PROCEEDINGS OF SCIENCE

b-quark mass and B decay constant from $\mathrm{N_f} = 2$ lattice QCD simulations

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We report about the status of the on-going project tackled by the ALPHA Collaboration to measure from lattice QCD simulations with $\mathrm{N_f} = 2$ dynamical quarks the b quark mass and the B meson decay constant. We have performed the computation in the framework of Heavy Quark Effective Theory expanded at $1/m$ order; we have matched it to QCD using a non-perturbative approach. We have considered several lattice spacings and light quark masses in order to control cut-off effects and extrapolate to the chiral limit.

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1. Introduction

The impressive luminosity obtained at LHCb and the high activity to develop new e^+e^- machines (Super KEKB and Super B) open exciting perspectives to use low energy processes as a probe of New Physics scenarios. Indeed, rare hadron decays offer a rich set of constraints because they are mediated by quantum loops, where high energy particles circulate, or by new couplings at tree level. Studying those transitions is then a complementary approach to the direct search of new particles from the electroweak scale up to the TeV scale performed at ATLAS and CMS. A way to reach that goal is for instance to estimate the contribution beyond the Standard Model to branching ratios of B decays. The inclusive $b \to s\gamma$ decay has been very attractive because it is highly sensitive to penguin diagrams. However a major source of uncertainty comes from the b quark mass: m_b enters the analytical expression of the decay at the fifth power [1]. The puzzle "$(\sin 2\beta,\ BR(B \to \tau\nu))$" remains: a compatible value of the CKM matrix element V_{ub} extracted from $B \to \pi l \nu$ and $B \to \tau\nu$ would mean a value of f_B 30 % larger than its updated one, or the exchange of a charged Higgs through a right-handed current, to enhance the helicity suppressed leptonic B decay with respect to its magnitude within the Standard Model.

In both cases a theoretical input is needed to normalize the width: m_b and f_B. Lattice QCD reveals to be a powerful approach to determine them with an accuracy of few %, in order to be competitive with the precision on experimental data. Nevertheless an issue is the control on cut-off effects: the Compton length of the b quark is typically smaller than the lattice spacing, making the situation particularly uneasy. Different strategies have been explored by the lattice community to deal with the discretisation errors at a minimal cost. The ALPHA Collaboration has proposed to use the framework of Heavy Quark Effective Theory (HQET), expanded up to the first order in $1/m$, with a non perturbative determination of the couplings [2]. That program has been now achieved [3] and we are in a position to extract the hadronic quantities from our set of simulations.

In the next sections we will present the results obtained by analysing 7 ensembles of gauge configurations build within the CLS effort [4] with $\mathrm{N_f}=2$ $O(a)$ improved Wilson-Clover fermions. We have considered 3 lattice spacings ($a = 0.05$ fm, 0.065 fm and 0.075 fm) and pion masses in the range [250-400] MeV to extrapolate to the continuum and chiral limits. All the ensembles have an extension $Lm_\pi > 4$, so that we neglect finite volume effects.

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2. B spectrum and b quark mass

The HQET Lagrangian reads up to $\mathscr{O}(1/m)$ $\mathscr{L}^{\mathrm{HQET},1/\mathrm{m}}(x) = \sum_i \omega_i \mathscr{O}^i(x)$ where $[\mathscr{O}^i] \leq 5$. The HQET couplings ω_i depend implicitly on the cut-off a, on the RGI heavy quark mass $z \equiv LM$ (expressed in volume units) that is used to perform the matching between HQET and QCD, and on the static quark action (HYP1 or HYP2) that one writes in function of the HYP-smearing parameters α_i. L is defined through the renormalised strong coupling in the Schödinger Functional scheme by $\bar{g}^2(L) = 2.989$ [5]. Finally the B meson mass depends also on the light quark mass. We have extrapolated to the continuum and chiral limits the B meson mass using the LO formula

$$m_B^{0(1)}(a,z,m_\pi,\alpha_i) = m_B^{0(1)}(z) + b^{0(1)} m_\pi^2 + c^{0(1)}(\alpha_i)a^2, \tag{2.1}$$

