Analysis of the anomalous-dimension matrix of n-jet operators at 4 loops

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Recently, an all-order conjecture for the anomalous-dimension matrix of n-jet operators in SCET was proposed, which allows one to predict the structure of the infrared divergences of dimensionally regularized, massless gauge-theory scattering amplitudes with an arbitrary number of legs and loops. The conjecture is severely constrained by soft-collinear factorization, non-abelian exponentiation, and the behavior of amplitudes in collinear limits. Using these constraints, a diagrammatic analysis has shown that the anomalous dimension involves only two-parton correlators up to three loop order. The only exception is given by a single color structure multiplying a function of conformal cross ratios depending on the momenta of four external partons, which would have to vanish in all two-particle collinear limits. We extend this analysis by completing the diagrammatic analysis at four loop, and we find that additional functions which vanish in all two-particle collinear limits may arise.

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1. Infrared divergences of massless gauge-theory scattering amplitudes

The problem of predicting the structure of infrared singularities of on-shell *n*-particles scattering amplitudes in massless QCD simplifies, if one realizes that they can be put in one-to-one correspondence with UV divergences of operators defined in soft-collinear effective field theory (SCET) [1, 2, 3].

This idea implies that infrared divergences can be studied by means of standard renormalization-group techniques. The IR divergences of *n*-point scattering amplitudes can be absorbed into a multiplicative renormalization factor **Z**, which is related to an anomalous dimension **Γ**. The **Z**-factor and the anomalous dimension **Γ** are matrices in color space, i.e. they mix amplitudes with the same particle content but different color structure.

The predictive power of this approach relies on the fact that the anomalous dimension **Γ** is strictly constrained by the structure of the effective field theory, as well as by other elements, like the non-abelian exponentiation theorem and the two parton collinear limit. In [1, 2, 3] a form of the anomalous dimension **Γ** compatible with all these constraints has been derived, whose central feature is that only pairwise correlations among charges and momenta of different partons are allowed. Using the color-space formalism it explicitly reads

$$\Gamma(\{\underline{p}\},\mu) = \sum_{(i,j)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2} \gamma_{\text{cusp}}(\alpha_s) \ln \frac{\mu^2}{-s_{ij}} + \sum_i \gamma^j(\alpha_s), \tag{1.1}$$

where $s_{ij} \equiv 2\sigma_{ij} p_i \cdot p_j + i0$, and the sign factor $\sigma_{ij} = +1$ if the momenta p_i and p_j are both incoming or outgoing, and $\sigma_{ij} = -1$ otherwise. The sum runs over the *n* external partons, and we refer to [3] for further detail.

One of the major statements of [1, 2, 3] is that the conjecture (1.1) should hold to all orders in perturbation theory. If true, this is an intriguing result, because it implies a semi-classical origin of IR singularities, and it allows to shed new light on the deeper structure of the strong interaction. For instance, an interesting consequence is the prediction that the cusp anomalous dimension of quarks and gluons should be equal to the quadratic Casimir operator C_R in the fundamental or adjoint representation times a universal coefficients. In other words, the cusp anomalous dimension of quarks and gluons should obey Casimir scaling to all orders in perturbation theory. Such a prediction is highly non-trivial, because it is expected not to hold anymore at the non-perturbative level, which is supported by recent investigations based on the AdS/CFT correspondence [7, 8, 9].

The validity of (1.1) was studied explicitly to three-loop order by means of a diagrammatic analysis, [3, 4]. It was found that only one new color structure can arise, at three loop, which has to depend on the momenta of four external partons. In order to be consistent with all the constraints, however, the corresponding coefficient function, which encode the momentum dependence, must be highly nontrivial, as it must vanish in all collinear limits. Explicitly the additional term reads

$$\Delta\Gamma(\{\underline{p}\},\mu) = \sum_{(i,j,k,l)} f^{ade} f^{bce} \boldsymbol{T}_i^a \boldsymbol{T}_j^b \boldsymbol{T}_k^c \boldsymbol{T}_l^d \, F(\beta_{ijkl}, \beta_{iklj} - \beta_{iljk}), \tag{1.2}$$

