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Soft gluon resummation in $t\bar{t}$ production at hadron colliders

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We discuss results for the total and differential cross sections in top-quark pair production at hadron colliders. The focus is on a comparison of the next-to-leading order results with those based on soft gluon resummation to next-to-next-to-leading logarithmic accuracy.

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1. Introduction

Top-quark pair production is a benchmark process at hadron colliders such as the Tevatron and LHC. Its special role in the physics program of these experiments makes it crucial to have precise QCD predictions for the total and differential cross sections. The starting point for such predictions is the next-to-leading order (NLO) calculations of the total and differential cross sections carried out more than two decades ago [1]. Since higher-order corrections to these results as estimated through scale variations are expected to be as large as 10-15%, it would be desirable to extend the calculations beyond NLO. Here there are two paths. One is to calculate the full next-to-next-to-leading order (NNLO) cross section. This is indeed an active area of research and was discussed at this conference by Andreas von Manteuffel. Another is to use techniques from soft gluon resummation to calculate what are argued to be the dominant corrections at NNLO and beyond. Such resummed calculations are the subject of this talk.

2. Soft gluon resummation

Soft gluon resummation is a rich field with a long history and it is far beyond the scope of this talk to give a proper overview. We aim instead to briefly explain the main ideas and how they are applied in the literature.

The basic idea of resummation can be conveyed through the following schematic picture. Quite generically, (differential) partonic cross sections $d\hat{\sigma}$ receive double logarithmic corrections of the form $\alpha_s^n \ln^{m\leq 2n}\lambda$ at each order in perturbation theory, where λ is a variable which vanishes in the limit where real gluon radiation is soft. Resummation is essentially a re-organization of the perturbative series appropriate for the parametric counting $L \equiv \ln\lambda \sim 1/\alpha_s$. In particular, one can show that the partonic cross sections can be written in the form[1]

$$d\hat{\sigma} = \exp[\alpha_s L^2 g_1 + \alpha_s L g_2 + \alpha_s^2 L g_3 + \ldots] \times C(\alpha_s) + \mathcal{O}(\lambda)\,. \tag{2.1}$$

In words, the logarithmic corrections exponentiate up to power corrections in λ. The functions g_i (anomalous dimensions) and C (matching functions) can be calculated order by order in α_s. Every time one adds a higher-order function g_i an infinite series of logarithmic terms is resummed into the exponent, which explains the nomenclature of the technique. Roughly speaking, including the piece in the exponent proportional to g_1 is called leading-logarithmic order (LL), including that proportional to g_2 next-to-leading-logarithmic order (NLL), and so on. The current state of the art in top-quark pair production is NNLL.

Soft gluon resummation can be thought of as a universal technique which takes a specific form for different observables. In top-quark pair production at hadron colliders, all applications in the literature can be grouped into one of the three cases listed in Table 1, which shows specific (differential) cross sections along with a parameter which vanishes in the soft limit (this plays the role of the λ in (2.1)). The list looks slightly arbitrary at a first glance but there is a logic behind it. At Born level, top-quark pair production is a two-to-two process depending on Mandelstam

[1] This is schematic both because the exponentiation takes place in an integral transform space, either Mellin or Laplace, and because the formulas involve matrices in color space rather than simple functions.

Name	Observable	Soft limit
pair-invariant-mass (PIM)	$d\sigma/dM_{t\bar{t}}d\theta$	$(1-z) = 1 - M_{t\bar{t}}^2/\hat{s} \to 0$
single-particle-inclusive (1PI)	$d\sigma/dp_T dy$	$s_4 = \hat{s} + \hat{t}_1 + \hat{u}_1 \to 0$
production threshold	σ	$\beta = \sqrt{1-4m_t^2/\hat{s}} \to 0$

Table 1: The three cases in which soft gluon resummation has been applied. The first column indicates the name often used in the literature, the second the observable to which it applies, and the third the partonic variable associated with large logarithmic corrections in the soft limit.

