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Pion Elastic and $\pi \to \gamma\gamma^*$ Transition Form Factors at Large Momentum Transfers

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We present the results of our recent analyses of the form factors $F_\pi(Q^2)$ and $F_{P\gamma}(Q^2)$, $P=\pi,\eta,\eta'$, within the local-duality (LD) version of QCD sum rules [1, 2]. To probe the expected accuracy of this method, we consider, in parallel to QCD, a quantum-mechanical (QM) potential model. In the latter case, the exact form factor may be calculated from the solutions of the Schrödinger equation and confronted with the result from the QM LD sum rule. We find that the LD sum rule is expected to yield reliable predictions for both $F_\pi(Q^2)$ and $F_{\pi\gamma}(Q^2)$ in the region $Q^2 \geq 5$–6 GeV^2. Moreover, in this region the accuracy of this approach improves rather fast with increasing Q^2. For the elastic form factor $F_\pi(Q^2)$, we are therefore forced to conclude that large deviations from the LD limit in the region $Q^2 = 20$–50 GeV^2 reported in some recent theoretical studies seem to us unlikely. The data on the $\eta,\eta' \to \gamma\gamma^*$ transition form factors meet pretty well the predictions of the "LD model." Interestingly, recent BABAR results for the $\pi^0 \to \gamma\gamma^*$ transition form factor hint at an LD violation rising with Q^2; this is at odds with the η,η' cases and all our experience from quantum mechanics.

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1. Introduction

The pion is full of surprises: In spite of the long history of theoretical studies of the pion elastic form factor, no consensus on its behaviour in the region $Q^2 \approx 5$–50 GeV^2 has been reached (Fig. 1); recent BABAR results on the $\pi \to \gamma\gamma^*$ form factor [5] imply a large violation of pQCD factorization in a range of Q^2 up to 40 GeV^2. In [1, 2], we investigated $F_\pi(Q^2)$ and $F_{P\gamma}(Q^2)$ by local-duality (LD) QCD sum rules [6]; their attractive feature is to offer the possibility to study form factors of hadrons without knowing subtle details of their structure and to consider different hadrons on equal footing.

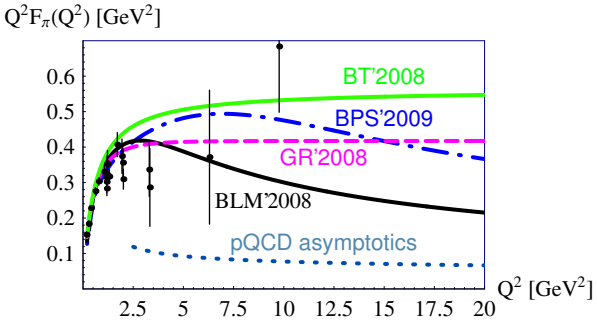

Figure 1: Pion elastic form factor $F_\pi(Q^2)$: recent theoretical predictions [1, 3] vs. experimental data [4].

2. Local-Duality Sum Rules in QCD

LD sum rules are dispersive sum rules in the limit of infinite Borel mass parameter: all power corrections vanish and all details of nonperturbative dynamics are subsumed in a single quantity, the *effective threshold* $s_{\rm eff}(Q^2)$. The basic objects for finding form factors are three-point functions: for the pion elastic form factor the $\langle AVA\rangle$ correlator, for the transition form factor the $\langle AVV\rangle$ correlator, with A the axialvector and V the vector current. Upon implementing standard quark–hadron duality, sum rules relate these pion form factors to the low-energy portions of the perturbative contributions:

$$F_\pi(Q^2) = \frac{1}{f_\pi^2}\int\limits_0^{s_{\rm eff}(Q^2)} {\rm d}s_1 \int\limits_0^{s_{\rm eff}(Q^2)} {\rm d}s_2\, \Delta^{(AVA)}_{\rm pert}(s_1,s_2,Q^2)\,, \qquad F_{\pi\gamma}(Q^2) = \frac{1}{f_\pi}\int\limits_0^{\bar{s}_{\rm eff}(Q^2)} {\rm d}s\, \sigma^{(AVV)}_{\rm pert}(s,Q^2)\,, \tag{2.1}$$

