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Center-symmetric effective theory for two-color QCD with massive quarks at nonzero chemical potential

Tomáš Brauner*
Faculty of Physics, University of Bielefeld, Germany
Department of Theoretical Physics, Nuclear Physics Institute ASCR, Řež, Czech Republic
E-mail: tbrauner@physik.uni-bielefeld.de

We revisit the center-symmetric dimensionally reduced effective theory for two-color Yang–Mills theory at high temperature. This effective theory includes an order parameter for center symmetry breaking/restoration and thus allows to broaden the range of validity of the conventional three-dimensional effective theory (EQCD) to lower temperatures, towards the confining phase transition. We extend the previous results by including in the effective theory the effects of massive quarks with nonzero baryon chemical potential. The parameter space of the theory is constrained by leading-order matching to the Polyakov loop effective potential of two-color QCD. Two-color QCD has attracted considerable interest due to the absence of the sign problem, and hence the possibility to probe its phase diagram at nonzero baryon density using standard Monte Carlo simulations. Our effective theory can provide model-independent predictions for the physics above the deconfinement transition, thus bridging the gap between large-scale numerical simulations and semi-analytical calculations within phenomenological models.

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*Speaker.

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Introduction. The calculation of thermodynamic properties of quantum field theories at high temperature is streamlined by the dimensional reduction formalism, which constructs an effective three-dimensional field theory for the zero Matsubara modes of the fields. This allows for a clear separation of physical effects coming from different energy scales: the hard scale of order the temperature T, and the soft scale of order gT where g is a dimensionless coupling of the theory. The dimensionally reduced effective theory of QCD, the electrostatic QCD (EQCD), however, suffers from a serious drawback: it gives up the center symmetry which is vital for understanding the confinement phase transition.

A modification of EQCD that admits an implementation of the center symmetry was proposed in Ref. [1] for the case of the three-color pure Yang–Mills theory. The new effective theory, dubbed ZQCD, possesses a scalar field representing a coarse-grained Polyakov loop, whose expectation value serves as an order parameter for center symmetry breaking. The same approach was applied to two-color pure Yang–Mills theory in Ref. [2] which is technically considerably easier to deal with. It was shown that the parameters of the effective theory can be determined partially by perturbative matching to EQCD, and the rest by a nonperturbative simulation of the effective theory. ZQCD reproduces successfully the second-order deconfining phase transition and, once all its parameters are known, can be used to study the thermodynamics in the vicinity of the transition.

In this contribution we extend the results of Ref. [2] by including the effects of dynamical quarks with arbitrary masses and chemical potentials. As a byproduct, we correct an algebraic error in the matching conditions of Ref. [2] caused by missing a contribution to an effective operator of the EQCD type.

The effective theory. The degrees of freedom of ZQCD are the magnetic gluon field $\boldsymbol{A}(\boldsymbol{x})$, and the coarse-grained Polyakov loop field $\mathscr{Z}(\boldsymbol{x})$. In the underlying microscopic theory, that is QCD, the Polyakov loop is a unitary matrix. Coarse graining over the length scale $1/T$ turns it into a matrix which is unitary up to a multiplicative real factor. [This is only true for SU(2) matrices, which makes the case of two colors particularly easy to deal with.] Therefore, we parameterize $\mathscr{Z}(\boldsymbol{x})$ as $\mathscr{Z} = (\Sigma + \mathrm{i}\vec{\Pi}\cdot\vec{\sigma})/2$ where σ_a are the Pauli matrices. The most general Lagrangian density with operators up to fourth order in the fields, compatible with the underlying SU(2) gauge invariance, reads

