mathbb{Z}_2^{(k),\,k=1\,\text{or}\,3}$	$-\sum_{i=1,3}\frac{b^{\mathscr{M},(i)}_{aa'}}{4\pi^2}\ln\eta(i\tilde{v}_i)$ $+\frac{1}{8\pi^2}\ln\left(2^4\,\frac{v_1v_3V^{(1)}_{aa'}V^{(3)}_{aa'}}{(v_2V^{(2)}_{aa'})^2}\right)$	$(\phi^{(1)},\phi^{(2)},\phi^{(3)})$ $\sum_{i=1}^3\phi^{(i)}=0$	$\frac{\ln(2)}{16\pi^2}\left(\lvert\tilde{I}^{\Omega\mathscr{R}}_a\rvert+\lvert\tilde{I}^{\Omega\mathscr{R}\mathbb{Z}_2^{(2)}}_a\rvert\right)$

Table 2: Classification of the Möbius strip contributions $\delta^{\text{loop},\mathscr{M}}_{a'}\,\mathrm{f}_{SU(N_a)}$ to the holomorphic gauge kinetic functions on D6-branes on $T^6/\mathbb{Z}_{2N}$ with $\tilde{c}_{\phi=0}=1$ and $\tilde{c}_{\phi\neq 0}=0$ in the upper left entry, see [7] for the full classification on all other torus and orbifold backgrounds.

the individual $U(1)_i$ factors plus pairwise mixing terms [7],

$$\mathrm{f}^{\text{tree}}_{U(1)_X}=\sum_i x_i^2\,\mathrm{f}^{\text{tree}}_{U(1)_i},\qquad \delta^{\text{loop}}_{\text{total}}\,\mathrm{f}_{U(1)_X}=\sum_i x_i^2\,\delta^{\text{loop}}_{\text{total}}\mathrm{f}_{U(1)_i}+4\sum_{i<j}x_ix_jN_i\left(\delta^{\text{loop},\mathscr{A}}_{j'}\,\mathrm{f}_{SU(N_i)}-\delta^{\text{loop},\mathscr{A}}_{j}\,\mathrm{f}_{SU(N_i)}\right).$$

This formula describes the potential one-loop kinetic mixing of the SM $U(1)$ with some hidden Z' boson from the open string sector on D6-branes.

The Standard Model example with hidden $Sp(6)_h$ on fractional D6-branes on $T^6/\mathbb{Z}_6'$ [1]. The right-handed quarks $Q_R^{1,2,3}$ of this model are split into two generations $Q_R^{1,2}$ localised at intersections of the QCD and 'right' stack ac at angle $\pi(0,\frac{1}{2},\frac{-1}{2})$ and with Kähler metrics $K_{Q_R^{1,2}}=f(S,U)\sqrt{\frac{4\pi}{\sqrt{3}v_2v_3}}$, whereas the third generation Q_R^3 is localised at an $a(\theta^2c)$ intersection at angle $\pi(\frac{-1}{3},\frac{-1}{6},\frac{1}{2})$ and has the Kähler metric $K_{Q_R^3}=f(S,U)\sqrt{\frac{10}{v_1v_2v_3}}$. The three right-handed lepton generations $L_R^{1,2,3}$ are localised at $c(\theta d)$ intersections of the 'right' and 'leptonic' stack at angle $\pi(\frac{-1}{6},\frac{-1}{3},\frac{1}{2})$ and have the Kähler metrics $K_{L_R^{1,2,3}}=f(S,U)\sqrt{\frac{10}{v_1v_2v_3}}$, cf. [7].

The tree-level QCD coupling is proportional to the one bulk complex structure U of $T^6/\mathbb{Z}_6'$, and the one-loop correction is a sum over contributions from all D6-branes a,b,c,d,h [7], $\delta^{\text{loop}}_{\text{total}}\mathrm{f}_{SU(3)_a}=\frac{1}{2\pi^2}\ln\left[\frac{\eta(i\tilde{v}_1)}{\eta(iv_1)}\frac{1}{\eta(iv_3)^4}\left(e^{-\pi v_3/4}\,\frac{\vartheta_1(\frac{1-iv_3}{2},iv_3)}{\eta(iv_3)}\right)^{-3/2}\left(2^{15/2}\,\frac{\sqrt{3}v_1v_3}{v_2^2}\,r\right)^{1/4}_{r=1/\sqrt{3}}\right]$, together with their orbifold and orientifold images, where the orbifold invariant orbit of a is of the special type $\uparrow\uparrow\Omega\mathscr{R}\mathbb{Z}_2^{(1)}$ in table 2.

3. Conclusions

By comparison of CFT results with canonical supergravity expressions, we obtained the perturbatively exact holomorphic gauge kinetic functions for $SU(N)$ and anomaly-free $U(1)$ gauge groups as well as the tree-level Kähler metrics for charged matter on D6-branes in $T^6/\mathbb{Z}_{2N}$ orientifold backgrounds, all of which are required for an extension of D-brane model building from massless particle spectra to their effective low-energy field theory. As examples, we explicitly computed the Kähler metrics for right-handed quarks and leptons and derived the one-loop holomorphic gauge kinetic function for the QCD gauge factor of the SM with 'hidden' $Sp(6)_h$ on $T^6/\mathbb{Z}_6'$.

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