

## Global fit to CKM data

---

**V. Niess\***

*On Behalf of the CKMfitter Group*

*Laboratoire de Physique Corpusculaire de Clermont-Ferrand,  
Université Blaise Pascal (UMR 6533 CNRS-IN2P3)  
24 Avenue des landais, F-63177 Aubière Cedex, FRANCE*

*E-mail: niess@in2p3.fr*

We present updated results for the CKM matrix elements from a global fit to flavour physics data within the standard model theoretical context. We emphasise recent improvements on the determination of the CKM parameters  $\lambda$  and  $A$  as well as on the angle  $\gamma$  of the unitarity triangle.

*The 2011 Europhysics Conference on High Energy Physics, EPS-HEP 2011,  
July 21-27, 2011  
Grenoble, Rhône-Alpes, France*

---

\*Speaker.

## 1. Introduction

The Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] is a unitary 3 by 3 matrix describing the mixing of quarks of different generations in the Standard Model (SM), through weak charged current transitions. The matrix is parametrised by 4 real parameters, free in the theory, constrained by experimental observations. One of these parameters is a non-vanishing phase which is currently the only established source of violation of the Charge-Parity symmetry (CP). We use a Wolfenstein improved parametrisation [2, 3] of the CKM matrix where the four CKM parameters  $\lambda, A, \bar{\rho}$  and  $\bar{\eta}$  are rephasing invariant. Furthermore, the CKM matrix elements can be expanded in powers of the Cabibbo angle  $\lambda = \sin(\theta_C)$ , while preserving the matrix unitarity up to any order of the series. The parameter set  $(\bar{\rho}, \bar{\eta})$  defines the apex of a Unitarity Triangle (UT) related to the  $B_d$  meson decays, which exhibit sizable CP violation effects. We follow the  $\alpha, \beta, \gamma$  notation for the angles of this triangle.

## 2. Methodology

We perform a global fit of the CKM parameters to a selected set of observables for which both accurate experimental data and theoretical predictions are available. A compilation of the observables used together with a more in depth discussion on numeric values can be found in [4]. In the following we emphasise only recent updates. Let us nevertheless recall that one of the main challenges of such an exercise is that the extraction of the electroweak CKM parameters of interest is complicated by strong interaction effects, and so requires hadronic inputs. These hadronic inputs are currently best estimated from lattice QCD (LQCD) simulations. They severely contribute to the CKM parameter uncertainties and are now dominated by systematic uncertainties.

The fit procedure and the interpretation of the fits results rely on frequentist hypothesis testing tools. We first distinguish two categories of uncertainties: statistical ones, which are assumed to be Gaussian distributed with known standard deviation and correlations, and theoretical systematics, for which no statistical distribution can be safely assumed. Hence, the latter are considered as additional nuisance parameters bounded to a theoretical interval, following the so called *RFit* scheme [3]. We then perform frequentist hypothesis tests to build up statistical significance or  $p$ -value functions from which confidence intervals are derived. The test statistic we use is the logarithm of the maximum likelihood ratio. For most cases, we assume Wilks's asymptotic regime for the test statistic distribution, leading to profile likelihood confidence intervals [5].

## 3. Improved treatment of $|V_{us}|$

The matrix element  $|V_{us}|$  is best determined from  $K_{\ell 3}$  semileptonic decays and previously only this constraint was used in the global CKM fit. Following the work of [6] we recently [4] included constraints from  $K_{\ell 2}$  and  $\pi_{\ell 2}$  leptonic decays as well as the  $\tau$  decays to  $K$  and  $\pi$  mesons. The combination of all these inputs leads to a significant 25% accuracy improvement on the determination of  $|V_{us}|$  which directly repercuts on the CKM parameters  $\lambda = 0.22518_{-0.00077}^{+0.00036}$  and  $A = 0.816_{-0.021}^{+0.011}$ . The impact on  $|V_{ud}|$  is negligible, since it is already strongly constrained by super-allowed  $\beta$  decays.

#### 4. Improved treatment of the angle $\gamma$

The angle  $\gamma$  of the UT is currently the less constrained on the 3 angles. It is best derived from interferences between  $B^- \rightarrow D^{(*)0}K^{(*)-}$  and  $B^- \rightarrow \bar{D}^{(*)0}K^{(*)-}$  decays. The  $D^0$  final state arises from the leading  $b \rightarrow c$  transition whereas the  $\bar{D}^0$  is produced by a CKM and colour suppressed  $b \rightarrow u$  transition. We perform a combination of 3 established methods exploiting different  $D^0$  final states: GLW [7], ADS [8] and GGSZ [9]. The angle  $\gamma$  is fitted simultaneously with hadronic quantities. The determination of  $\gamma$  is "theoretically clean" -because contributions to the amplitudes are dominated by tree level transitions- however it critically depends on a nuisance parameter: the colour suppression ratio,  $r_b$ , of the  $b \rightarrow u$  to the  $b \rightarrow c$  transition amplitudes. The smaller the value of  $r_b$  the larger the uncertainty on  $\gamma$ . Therefore, the computation of the p-value for  $\gamma$  deserves a full frequentist treatment, with toy Monte-Carlo sampling of the test statistic distribution and a well defined treatment of nuisance parameters.

