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Spectrum of quarks in QCD_2

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The properties of the gauge invariant two-point quark Green's function are studied in the large-N_c limit of two-dimensional QCD. The analysis is done by means of an exact integrodifferential equation. The Green's function is found to be infrared finite, with singularities in the momentum squared variable represented by an infinite number of threshold type branch points with a power -3/2, starting at positive mass squared values, with cuts lying on the positive real axis. The expression of the Green's function is analytically determined.

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We report in this talk results concerning the gauge invariant two-point quark Green's function [1, 2] considered in the large N_c limit of two-dimensional QCD [3, 4, 5].

Defining the gauge invariant quark Green's function with a path-ordered gluon field phase factor along a straight line segment, one can establish an exact integrodifferential equation for the latter (in general in four-dimensional QCD and for any N_c), where the kernel is made of a series of Wilson loops [6, 7, 8] along polygonal contours with a certain number of functional derivatives acting on the contours [9].

Many simplifications occur in two-dimensional QCD at large N_c. This theory is expected to have the essential features of confinement observed in four dimensions, with the additional simplification that asymptotic freedom is realized here in a trivial way, since the theory is super-renormalizable. For simple contours, Wilson loop averages in two-dimensional Yang-Mills theory are exponential functionals of the areas enclosed by the contours [10, 11, 12]. Furthermore, at large N_c, crossed diagrams and quark loop contributions disappear.

It turns out that in two dimensions and at large N_c the action of the kernel of the above quoted integrodifferential equation can explicitly be evaluated. The equation then reduces to the following form [13]:

$$(i\gamma.\partial - m)S(x) = i\delta^2(x) - \sigma\gamma^\mu(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})x^\nu x^\beta \times \left[\int_0^1 d\lambda\,\lambda^2 S((1-\lambda)x)\gamma^\alpha S(\lambda x) + \int_1^\infty d\xi\, S((1-\xi)x)\gamma^\alpha S(\xi x)\right], \tag{1}$$

where S is the gauge invariant quark Green's function with a phase factor along a straight line segment, x the ralative coordinate of the quark and antiquark fields and σ the string tension.

The equation is solved by decomposing S into Lorentz invariant parts, here in momentum space:

$$S(p) = \gamma.pF_1(p^2) + F_0(p^2), \tag{2}$$

or, in x-space:

$$S(x) = \frac{1}{2\pi}\Big(\frac{i\gamma.x}{r}\widetilde{F}_1(r) + \widetilde{F}_0(r)\Big), \qquad r = \sqrt{-x^2}. \tag{3}$$

One obtains, with the introduction of the Lorentz invariant functions, two coupled equations. Their resolution proceeds through several steps, mainly based on the analyticity properties resulting from the spectral representation of S [9, 13]. The solutions are obtained in explicit form for any value of the quark mass m.

The covariant functions $F_1(p^2)$ and $F_0(p^2)$ are, for complex p^2:

$$F_1(p^2) = -i\frac{\pi}{2\sigma}\sum_{n=1}^{\infty} b_n \frac{1}{(M_n^2 - p^2)^{3/2}}, \tag{4}$$

$$F_0(p^2) = i\frac{\pi}{2\sigma}\sum_{n=1}^{\infty} (-1)^n b_n \frac{M_n}{(M_n^2 - p^2)^{3/2}}. \tag{5}$$

The masses M_n $(n = 1, 2, \dots)$ have positive values greater than the quark mass m and are labelled with increasing values with respect to n; their squares represent the locations of branch point singularities with power $-3/2$, with cuts lying on the positive real axis of the complex plane of p^2.

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The masses M_n and the coefficients b_n satisfy an infinite set of coupled algebraic equations that are solved numerically. Their asymptotic behaviors for large n, such that $\sigma\pi n \gg m^2$, are:

$$M_n^2 \simeq \sigma\pi n, \qquad b_n \simeq \frac{\sigma^2}{M_n}. \tag{6}$$

In x-space, the solutions are:

$$\widetilde{F}_1(r) = \frac{\pi}{2\sigma}\sum_{n=1}^{\infty} b_n e^{-M_n r}, \quad \widetilde{F}_0(r) = \frac{\pi}{2\sigma}\sum_{n=1}^{\infty} (-1)^{n+1} b_n e^{-M_n r}. \tag{7}$$

At high energies, the solutions satisfy asymptotic freedom [14].

In conclusion, the spectral functions of the quark Green's function are infrared finite and lie on the positive real axis of p^2. No singularities in the complex plane or on the negative real axis have been found. This means that the quarks contribute like physical particles with positive energies. (In two dimensions there are no physical gluons.)

The singularities of the Green's function are represented by an infinite number of threshold type singularities, characterized by a power of $-3/2$ and positive masses M_n ($n = 1, 2, \ldots$). The corresponding singularities are stronger than simple poles and this feature might be at the origin of the unobservability of quarks as asymptotic states.

The threshold masses M_n represent dynamically generated masses, since they are not present in the QCD Lagrangian. They survive even when the quark mass is zero. They play the role of gauge invariant effective masses of quarks.

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