variables which satisfy $\hat{s} + \hat{t}_1 + \hat{u}_1 = 0$. The most general partonic cross section is thus double differential. Beyond Born level, one must integrate over phase space with respect to extra real emissions to get a double differential cross section. The basic choices are to observe properties of the top-quark pair, as in PIM kinematics, or properties of the top (or anti-top) quark, as in 1PI kinematics. Results in PIM or 1PI kinematics are obtained by integrating over a different portion of the fully differential phase space and are thus independent calculations. A final choice is to integrate over the whole phase space and calculate the partonic cross section in the production threshold limit. As far as soft gluon resummation is concerned, results in the production threshold limit are a special case of the PIM and 1PI results and do not contain independent information. However, in the limit $\beta \to 0$ one must also consider Coulomb terms of the form $\ln^m \beta/\beta^n$, and a full treatment of all singular terms in that limit involves a joint soft and Coulomb resummation [2, 3].

In each of the soft limits in Table 1 resummed calculations for the partonic cross sections are available to NNLL accuracy. Hadronic cross sections are obtained from the partonic ones through convolutions with partonic distribution functions (PDFs). As an example, the pair invariant-mass distribution is given by

$$\frac{d\sigma}{dM_{t\bar{t}}} = \sum_{i,j=q,\bar{q},g} \int_\tau^1 \frac{dz}{z} \Phi_{ij}(\tau/z,\mu_F) \frac{d\hat{\sigma}_{ij}(z, M_{t\bar{t}}, \mu_F, \mu_R, \alpha_s(\mu_R))}{dM_{t\bar{t}}}, \tag{2.2}$$

where Φ_{ij} is the parton luminosity function. In the limit of very large invariant mass, the variable $\tau \equiv M_{t\bar{t}}^2/s \to 1$, which implies that the partonic variable $z \equiv M_{t\bar{t}}^2/\hat{s} \to 1$, so the most singular terms in the partonic threshold limit dominate the cross section at each order in α_s and resumming them is an improvement. However, for the total cross section or for smaller values of the invariant mass, the convolution integral involves values of z close to zero as well as those close to unity, and it is not obvious that the singular terms in the $z \to 1$ dominate over the less singular ones. The typical argument, referred to as "dynamical threshold enhancement," is that the parton luminosity functions fall off so steeply as a function of their first argument that the convolution integral is saturated by values of z close to unity, in other words by the partonic threshold region, no matter what the lower limit of integration. Similar arguments can be applied to the 1PI and production thresholds, although it cannot be overemphasized that the power corrections away from partonic threshold are different in each case and must be studied carefully. Reliably estimating the size of these corrections is the most important issue in phenomenological applications of soft gluon resummation.

3. Total and differential cross sections

In this section we cover some phenomenological applications. We first compare resummed and fixed-order calculations of the total production cross section. Results in the pole scheme for the top-quark mass are summarized in Table 2. In addition to the NLO results, we show different "approximate NNLO" implementations of the resummed results: production threshold results as obtained by the HATHOR program [4] with the default settings; 1PI results as obtained in [5]; $1PI_{SCET}$ and PIM_{SCET} results as obtained in [6], combined into a final result for the cross section using the procedure and computer program presented in [7]. By default, we set $\mu = \mu_R = \mu_F = m_t$, with $m_t = 173$ GeV. We display NLO results using MSTW2008 NLO PDFs, while for approximate NNLO results we use MSTW2008 NNLO PDFs. Uncertainties in the HATHOR and 1PI results from [5] are estimated by varying μ up and down by a factor of two, while uncertainties in [7] are estimated by independent variations of μ_R and μ_F by factors of two, along with a scan over the values of the cross section in PIM and 1PI kinematics.