with double and single spectral densities $\Delta^{(AVA)}_{\rm pert}$ and $\sigma^{(AVV)}_{\rm pert}$ of the perturbative three-point graphs; as soon as the effective thresholds $s_{\rm eff}(Q^2)$ and $\bar{s}_{\rm eff}(Q^2)$ have been fixed, extraction of the form factors is straightforward. Formulating reliable criteria for fixing the thresholds is, however, a very difficult task, discussed in great detail in [7]. For $Q^2 \to \infty$, the form factors satisfy the factorization theorems

$$Q^2 F_\pi(Q^2) \to 8\pi\,\alpha_s(Q^2)\, f_\pi^2\,, \qquad Q^2 F_{\pi\gamma}(Q^2) \to \sqrt{2}\, f_\pi\,, \qquad f_\pi = 130\ {\rm MeV}\,. \tag{2.2}$$

Owing to some properties of the spectral densities, this behaviour is correctly reproduced by (2.1) if

$$s_{\rm eff}(Q^2\to\infty) = \bar{s}_{\rm eff}(Q^2\to\infty) = 4\pi^2 f_\pi^2\,. \tag{2.3}$$

For finite Q^2, however, the effective thresholds $s_{\rm eff}$ and $\bar{s}_{\rm eff}$ depend on Q^2 and differ from each other [7]; the "conventional LD model" assumes (2.3) to hold down to "not too small" values of Q^2 [6].

Needless to say, such conventional LD model for effective thresholds is an approximation not taking into account details of the confining dynamics. Its only relevant feature is factorization of hard form factors. Thus, it can be checked in quantum mechanics, using potentials of Coulomb-plus-confining shape for the pion's elastic form factor and of purely confining shape for its transition form factor.

3. Exact vs. Local-Duality Form Factors in Quantum-Mechanical Potential Models

Quantum-mechanical (QM) potential models provide a possibility to test the accuracy of an LD model by comparing the exact form factors, obtained from the solution of the Schrödinger equation, with the outcomes of the QM LD model constructed in precisely the same way as in QCD. Figure 2 shows the exact effective thresholds $k_{\rm eff}$ that reproduce the exact form factors via the LD expression. Irrespective of the confining interaction $V_{\rm conf}(r)$, the precision both of the LD approximation for the effective threshold and of the LD elastic form factor increases with Q^2 in the region $Q^2 \geq 5$–$8\ {\rm GeV}^2$; for the transition form factor, the LD approximation starts to work well at even smaller values of Q^2.

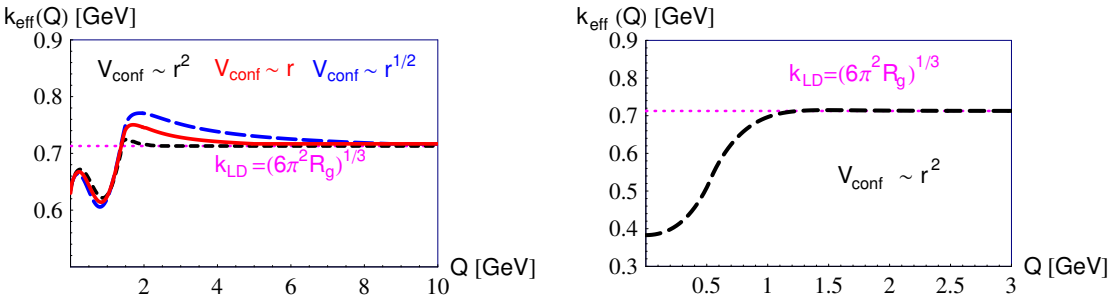

Figure 2: QM exact effective thresholds for elastic (left) and transition (right) form factors for different $V_{\rm conf}$.

4. The Pion Elastic Form Factor $F_\pi(Q^2)$ [1]

Let us introduce the notion of an *equivalent effective threshold*, defined as that quantity $s_{\rm eff}(Q^2)$