We recently changed from the supremum p-value to another one called the  $p_\beta$  p-value [10]. The latter p-value makes a more powerful use of the data in order to constrain nuisance parameters from an auxiliary test statistic, while ensuring frequentist coverage. Nuisance parameters are constrained to a  $3.3\sigma$  confidence interval based on their likelihood. Furthermore, the latest results from Belle [11] and CDF [12] for  $D \rightarrow K\pi$  final state (ADS) have been included in the present fit, providing additional rejection of small  $r_b$  values. Altogether, we get a significant improvement on the determination of the  $\gamma$  angle as:  $\gamma = (68_{-14}^{+13})^\circ$ . Its indirect global fit prediction is  $\gamma = (67.2_{-4.4}^{+2.9})^\circ$  in excellent agreement with direct measurements. Note that both the improvement in the statistical method and in the accuracy of experimental data over the last year contributed to reduce the error on  $\gamma$ . Indeed, with the current wealth of data the test static distribution of  $\gamma$  is close to the asymptotic pivotal distribution stated by Wilks's theorem, such that a naive profiling leads to very similar results than an exact frequentist treatment. This is not the case if we perform the same fit with the data available for the CKM 2008 conference.

#### 5. Global fit of the Unitarity Triangle

The apex of the UT is constrained from side and angle measurements. The fit is dominated by constraints from  $\sin(2\beta)$ ,  $\alpha$  and  $\Delta m_d/\Delta m_s$ . These 3 inputs are in excellent agreement, clearly establishing the KM mechanism as the dominant source of CP violation in the  $B$ 's system. We find ( $\bar{\rho} = 0.144_{-0.018}^{+0.027}$ ,  $\bar{\eta} = 0.343_{-0.014}^{+0.014}$ ). Predictions for numerous other observables can be found in [4, 13].

Looking at individual fit observables, the only significant deviation we do see amounts to an ongoing  $2.8\sigma$  discrepancy between  $\sin(2\beta)$  measurement derived from charmonium  $B$  decays and  $BR[B \rightarrow \tau\nu]$ , already widely discussed in previous papers [14]. Let us briefly recall that the combination of these two inputs defines a solution for the UT apex that is incompatible with other observables:  $\Delta m_d/\Delta m_s$  and  $\alpha$ . However, when discarding either  $BR[B \rightarrow \tau\nu]$  or  $\sin(2\beta)$  one can perfectly fit the remaining observables. Hence there is an inconsistency between  $BR[B \rightarrow \tau\nu]$  (too high) and  $\sin(2\beta)$  (too low) experimental values or with our theoretical interpretation of these observables. However, for each observable the measurements seem consistent as well as for the related hadronic quantities from LQCD. We investigated two New Physics (NP) scenarios, where

$B^+ \rightarrow \tau^+ \nu$  transitions receive sizable contributions mediated by a Type II charged-Higgs,  $H^+$ , instead of a  $W^+$  boson and a generic scenario of NP in the mixing of neutral mesons [15]. The first scenario is disfavoured considering a combined analysis of various flavour related observables [6]. In the second scenario, an additional  $\sim 10^\circ$  NP phase in the  $B_d$  mixing could accommodate this discrepancy.

## 6. Conclusion

From EPS 2001 to EPS 2011 the accuracy on the UT apex has improved by a factor of  $\sim 7$ . The Kobayashi-Maskawa (KM) mechanism is obviously at work, though there is an ongoing discrepancy between  $BR[B \rightarrow \tau \nu]$  and  $\sin(2\beta)$  derived from charmonium  $B$  decays. This discrepancy should encourage further theoretical and experimental investigations. An additional consistency test of the KM mechanism will soon be provided by the accurate direct measurement of the angle  $\gamma$ :  $\sim 5^\circ$  at LHCb with  $2 \text{ fb}^{-1}$  [16], and  $\sim 2^\circ$  at Belle II with  $50 \text{ ab}^{-1}$  [17].

## References

- [1] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).  
M. Kobayashi, T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
- [2] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).
- [3] J. Charles, et al. (CKMfitter Group), Eur. Phys. J. C **41**, 1-131 (2005).
- [4] J. Charles, et al. (CKMfitter Group), Phys. Rev. D **84**, 033005 (2011).
- [5] F. James, Statistical Methods in Experimental Physics, 2<sup>nd</sup> edition, World Scientific (2006).
- [6] O. Deschamps, et al. (CKMfitter Group), Phys. Rev. D **82**, 073012 (2010).
- [7] M. Gronau, D. London, Phys. Lett. **B253**, 483 (1991); M. Gronau, D. Wyler, Phys. Lett. **B256**, 172 (1991).
- [8] D. Atwood, et al., Phys. Rev. Lett. **B341**, 372 (1995); D. Atwood, I. Dunietz, A. Soni, Phys. Rev. Lett. **78**, 3257 (1997); D. Atwood, I. Dunietz, A. Soni, Phys. Rev. Lett. **63**, 036005 (2001).
- [9] A. Giri, et al., Phys. Rev. D **68**, 0504018 (2003).
- [10] R. L. Berger, D. D. Boos, J. Am. Stat. Assoc. **89**, 1012-1016 (1994).
- [11] Y. Horri, et al. (Belle Collaboration), Phys. Rev. Lett. **106**, 231803 (2011).
- [12] T. Aaltonen, et al. (CDF Collaboration), arXiv:1108.5765 [hep-ex] (2011).
- [13] CKMfitter website at URL: <http://ckmfitter.in2p3.fr>.
- [14] V. Niess (CKMfitter Group), PoS (FPCP 2009) **049** (2009); J. Ocariz (CKMfitter Group), PoS (EPS-HEP 2009) **159** (2009); S. Descotes-Genon (CKMfitter Group), PoS (FPCP 2010) **038** (2010); S. T'Jampens (CKMfitter Group), PoS (ICHEP 2010) **269** (2010).
- [15] A. Lenz, et al., Phys. Rev. D **83**, 036004 (2011).
- [16] O. Deschamps, et al. (LHCb Collaboration), PoS (FPCP 2009) **021** (2009).
- [17] A. Abe, et al. (Belle II Collaboration), arXiv:1011.0352 [hep-ex].