$$\begin{aligned} \mathscr{L} &= g_3^{-2}\left[\tfrac{1}{2}\,\mathrm{tr}\,F_{ij}^2 + \mathrm{tr}\big(D_i\mathscr{Z}^\dagger D_i\mathscr{Z}\big) + V(\mathscr{Z})\right], \\ V(\mathscr{Z}) &\equiv b_1\Sigma^2 + b_2\vec{\Pi}^2 + c_1\Sigma^4 + c_2(\vec{\Pi}^2)^2 + c_3\Sigma^2\vec{\Pi}^2 + d_1\Sigma^3 + d_2\Sigma\vec{\Pi}^2, \end{aligned} \tag{1}$$

where g_3 is the three-dimensional coupling, D_i the (adjoint) covariant derivative, and $F_{ij} \equiv \partial_i A_j - \partial_j A_i - [A_i, A_j]$. The Z_2 symmetry of two-color Yang–Mills theory is respected by all operators but those proportional to $d_{1,2}$. These incorporate on the ZQCD level the effects of dynamical quarks. The effective couplings are next split into 'hard' and 'soft' parts as $b_1 = \frac{1}{2}h_1$, $b_2 = \frac{1}{2}(h_1 + g_3^2 s_1)$, $c_1 = \frac{1}{4}h_2 + g_3^2 s_3$, $c_2 = \frac{1}{4}(h_2 + g_3^2 s_2)$, $c_3 = \frac{1}{2}h_2$, $d_1 = \frac{1}{2}g_3^2 s_4$, $d_2 = \frac{1}{2}g_3^2 s_5$. The point of this splitting is that the hard part of the potential is invariant under the extended $\mathrm{SU}(2)\times\mathrm{SU}(2)$ symmetry, which is spontaneously broken by nonzero vacuum expectation value of $\mathscr{Z}$ to an SU(2) subgroup. As a consequence, three of the four degrees of freedom in $\mathscr{Z}$, to be later identified with the A_0 field of EQCD, remain naturally light, only getting their masses from the soft terms in the potential.

Perturbative matching. Demanding that the parameters of the potential $V(\mathscr{Z})$ take such values that the field $\mathscr{Z}$ develops a nonzero vacuum expectation value (which is necessary for center symmetry to be spontaneously broken and thus for deconfinement to take place), we parameterize the field as $\mathscr{Z} = \frac{1}{2}(v + g_3\phi)\exp(\mathrm{i}g_3\vec{\chi}\cdot\vec{\sigma}/v)$. Comparing this to the expression for the Polyakov loop in the full theory, it is natural to identify χ with the adjoint scalar A_0 of EQCD. Comparing their properties under center symmetry transformations then leads to $v = 2T$. To leading order in the coupling, this implies the first matching condition,

$$h_1 + 4T^2 h_2 = 0. \tag{2}$$

The next step is to determine the values of the soft parameters s_i. This is accomplished by integrating out the heavy amplitude mode ϕ and Taylor expanding in powers of $\vec{\chi}$, and comparing the resulting theory to EQCD. Integrating out ϕ is greatly facilitated by using the exponential parameterization of $\mathscr{Z}$ defined above; to leading order in the coupling expansion, one simply drops all terms in the Lagrangian containing ϕ. This results in the following values of the mass and quartic coupling, corresponding to the operator $(\vec{\chi}^2)^2/8$,

$$\begin{aligned} m_\chi^2 &= g_3^2\left(s_1 - 4s_3v_0^2 - \frac{3}{2}s_4v_0 + s_5v_0\right), \\ \lambda &= g_3^4\left(-\frac{4s_1}{3v_0^2} + 2s_2 + \frac{40s_3}{3} + \frac{7s_4}{2v_0} - \frac{10s_5}{3v_0}\right). \end{aligned} \tag{3}$$

