

## Neutron EDM in Four Generation Standard Model

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Fanrong Xu<sup>\*†</sup>

*Department of Physics, National Taiwan University, Taipei, Taiwan 10617*

*E-mail: frxu@ntu.edu.tw*

A fourth generation of quarks may provide sufficient  $CP$  violation for the baryon asymmetry of the Universe. We estimate the neutron electric dipole moment in the presence of the fourth generation, and find it would be dominated by the strange quark chromoelectric dipole moment, assuming it does not get wiped out by a Peccei-Quinn symmetry. With three loop analytical expression, 500 GeV heavy quark mass, and a Jarlskog CPV factor which is consistent well with LHC 2011 summer data, the neutron EDM is found around  $10^{-31}$  e cm, still far below the  $10^{-28}$  e cm reach of next generation of experiments.

**PACS numbers** 13.40.Em 11.30.Er 14.65.Jk

*The 2011 Europhysics Conference on High Energy Physics-HEP 2011,  
July 21-27, 2011  
Grenoble, Rhône-Alpes France*

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<sup>\*</sup>Speaker.

<sup>†</sup>I thank my collaborators Junji Hisano and Wei-Shu Hou on arXiv:1107.3642 [hep-ph], which would appear in Phys. Rev. D.

## 1. INTRODUCTION

It was pointed out that, by extending to four quark generations[1], SM4, we may have enough CPV phase for Baryon Asymmetry of the Universe (BAU). The long quest for neutron electric dipole moment (nEDM) has been motivated by BAU, as the latter implies the existence of new CPV sources beyond SM. The gap between theory and experiment is large, as the prediction given in SM is  $d_d \sim 10^{-34} e\text{cm}$  [2] with one to two orders of magnitude enhancement due to long distance (LD) effect [3], while current limit is  $2.9 \times 10^{-26} e\text{cm}$  at 90% C.L. from the RAL-Sussex-ILL experiment [4]. Furthermore, there is a renewed effort, by several groups in the world, to push nEDM first towards  $\mathcal{O}(10^{-27}) e\text{cm}$ , then eventually down to  $10^{-28} e\text{cm}$ . Driven by these new experiments, and given the large jump in CPV, it is of interest to ask what nEDM value one might expect for SM4.

## 2. Some Relevant Formulas

Starting from the effective Lagrangian of all CPV operators up to dimension 5, the neutron EDM was evaluated in the QCD sum rule framework [5].

$$d_n = (0.4 \pm 0.2) \left[ 1.9 \times 10^{-16} \bar{\theta} e\text{cm} - 0.08e\tilde{d}_s + 1.8e\tilde{d}_d - 1.4e\tilde{d}_u + (4d_d - d_u) \right]. \quad (2.1)$$

The large factor of 3 uncertainty inherent in the overall  $0.4 \pm 0.2$  coefficient reflects the large hadronic uncertainty. Thus, our estimates that follow are only aimed at the order of magnitude.

The interesting subtlety is that, when a Peccei-Quinn symmetry is invoked to remove the  $\bar{\theta}$  term (setting it to zero), it induces additional CPV terms [7] to the axion potential that cancels the strange quark CEDM (sCEDM) contribution [6] at leading-order of the expansion of nucleon current. While remarkable, as we shall see, the sCEDM is of the greatest interest in SM4. Furthermore, three decades of axion search has so far come to naught. Given that there are models of spontaneous CPV, we shall ignore the  $\bar{\theta}$  term while keeping the qCEDM terms.

Analyzing the flavor structure of a typical three loop diagram shows why the strange CEDM is highlighted, despite a smaller coefficient in Eq. (2.1). We shall consider typical loop momenta would be at rather heavy  $t'$  and  $b'$  scale. Therefore, one can take  $c = u \equiv \mathbf{u}$ ,  $d = s = b \equiv \mathbf{d}$ , and easily see that the (C)EDM of the  $u$  quark vanishes. For  $f = d, s$ , we have

$$i \sum_{j,k,l} \text{Im}(V_{uj}^* V_{kj} V_{kl}^* V_{ul}) f j k l f = i \text{Im}(V_{if}^* V_{ib} V_{i'b}^* V_{i'f}) f [t(\mathbf{d} - b')t' - t'(\mathbf{d} - b')t + t'(\mathbf{d} - b')\mathbf{u} - \mathbf{u}(\mathbf{d} - b')t' + \mathbf{u}(\mathbf{d} - b')t - t(\mathbf{d} - b')\mathbf{u}] f. \quad (2.2)$$

The  $s$  quark CEDM arising from the two- $W$  loop plus one gluon loop diagram was estimated [8] using the external field method, with the result of

$$\tilde{d}_s^{(g)} = -\mathcal{J}_s m_s \frac{G_F \alpha_s \alpha_W}{\sqrt{2} (4\pi)^4} \frac{5N_c}{6} \frac{m_t^2}{M_W^2} \frac{1}{2!} \log^2 \left( \frac{m_{t'}^2}{m_t^2} \right), \quad (2.3)$$

where Jarlskog CPV factor  $\mathcal{J}_f \equiv \text{Im}(V_{if}^* V_{ib} V_{i'b}^* V_{i'f}) \cong -\text{Im}(V_{if}^* V_{ib} V_{i'b}^* V_{i'f})$  is introduced.