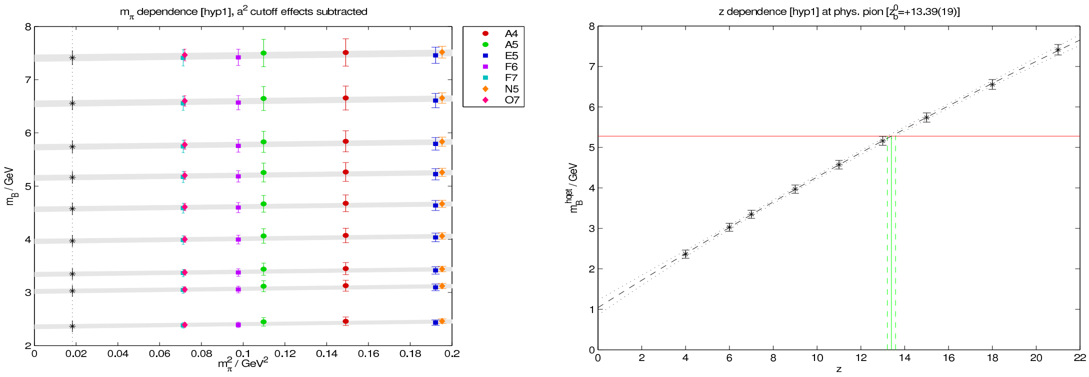

Figure 1: $m_B(z)$ vs. m_π^2 and its extrapolation to the chiral and continuum limits (left panel); $m_B(z)$ vs. z and its interpolation to $z_b \equiv LM_b$ (right panel).

where the exponent (0) or (1) refers to the order in $1/m$ for which the computation is made. The NLO formula consists in subtracting the term $\frac{3\hat{g}^2 m_\pi^3}{16\pi f_\pi^2}$, with $\hat{g} = 0.51(2)$ [6]. We have applied a quadratic fit for m_B in function of z: $m_B = m_0 + m_1 z + m_2 z^2$. We have shown in Figure 1 the pion mass dependence (left panel) and the heavy quark mass dependence (right panel) of that quantity. We are then in a position to extract the b quark mass, after a perturbative conversion from RGI to $\overline{\mathrm{MS}}$ scheme. We have indeed applied a quadratic interpolation to $\overline{m_b}(\overline{m_b})$ by matching $m_B(z)$ to m_B^{exp} and have obtained as a preliminary result:

$$[\overline{m_b}(\overline{m_b})]^{\mathrm{stat}} = 4.22(24)\,\mathrm{GeV}, \quad [\overline{m_b}(\overline{m_b})]^{\mathrm{stat}+1/\mathrm{m}} = 4.20(25)\,\mathrm{GeV}, \tag{2.2}$$

where the error combines the statistical error, the (negligible) uncertainty on the chiral extrapolation, an error on the RGI quark mass renormalisation constant Z_m^{RGI} [7] and the uncertainty on L whose the value in physical units is obtained from f_K [8]. This is consistent with the PDG value $\overline{m_b}(\overline{m_b}) = 4.19^{+18}_{-6}$ GeV.

3. *B* decay constant

The B decay constant depends on the HQET couplings of the Lagrangian and of the temporal axial current, but also on improvement coefficients $b_A^{\mathrm{stat}}(\alpha_i)$ and $c_A^{\mathrm{stat}}(\alpha_i)$ that one knows in perturbation theory [9]. We have extrapolated f_B to the continuum and chiral limits using the NLO formula

$$[f_B\sqrt{m_B/2}]^{0(1)}(a,z,\alpha_i,m_\pi) = f_B^{0(1)}(z)\left(1 - \frac{3}{4}\frac{1+3\hat{g}^2}{16\pi^2 f_\pi^2} m_\pi^2 \ln m_\pi^2 + f'^{0(1)} m_\pi^2\right) + h^{0(1)}(\alpha_i)a^2. \tag{3.1}$$