where $\beta_{iklj} = \ln \frac{(-s_{ij})(-s_{kl})}{(-s_{ik})(-s_{jl})}$ is the logarithm of the conformal cross ratio. An example of such a function [4] is $F(x,y) = x^3(x^2 - y^2)$, but it was recently excluded using additional constraints obtained exploiting the high-energy Regge limit of the anomalous dimension **Γ** [5, 6].
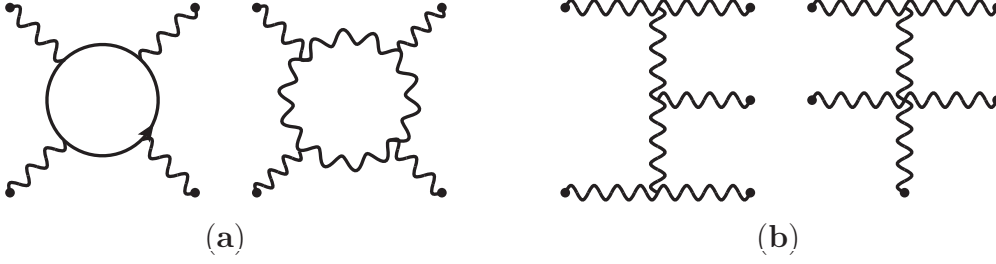
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Figure 1: Gluon webs contributing to the soft anomalous dimension matrix at four loops.

In this talk we report about work in progress [15]. We extend the diagrammatic analysis [3] to four loops in perturbation theory, in order to look for new color structures which could arise and break the simple structure of the anomalous dimension (1.1). In particular, we look for new terms which could lead to a violation of the Casimir scaling of the cusp anomalous dimension of quarks and gluons. We refer to [15] for further details.

2. Diagrammatic analysis at four loops

The correspondence between infrared singularities of gauge scattering amplitudes and UV poles of matrix elements of SCET operators can be seen by considering off-shell n-parton Green's function with large momentum transfer $s_{ij} = (p_i \pm p_j)^2$ and small off-shellness $p_i^2 \ll s_{ij}$. In the effective theory these Green's functions are represented by the matrix element of UV renormalized n-jet operators:

$$\mathcal{G}_n(\{p\}) = \lim_{\varepsilon \to 0} \sum_i \mathcal{C}_{n,i}(\mu) \langle \mathcal{O}_{n,i}^{\mathrm{ren}}(\mu) \rangle = \lim_{\varepsilon \to 0} \sum_{i,j} \mathcal{C}_{n,i}(\mu) Z_{ij}(\mu,\varepsilon) \langle \mathcal{O}_{n,j}^{\mathrm{bare}}(\varepsilon) \rangle. \tag{2.1}$$

In the second identity we have written explicitly the renormalization factor $\mathbf{Z}$ of the bare SCET operators. To obtain on-shell n-parton scattering amplitudes one takes the limit $p_i^2 \to 0$. In this way, on the one hand infrared divergences are introduced in the Green's function, which are regularized in $d = 4 - 2\varepsilon$. On the other, the matrix elements of the bare operators become trivial, because the collinear and soft scale are set to zero, and all loops integrals in the effective theory become scaleless and vanish. The n-parton scattering amplitude free of infrared divergences is therefore equal to the Wilson coefficient $\mathcal{C}_n$,

$$\mathcal{C}_{n,i}(\{p\},\mu) = \lim_{\varepsilon \to 0} \sum_{i,j} \left(Z^{-1}\right)_{ij} \mathcal{G}_{n,j}(\varepsilon\{p\}), \tag{2.2}$$

times trivial color and Dirac structures from the operator matrix element. The logarithm of the renormalization factor $\mathbf{Z}$ is related via $\Gamma = -d\ln\mathbf{Z}/d\ln\mu$ to the anomalous dimension matrix Γ governing the RG evolution equation of the n-jet SCET operators $\mathcal{O}_n^{\mathrm{ren}}$. $\Gamma = \Gamma_{c+s}$ is determined by the collinear and soft modes, and collinear-soft factorization assures that $\Gamma_{c+s} = \Gamma_s + \sum_i \Gamma_c^i$. The collinear anomalous dimension is known, $\Gamma_c^i = -\Gamma_{\mathrm{cusp}}^i L_i + \gamma_c^i$, where $L_i = \ln \mu^2/p_i^2$ is a collinear logarithm, therefore the conjecture (1.1) becomes a prediction for Γ_s, with the constraint

$\partial \Gamma_s(\mu)/\partial L_i = \Gamma_i^{\rm cusp}$. This important constraint originates from the requirement that collinear log-arithms must cancel between Γ_s and $\sum_i \Gamma_c^i$, in order to match the dependence on the hard scale alone in $\Gamma = \Gamma_{c+s}$. As a consequence, the conjecture (1.1) can be proven order by order based on a diagrammatic analysis of the anomalous dimension of the soft function. The latter is given by the vacuum expectation value of n soft Wilson lines. The non-abelian exponentiation theorem guarantees that the corresponding anomalous dimension receives contributions only from single-connected gluon webs attached to the n Wilson lines of the soft operator. An additional constraint comes from the two-parton collinear limit: when two partons become collinear, a n-parton am-plitude splits into a $(n-1)$-amplitude times a splitting function. One finds that the anomalous dimension of the splitting function can only depend on the color and momenta of the two collinear partons.