On the other hand, the values of these parameters in EQCD are straightforwardly determined by Taylor expanding the one-loop center-symmetric effective potential of QCD, derived by Weiss [3]. Writing $A_0 = \vec{a}\cdot\vec{\sigma}/2$, this takes the form

$$\begin{aligned} V_{\mathrm{Weiss}}(\vec{a}) =& \frac{4}{3}\pi^2T^4\left\langle\frac{g|\vec{a}|}{2\pi T}\right\rangle^2\left(1 - \left\langle\frac{g|\vec{a}|}{2\pi T}\right\rangle\right)^2 + \\ &+ \frac{4T^2}{\pi^2}\sum_{j=1}^{N_\mathrm{f}} m_j^2 \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}K_2(n\beta m_j)\cosh(n\beta\mu_j)\cos\frac{ng|\vec{a}|}{2T}, \end{aligned} \tag{4}$$

where m_j and μ_j are quark masses and chemical potentials. Obviously, EQCD provides two matching conditions which essentially constrain the parameters $s_{1,2,3}$ with corrections induced by explicit center symmetry breaking. The fact that there are three parameters but only two conditions does not present a problem. Once the heavy mode ϕ is integrated out, the ZQCD fields satisfy the "unitarity" constraint $\Sigma^2 + \vec{\Pi}^2 = 1$. Thus, out of the operators corresponding to $s_{1,2,3}$, only two are linearly independent. In other words, the linear combination of $s_{1,2,3}$ that is left unconstrained by EQCD, has no effect on low-energy physics at the soft scale gT.

Domain wall and bubble solutions. To fix the remaining parameters, one has to deal with observables related to the center symmetry and its explicit breaking by dynamical quarks. Consider first the case of exact center symmetry. The theory then possesses a one-dimensional field configuration of the domain wall type, which interpolates between the two Z_2 minima of the effective potential. The domain wall solution in the Yang–Mills theory was found in Ref. [4]. Since, as explained above, the two linear combinations of $s_{1,2,3}$ that affect the low-energy physics can be fixed using EQCD, finding the domain wall solution in ZQCD gives us no new information, and represents a genuine prediction of the theory. Numerical solution of the classical equation of motion of the gauge field yields a domain wall with a tension exceeding by 8% the value in QCD.

In presence of dynamical quarks, the domain wall is no longer stable, but there is still a static three-dimensional spherically symmetric configuration, corresponding to a bubble of the stable vacuum in a metastable environment. In the limit of weak explicit breaking of the center symmetry (large quark masses), the bubble solution can be found in the thin-wall approximation: its profile is given by the one-dimensional domain wall, and the radius is only sensitive to the energy density difference of the stable and the metastable vacuum. From Eq. (4), one then finds another matching condition,

$$-s_4 v^3 = \frac{8T^4}{\pi^2} \sum_{j=1}^{N_f} (\beta m_j)^2 \sum_{\text{odd } n} \frac{1}{n^2} K_2(n\beta m_j) \cosh(n\beta\mu_j). \tag{5}$$

To the order we work at here, the last soft parameter in the Lagrangian, s_5, does not show up in physical observables pertaining to the structure of the minima of the effective potential.

Summary and conclusions. We have extended the three-dimensional effective theory for two-color QCD proposed before [2] by including the effects of explicit center symmetry breaking due to dynamical quarks. As a byproduct, we corrected an algebraic error in the previous work that appeared in the matching condition for the quartic coupling λ, analogous to our Eq. (3). The soft parameters of the effective theory that affect the low-energy physics are now completely fixed. In principle, there is still one extra, hard parameter whose value needs to be determined. This was done previously [2] by nonperturbative numerical lattice simulation of the effective theory. The value of the critical temperature of the deconfining phase transition was used as the necessary matching condition. Nevertheless, again, it was demonstrated that the precise value of this remaining hard parameter has a very small effect on low-energy physics.

Now that the effective theory is completely fixed, it can be used to make predictions. Although two-color QCD has no sign problem, and thus can be simulated on the lattice without essential difficulties, we still believe ZQCD can provide a useful semi-analytic insight into the thermodynamics around and above the deconfinement transition. An extended discussion of the applications as well as full details of our calculations will be reported in a forthcoming publication.

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