	Tevatron	LHC (7 TeV)
NLO	$6.74^{+0.36+0.37}_{-0.76-0.24}$	160^{+20+8}_{-21-9}
Aliev et. al. [4]	$7.13^{+0.31+0.36}_{-0.39-0.26}$	164^{+3+9}_{-9-9}
Kidonakis [5]	$7.08^{+0.00+0.36}_{-0.24-0.24}$	163^{+7+9}_{-5-9}
Ahrens et. al. [7]	$6.65^{+0.08+0.33}_{-0.41-0.24}$	156^{+8+8}_{-9-9}

Table 2: Results for the total cross section in pb at NLO and within the various NNLO approximations. The first uncertainty is related to perturbative uncertainties, and the second is the PDF error using the MSTW2008 PDF sets [8] at 90% CL.

An examination of the numbers in the table reveals the following features. First, the perturbative uncertainties in the NLO result are on the order of 20% at both the Tevatron and the LHC. This is a bit larger than the PDF uncertainty in both cases, although especially at the LHC one may obtain rather different results with other PDF sets, we refer the reader to [9] for a recent discussion of this issue. Second, the perturbative uncertainties in the approximate NNLO results as obtained through the individual calculations are invariably smaller than at NLO–depending on the implementation, the uncertainties are reduced by a factor of roughly two to three, and are thus under the PDF uncertainties. At the LHC, the different NNLO approximations are in relatively good agreement, though the cross section of [7] is somewhat smaller than in [4, 5]. At the Tevatron, on the other hand, the results from [4, 5] are significantly larger than those from [7]. In fact, the range of values spanned by the three different approximate NNLO results at the Tevatron is about as large as that spanned by the NLO calculation. Given the discrepancy, one is faced with the choice of estimating the theory uncertainties through the NLO calculation, with the spread of approximate NNLO values from the three different calculations, or by a particular NNLO approximation alone. The authors of [4, 5, 7] all give arguments in favor of their particular implementation of soft-gluon resummation, but it is beyond the scope of the talk to properly summarize them. We refer the reader to [10] for more details.

Next, we very briefly mention predictions for differential cross sections. Particularly interesting are the differential cross section and forward-backward asymmetry as a function of the invariant

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mass $M_{t\bar{t}}$ of the top-quark pair. We compare in Figure 1 the NLO and NLO+NNLL results from [11] with experimental data from the CDF collaboration [12]. There is good agreement between theory and experiment for the differential distribution, but the CDF measurement of the forward-backward asymmetry $A_{\rm FB}^{t\bar{t}}$ in the high invariant-mass bin is much higher than the theory result. A smaller discrepancy is found in the D0 results [13] presented at this conference.

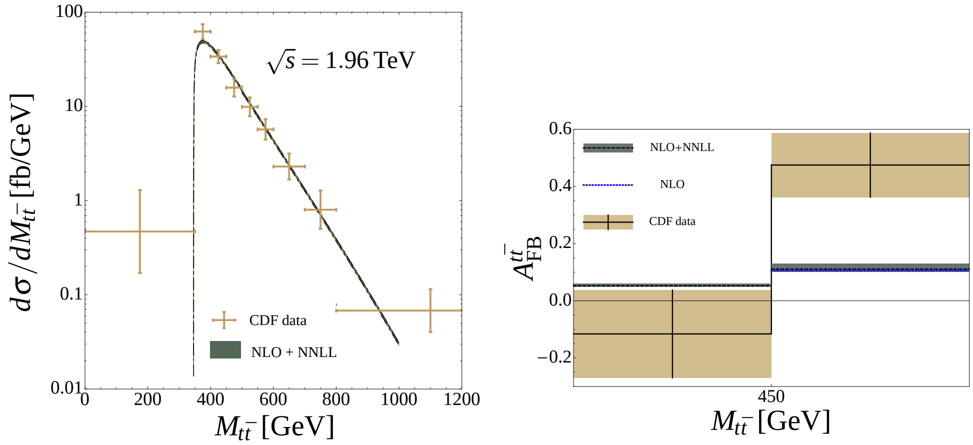

Figure 1: The differential cross section and forward-backward asymmetry as a function of $M_{t\bar{t}}$.

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