Replacing the gluon by a  $Z$  boson loop, we can also realize that the  $Z$ -loop gives important contribution. By an ingenious argument of limiting to large loop momenta and involving longitudinal vector bosons, the authors of Ref. [9] were able to reduce the three-loop calculation effectively

to calculating three one-loop integrals, and the core of it is an effective  $i \rightarrow fZ$  transition involving the heavy fourth generation quark in the loop. This is the familiar Z penguin [12, 13], and indeed it has been found [10] that  $b' \rightarrow bZ$  and  $b' \rightarrow bg$  transitions are not too different in strength. The upshot of the estimate (with the brutality of setting all logarithms to order 1) of Ref. [9] is

$$\tilde{d}_s^{(Z)} = -\mathcal{J}_s m_s \frac{G_F}{\sqrt{2}} \frac{\alpha_W^2}{(4\pi)^4} \frac{m_t^2 m_{t'}^2}{4M_W^4} \log\left(\frac{m_{t'}^2}{m_t^2}\right). \quad (2.4)$$

Comparing Eqs. (2.3) and (2.4), one can see from  $\alpha_W/M_W^2 = \sqrt{2}G_F/\pi = 1/\pi v^2$  that one is comparing  $5N_c\alpha_s/6$  with  $\lambda_{t'}^2/4\pi$ . The gluonic effect is enhanced by the color factor, but the Yukawa coupling grows with  $m_{t'}^2$ . Compared literally, they are actually comparable. On the other hand, in arriving at Eq. (2.4), one has set all logarithms to 1. In this spirit, both the double log (including the  $1/2!$ ) in Eq. (2.3) and the single log in Eq. (2.4) should be treated as order one. Then, the gluonic effect would be subdominant to the Z effect, for  $t'$  and  $b'$  masses of order 500 GeV (or higher), a nominal value used by Ref. [9], and which we shall use in the next section.

Given the roughness of these calculations, and the great difficulty in calculating genuine three electroweak loop diagrams, we shall take the estimate of Eq. (2.4) for our subsequent numerics.

### 3. Numerical Estimate

We shall use  $m_{t'} \simeq m_{b'} \simeq 500$  GeV as our nominal fourth generation quark mass. The other parameters are:  $m_t = 165.5$  GeV,  $m_u = 2.5$  MeV,  $m_d = 5$  MeV and  $m_s = 100$  MeV. And for the Jarlskog CPV factor  $\mathcal{J}_s$  and  $\mathcal{J}_d$ , we take the nominal fit [14] to flavor data performed for  $m_{t'} \simeq 500$  GeV, where  $V_{t'b} \simeq -0.1$ ,  $V_{t's} \simeq -0.06 e^{-i75^\circ}$ , and  $V_{t'd} \simeq -0.003 e^{-i18^\circ}$ , we get

$$\mathcal{J}_s = \text{Im}(V_{ts}^* V_{tb} V_{t'b}^* V_{t's}) \simeq 2.4 \times 10^{-4}, \quad \mathcal{J}_d = \text{Im}(V_{td}^* V_{tb} V_{t'b}^* V_{t'd}) \simeq 1.7 \times 10^{-7}. \quad (3.1)$$

Note that  $\mathcal{J}_s$  is consistent with LHC data released in 2011 summer, and could be measured [15] in the next two years at the LHC, but  $\mathcal{J}_d$  would be harder to disentangle.

Putting in numbers, we find from Eq. (2.4) that

$$\tilde{d}_s^{(4)} \simeq -4 \times 10^{-16} \text{ GeV}^{-1} \simeq -0.8 \times 10^{-29} \text{ cm}, \quad (3.2)$$

where the  $W$ - $W$ - $g$  3-loop effect of Eq. (2.3) is treated as subdominant. Treating the sCEDM as the leading effect in Eq. (2.1), then we have

$$d_n^{(4)} = (2.2 \pm 1.1) \times 10^{-31} \text{ e cm}. \quad (3.3)$$

We note that the estimation in (3.3) is lower than the original value in [9]. Besides changes in the numerical value for the Jarlskog invariant  $\mathcal{J}_s$ , the reason is that [9] relied on CP-odd meson-nucleon coupling induced by strange quark condensate estimated through chiral perturbation theory, while it is disfavored from the recent lattice evaluation.

#### 4. Discussion and Conclusion

If a PQ symmetry is operative in Nature, then the sCEDM effect is canceled [7, 6]. In this case, one has a reduced formula [5],

$$d_n^{\text{PQ}} = (0.4 \pm 0.2) \left[ 1.6e(2\tilde{d}_d + \tilde{d}_u) + (4d_d - d_u) \right], \quad (4.1)$$

i.e. only dependent on the naive constituents of the neutron.

Assuming again that the analogue of Eq. (2.4) dominates over the gluonic counterpart, we obtain  $d_d$  and  $\tilde{d}_d$  by simply shifting the CKM index, i.e. shifting from  $\mathcal{J}_s$  to  $\mathcal{J}_d$  in Eq. (3.1), and replacing  $m_s$  by  $m_d$ . We find

$$\tilde{d}_d^{(4)} \simeq -3 \times 10^{-34} \text{ cm}, \quad d_d^{(4)} \simeq -4 \times 10^{-34} \text{ e cm}, \quad (4.2)$$

$$d_n^{(4)\text{PQ}} = -(1 \pm 0.5) \times 10^{-33} \text{ e cm}, \quad (4.3)$$

where  $d_d$  contributes roughly twice as  $\tilde{d}_d$ . These should be taken as very rough estimates.

In conclusion, with four quark generations and with Peccei-Quinn symmetry operative, the neutron EDM is slightly enhanced above the SM value, but no more than an order of magnitude, hence much below the sensitivities of the next generation of experiments. If PQ symmetry is absent, then a large enhancement is possible through the  $s$  quark CEDM, which is correlated with possible effects in  $b \rightarrow s$  transitions that are of current interest. However, it is still unlikely that the neutron EDM could reach the  $10^{-28}$  e cm level sensitivity that may be probed during the next decade.

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