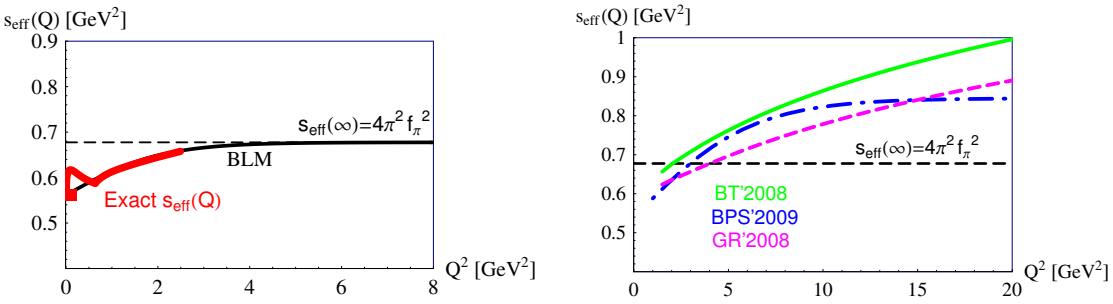

Figure 3: Equivalent effective thresholds $s_{\rm eff}$ for the pion elastic form factor extracted from the experimental data [4] vs. the improved LD model of [1] (left) and from the theoretical predictions depicted in Fig. 1 (right).

which reproduces by Eq. (2.1) some preset behaviour of a form factor. The exact effective threshold extracted from the data (Fig. 3) suggests that the LD limit might be reached already at relatively low Q^2, whereas its theoretical counterparts imply that the accuracy of the LD model still decreases with increasing Q^2 even at Q^2 as large as $Q^2 = 20\,\text{GeV}^2$, in conflict with our QM experience and the hints from the data at low Q^2. Future more accurate JLab data in the range up to $Q^2 = 8\,\text{GeV}^2$ will decide.

5. The $P \to \gamma\gamma^*$ ($P = \pi, \eta, \eta'$) Transition Form Factors $F_{P\gamma}(Q^2)$ [2]

For the η and η' decays, we are obliged to take properly into account both η–η' mixing and the presence of two — strange and nonstrange — LD form factors (for details, consult [8, 1]). Figure 4 shows the corresponding parameter-free predictions. There is an overall agreement between the LD model and the data. Surprisingly, for the pion transition form factor (Fig. 5) one observes a manifest disagreement with the BABAR data [5]. Moreover, in distinct conflict with both the η and η' results and our QM experience, these data suggest that the LD violations increase with Q^2 even in the range $Q^2 \approx 40\,\text{GeV}^2$! It is hard to find a compelling argument explaining why the nonstrange components in η and η', on the one hand, and in π^0, on the other hand, should exhibit such a different behaviour.

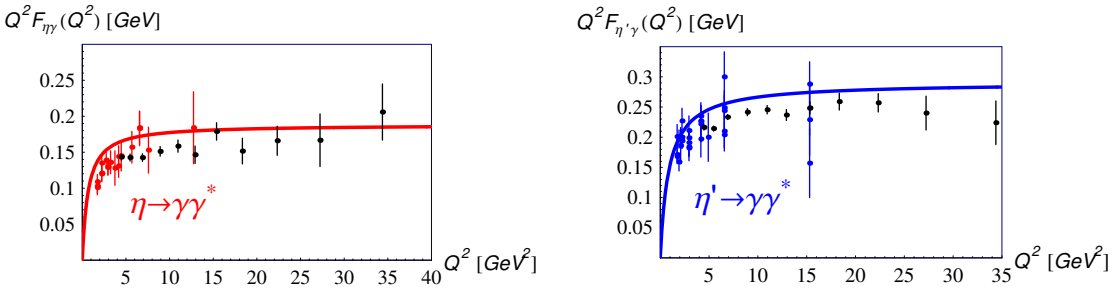

Figure 4: LD predictions for both η and η' transition form factors $F_{(\eta,\eta')\gamma}(Q^2)$ vs. experimental data [9, 10].

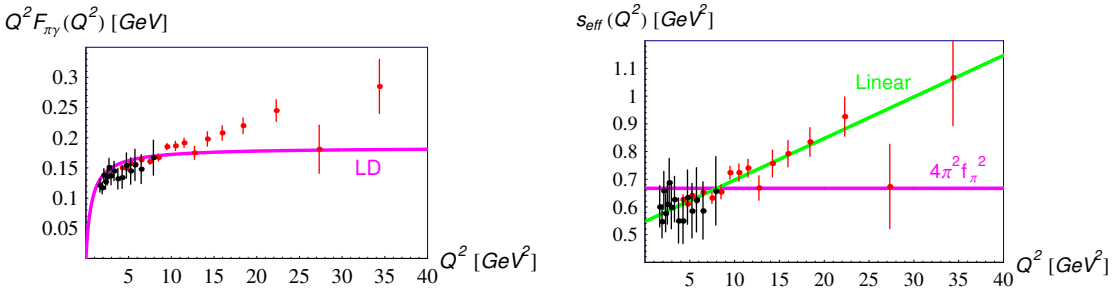

Figure 5: $\pi\gamma$ transition form factor $F_{\pi\gamma}(Q^2)$ vs. data [9, 5], and associated equivalent effective threshold s_{eff}.