At LO, the expression simplifies to

$$[f_B\sqrt{m_B/2}]^{0(1)}(a,z,\alpha_i,m_\pi) = f_B^{\mathrm{LO},0(1)}(z) + f^{\mathrm{LO},0(1)} m_\pi^2 + h^{\mathrm{LO},0(1)}(\alpha_i)a^2. \tag{3.2}$$

Using the scaling law of HQET [10] the heavy mass dependence is depicted by $[f_B\sqrt{m_B/2}](z) = f_0 + \frac{f_1}{z} + \frac{f_2}{z^2}$. We have shown on the left panel of Figure 2 the pion mass dependence of f_B. We have obtained $f_B^{\mathrm{stat}} = 190(4)(3)$ MeV and $f_B^{\mathrm{stat}+1/\mathrm{m}} = 178(8)(4)$ after an interpolation to the static

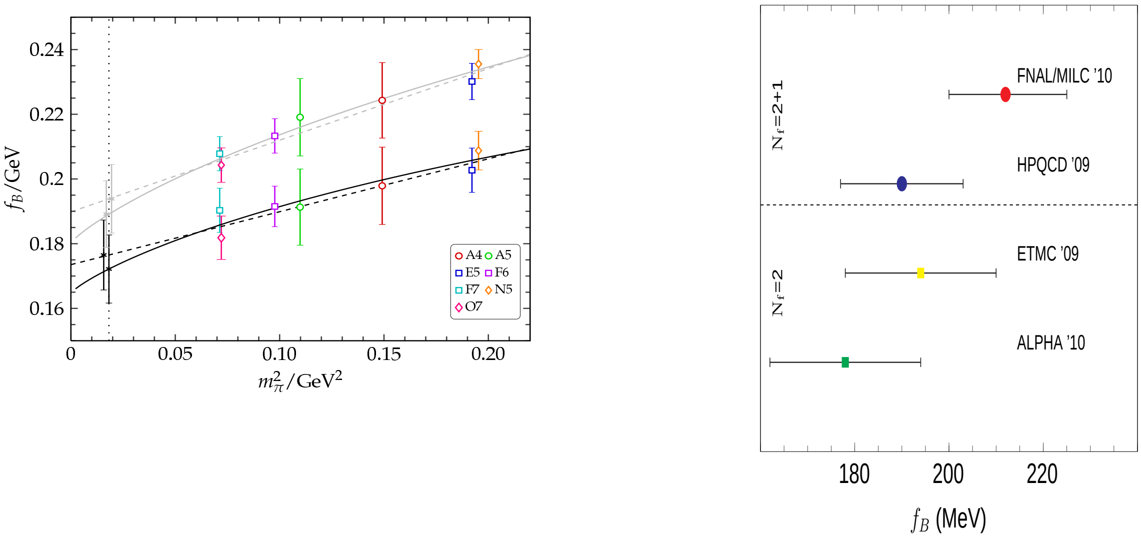

Figure 2: Extrapolation to the chiral and continuum limits of $[f_B\sqrt{m_B/2}](z)$ (left panel) and a compilation of several unquenched lattice estimates (right panel).

z_b^0 and $O(1/m)$ z_1^b *b* quark mass, respectively. The first error is statistical while the second is the discrepancy between the NLO (our preferred preliminary central value) and the LO estimates. We have shown on the right panel of Figure 2 a compilation of several recent lattice determinations. They converge to a rather low value, which is not in contradiction with the interpretation of a New Physics contribution for the branching ratio of $B \to \tau\nu$.

4. Conclusion

We have presented the status of the ALPHA project to extract from $\mathrm{N_f} = 2$ lattice simulations the *b* quark mass and *B* decay constant. Results are encouraging as far as the control on systematics is concerned. We plan to extend our approach to the computation of the $f_+^{B\to\pi l\nu}$ form factor at q^2_{max}.

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