Using all these results, new structures contributing to the anomalous dimension can arise at four loops, which are related to the webs in Fig. 1. The webs in (a) are proportional to higher Casimir invariants. The possibility of having new structures of this type, linear in the cusp angle, $\beta_{ij} \equiv \ln \mu^2/p_i^2 + \ln \mu^2/p_j^2 - \ln \mu^2/(-s_{ij})$, has been already excluded [3]. Here we focus on struc-tures which have a different momentum dependence, such as a dependence on the logarithm of the conformal cross ratio defined above. We find that the following structures are compatible with constraints from the soft-collinear factorization:

$$\Delta\Gamma = \sum_{(i,j)} \left[\mathscr{D}_{iijj}\, g_4(\alpha_s) + \mathscr{D}_{iiii}\, g_5(\alpha_s) \right] + \sum_{(i,j,k,l)} \mathscr{D}_{ijkl}\, G_1(\beta_{ijkl}, \beta_{iklj} - \beta_{iljk}), \qquad (2.3)$$

where $G_1(x,y) = G_1(-x,y)$ has to be even in its first argument to match the symmetries of $\mathscr{D}_{ijkl} = d_F^{abcd} \left(T_i^a T_j^b T_k^c T_l^d \right)_+$. Out of the terms proportional to $g_4(\alpha_s)$ and $g_5(\alpha_s)$, only the latter is compat-ible with the collinear constraint. However, since it does not have an explicit scale dependence, it would not affect the Casimir scaling of the cusp anomalous dimension. Considering then the term proportional to $G_1(x,y)$, we see that, similarly to the term proportional to $F(x,y)$ in (1.2), its presence is compatible with the collinear limit only if $G_1(x,y)$ vanishes in all two-particles collinear limits. An explicit example is $G_1(x,y) = x^2(x^2 - y^2)^2$, as found in [5] as well. How-ever, we find that this example, too, is ruled out by the constraint from Reggeization. In the high energy limit, in fact, $G_1(x,y) = x^2(x^2 - y^2)^2$ contains terms proportional to $\ln(s/(-t))^4$. If present, these leading logarithms (with the corresponding nontrivial color structure) would ap-pear as an additional contribution to the Regge trajectories of gluons, which are known to NLL [10, 11, 12, 13, 14], and those logarithms do not appear. Therefore, consistency with the Regge limit rules out $G_1(x,y) = x^2(x^2 - y^2)^2$. This of course does not exclude the existence of more complicated $G_1(x,y)$, in which the leading and the next-to-leading logarithms cancel.

The webs in Fig.1(b) involve up to five external partons, and have the following color structure:

$$\mathscr{T}_{ijklm} = -\mathscr{T}_{jilkm} = -\mathscr{T}_{klijm} = -\mathscr{T}_{ikjlm} = -\mathscr{T}_{ljkim} = f^{adx} f^{bcy} f^{exy} (T_i^a T_j^b T_k^c T_l^d T_m^e)_+ . \qquad (2.4)$$

We consider the contributions to the anomalous dimension compatible with the symmetries of this object, and we look for those terms which satisfy soft-collinear factorization as well. We find:

$$\begin{aligned} \Delta\Gamma = {} & \sum_{(i,j,k)} \mathscr{T}_{iijkk} \left[\bar{g}_1(\alpha_s)\beta_{ij} + \bar{g}_2(\alpha_s)\beta_{jk} \right] \\ & + \sum_{(i,j,k,l,m)} \mathscr{T}_{ijklm}\, G_2(\beta_{ijkm}, \beta_{ikmj} - \beta_{imjk}, \beta_{ijml}, \beta_{imlj} - \beta_{iljm}) . \end{aligned} \qquad (2.5)$$

Consistency with the collinear limit eliminates the terms proportional to $\bar{g}_1$ and $\bar{g}_2$. The last term involving a nontrivial function of the logarithms of the conformal cross ratios is possible only if the function G_2 vanishes in all the collinear limits. Its presence would however not affect the Casimir scaling of the cusp anomalous dimension.

In conclusion, we have analyzed possible new contributions to the soft anomalous dimension of n-jet operators in SCET at four loop in perturbation theory. We find that the only new color structures allowed would involve four partons, with momentum dependence encoded in functions of the logarithm of the conformal cross ratio. Those terms, however, would not affect the Casimir scaling of the cusp anomalous dimension of quarks and gluons.

Aknowledgments

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