

## The charm-quark contribution to $\varepsilon_K$ and $\Delta M_K$

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Neutral Kaon mixing plays an important role in the phenomenology of the standard model and its extensions because of its sensitivity to high-energy scales. In particular  $\varepsilon_K$ , parameterising indirect  $CP$  violation in the neutral Kaon system, serves as an important constraint on models of new physics and is well suited for the indirect search for heavy new particles. In order to exploit this potential, a precise prediction of the standard-model background is crucial. I give a short summary of the standard-model prediction of  $\varepsilon_K$ , and present our recent NNLO QCD calculation of the charm-quark contribution  $\eta_{cc}$  to the  $|\Delta S| = 2$  effective Hamiltonian. We find a large 36% shift with respect to the NLO value that leads to  $\eta_{cc} = 1.87(76)$ , shifting the standard-model prediction to  $|\varepsilon_K| = 1.81(28) \times 10^{-3}$ .

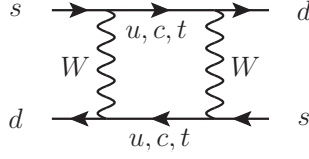
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**Figure 1:** LO box diagrams inducing the  $|\Delta S| = 2$  transition in the SM.

## 1. Introduction

Neutral Kaon mixing proceeds via the quark-level flavour-changing neutral current (FCNC)  $s - d$  transition. In the standard model of particle physics (SM) it is forbidden at tree level and induced by the weak interaction via the well-known box diagrams (see Fig. 1). The parameter  $\varepsilon_K$  describes indirect  $CP$  violation in the neutral Kaon system. The top-quark contribution to  $\varepsilon_K$  is parametrically suppressed by small CKM-matrix elements, whereas the hadronic matrix elements of non-SM operators are enhanced by QCD effects. This leads to an exceptional sensitivity to high-energy scales.

In the last decade there has been huge progress in reducing the theory uncertainty of the hadronic contribution to  $\varepsilon_K$ , in particular in the lattice computation of the hadronic matrix elements. We have in turn calculated the NNLO QCD corrections to the effective  $|\Delta S| = 2$  Hamiltonian. In this talk, I present our recent NNLO calculation of the charm-quark contribution  $\eta_{cc}$ : the correction is large and shifts the SM prediction to  $|\varepsilon_K| = 1.81(28) \times 10^{-3}$  [1]. This prediction is in slight disagreement with the precisely measured value  $|\varepsilon_K| = 2.228(11) \times 10^{-3}$  [2]. However, the remaining sizeable uncertainty of the SM prediction currently precludes us from inferring a clear sign of new physics in neutral Kaon mixing.

## 2. SM prediction of $\varepsilon_K$

The parameter  $\varepsilon_K$  is defined as the ratio of decay amplitudes<sup>1</sup>  $\varepsilon_K = \langle (\pi\pi)_{I=0} | K_L \rangle / \langle (\pi\pi)_{I=0} | K_S \rangle$  and vanishes in case of exact  $CP$  symmetry. It can be expressed by the following phenomenological formula [3]

$$\varepsilon_K = e^{i\phi_\varepsilon} \sin \phi_\varepsilon \left( \frac{\text{Im} M_{12}}{\Delta M_K} + \xi \right), \quad (2.1)$$

where  $\phi_\varepsilon = \arctan(2\Delta M_K / \Delta \Gamma_K) \approx 45^\circ$ .  $\xi = \text{Im} A_0 / \text{Re} A_0$ , where  $A_I = \langle (\pi\pi)_I | K^0 \rangle$ , is a non-perturbative correction of  $\mathcal{O}(5\%)$ . We have neglected terms proportional to the second power of the small quantities  $\phi = \arg(-M_{12}/\Gamma_{12})$  and  $|A_2|/|A_0|$ , and to the first power in  $\Gamma_L/\Gamma_S$ . Taking  $\Delta M_K$  and  $\phi_\varepsilon$  from experiment, we obtain a theory prediction of  $\varepsilon_K$  by computing  $\text{Im} M_{12}$  and  $\xi$ .

The effective Hamiltonian below the charm-quark scale, inducing the  $|\Delta S| = 2$  transitions in the SM, is given by

$$\mathcal{H}^{\Delta S=2} = \frac{G_F^2}{4\pi^2} M_W^2 [\lambda_c^2 \eta_{cc} S(x_c) + \lambda_t^2 \eta_{tt} S(x_t) + 2\lambda_c \lambda_t \eta_{ct} S(x_c, x_t)] b(\mu) \tilde{Q}_{S2} + \text{H.c.} + \dots, \quad (2.2)$$

<sup>1</sup>A very lucid discussion of the details can be found in Ref. [3].

where  $G_F$  is the Fermi constant,  $M_W$  the  $W$ -boson mass, and  $x_i = m_i^2/M_W^2$  ( $m_i$  being the quark masses). We have defined  $\lambda_i = V_{is}^* V_{id}$  and eliminated  $\lambda_u$  using the unitarity of the CKM matrix. The factor  $b(\mu)$  contains the remaining scale dependence.  $M_{12}$  is then given in terms of the effective Hamiltonian by  $M_{12} = \langle \bar{K}^0 | \mathcal{H}^{|\Delta S|=2} | K^0 \rangle / (2M_K)$ .