6. Summary and Conclusions

We reported the results of our investigation of the pion elastic [1] and the π^0, η, η' transition [2] form factors in the framework of QCD sum rules in LD limit. Our main observations are as follows:

1. For the elastic form factor, the (approximate) LD model is expected to work increasingly well in the region $Q^2 \geq 4$–$8\ \text{GeV}^2$, independently of the details of the confining interaction. For an arbitrary confining interaction, this LD model reproduces the true form-factor behaviour very precisely for $Q^2 \geq 20$–$30\ \text{GeV}^2$. Accurate data for the pion's form factor indicate that the LD value of its effective threshold, $s_{\text{eff}}(\infty) = 4\pi^2 f_\pi^2$, is reached already at relatively low momenta $Q^2 = 5$–$6\ \text{GeV}^2$; rendering large deviations from the LD limit for $Q^2 = 20$–$50\ \text{GeV}^2$ unlikely.

2. For all the $P \to \gamma\gamma^*$ transition form factors, the LD model should work well for Q^2 larger than a few GeV^2. Indeed, the LD model performs well for the $\eta \to \gamma\gamma^*$ and $\eta' \to \gamma\gamma^*$ form factors. For the $\pi \to \gamma\gamma^*$ form factor, however, BABAR data point to a *violation* of local duality, rising with Q^2, even at Q^2 as large as $40\ \text{GeV}^2$, corresponding to an effective threshold of linear rise. So far, this stunning puzzle withstood all attempts to find convincing theoretical explanations.

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References

[1] I. Balakireva, W. Lucha and D. Melikhov, arXiv:1103.3781 [hep-ph];
V. Braguta, W. Lucha, and D. Melikhov, Phys. Lett. B **661** (2008) 354.

[2] W. Lucha and D. Melikhov, arXiv:1110.2080 [hep-ph].

[3] S. J. Brodsky and G. F. de Téramond, Phys. Rev. D **77** (2008) 056007;
H. R. Grigoryan and A. V. Radyushkin, Phys. Rev. D **78** (2008) 115008;
A. P. Bakulev, A. V. Pimikov, and N. G. Stefanis, Phys. Rev. D **79** (2009) 093010.

[4] C. J. Bebek *et al.*, Phys. Rev. D **17** (1978) 1693;
T. Horn *et al.*, Phys. Rev. Lett. **97** (2006) 192001;
V. Tadevosyan *et al.*, Phys. Rev. C **75** (2007) 055205;
G. M. Huber *et al.*, Phys. Rev. C **78** (2008) 045203.

[5] BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. D **80** (2009) 052002.

[6] V. A. Nesterenko and A. V. Radyushkin, Phys. Lett. B **115** (1982) 410.

[7] W. Lucha, D. Melikhov, and S. Simula, Phys. Rev. D **76** (2007) 036002; Phys. Lett. B **657** (2007) 148; Phys. Atom. Nucl. **71** (2008) 1461; Phys. Lett. B **671** (2009) 445; Phys. Rev. D **79** (2009) 096011; J. Phys. G **37** (2010) 035003; Phys. Lett. B **687** (2010) 48; Phys. Atom. Nucl. **73** (2010) 1770; J. Phys. G **38** (2011) 105002; Phys. Lett. B **701** (2011) 82;
D. Melikhov, Phys. Lett. B **671** (2009) 450;
W. Lucha, D. Melikhov, H. Sazdjian, and S. Simula, Phys. Rev. D **80** (2009) 114028.

[8] V. V. Anisovich, D. I. Melikhov, and V. A. Nikonov, Phys. Rev. D **52** (1995) 5295; **55** (1997) 2918.

[9] CELLO Collaboration, H.-J. Behrend *et al.*, Z. Phys. C **49** (1991) 401;
CLEO Collaboration, J. Gronberg *et al.*, Phys. Rev. D **57** (1998) 33.

[10] BABAR Collaboration, P. del Amo Sanchez *et al.*, Phys. Rev. D **84** (2011) 052001.