In Eq. (2.2),  $\tilde{Q}_{S2} = (\bar{s}_L \gamma_\mu d_L)^2$  is the leading local four-quark operator that induces the  $|\Delta S| = 2$  transition, defined in terms of the left-handed  $s$ - and  $d$ -quark fields. Higher-dimensional operators are estimated to contribute less than 1% to  $\varepsilon_K$  [4]. The hadronic matrix elements of the operator  $\tilde{Q}_{S2}$  constitute the major part of the long-distance contributions to  $\varepsilon_K$ . They are parameterised by the bag factor

$$\hat{B}_K = \frac{3}{2} b(\mu) \frac{\langle \bar{K}^0 | \tilde{Q}_{S2} | K^0 \rangle}{f_K^2 M_K^2}, \quad (2.3)$$

where  $f_K$  is the Kaon decay constant, and  $b(\mu)$  is factored out of Eq. (2.2) in such a way that  $\hat{B}_K$  is a renormalisation-group invariant quantity. It has been calculated precisely using lattice QCD, with an total uncertainty of 4% or less [5]. The obtained values are consistent with the upper bound derived in the framework of large- $N$  QCD [6].

There are additional long-distance contributions which are not contained in  $\hat{B}_K$ , proportional to the dispersive and absorptive parts of the amplitude  $\int d^4x \langle \bar{K}^0 | \mathcal{H}^{\Delta S=1}(x) \mathcal{H}^{\Delta S=1}(0) | K^0 \rangle$ , respectively. The estimates of the parameter  $\xi$  [3, 7] and of the dispersive part of the amplitude contributing to  $\text{Im} M_{12}$  [8] have been combined with the experimental value of  $\phi_\varepsilon$  into the correction factor  $\kappa_\varepsilon = 0.94(2)$  [8], which multiplies the expression (2.1), in which we then set  $\xi = 0$  and  $\phi_\varepsilon = 45^\circ$ .

The short-distance contributions are contained in the loop functions  $S$  in Eq. (2.2), the  $\eta$  factors comprising the higher-order QCD corrections. The dominant term proportional to  $\lambda_t^2$  contributes approximately +75% to  $\varepsilon_K$ . The QCD corrections have been computed by a fixed-order matching calculation at the top-quark scale at NLO, yielding  $\eta_{tt} = 0.5765(65)$  [9]. The term proportional to  $\lambda_c \lambda_t$  contributes roughly +43% to  $\varepsilon_K$ . Our NNLO QCD calculation [10] leads to  $\eta_{ct} = 0.496(47)$ . The smallest contribution of about -18% is proportional to  $\lambda_c^2$ . The GIM mechanism causes the absence of a large logarithm  $\log(m_c/M_W)$  at LO and the mixing of  $|\Delta S| = 1$  into  $|\Delta S| = 2$  operators above the charm-quark scale. We performed a three-loop matching calculation at the charm-quark scale [1] and find  $\eta_{cc} = 1.87(76)$ . The large +36% shift with respect to the NLO result [11] and the large residual scale dependence at NNLO give rise to our assignment of a sizeable theory uncertainty. A conversion of our result to a suitable RI-SMOM renormalisation scheme [12] might improve the convergence of the perturbation series.

Using the numerical input values as in [1] and the NNLO values  $\eta_{ct} = 0.496(47)$  and  $\eta_{cc} = 1.87(76)$ , we obtain the SM prediction

$$|\varepsilon_K| = (1.81 \pm 0.14_{\eta_{cc}} \pm 0.02_{\eta_{tt}} \pm 0.07_{\eta_{ct}} \pm 0.08_{\text{LD}} \pm 0.23_{\text{parametric}}) \times 10^{-3}. \quad (2.4)$$

The error indicated by LD originates from the long-distance contributions, namely  $\hat{B}_K$  and  $\kappa_\varepsilon$ . A large part of the parametric error stems from  $|V_{cb}|$ , which enters the dominant top-quark contribution to the fourth power.

Finally, we remark in passing that using the NNLO value of  $\eta_{cc}$  and again the input as in Ref. [1], the short-distance contribution to  $\Delta M_K$  accounts for 86(34)% of the measured value [2].

### 3. Conclusion and outlook

I have presented the NNLO SM prediction of the important observable  $\varepsilon_K$ , with the result  $|\varepsilon_K| = 1.81(28) \times 10^{-3}$ . The uncertainty is dominated by the error on  $|V_{cb}|$  and the theory uncertainty of the perturbative contribution. The latter can possibly be decreased by transforming the result to a suitable RI-SMOM scheme at NNLO, or, in the long run, by computing the effects of a dynamical charm quark using lattice QCD.

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