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Right Unitarity Triangles and Tri-Bimaximal Mixing from Discrete Symmetries and Unification

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We discuss a recently proposed new class of flavour models which predicts both close to tri-bimaximal lepton mixing (TBM) and a right-angled Cabibbo-Kobayashi-Maskawa (CKM) unitarity triangle, $\alpha \approx 90^\circ$. The ingredients of the models include a supersymmetric (SUSY) unified gauge group such as $SU(5)$, a discrete family symmetry such as A_4 or S_4, a shaping symmetry including products of Z_2 and Z_4 groups as well as spontaneous CP violation. The vacuum alignment in such models allows a simple explanation of $\alpha \approx 90^\circ$ by a combination of purely real or purely imaginary vacuum expectation values (vevs) of the flavon fields responsible for family symmetry breaking.

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1. Motivation

The origin of the observed pattern of fermion masses, mixing angles and CP violating phases is a long standing puzzle in particle physics. The fact that leptonic mixing angles turned out to be close to TBM [1] has led to increasing interest in non-Abelian discrete family symmetries for flavour model building. Nevertheless, in many realistic models another shaping symmetry has to be invoked to forbid unwanted operators in the (super-)potential. These shaping symmetries can shed some light on the aspect of CP violation of the flavour puzzle.

Experimental results point towards a right-angled CKM unitarity triangle with $\alpha = (89.0^{+4.4}_{-4.2})^\circ$ [2]. On the other hand for hierarchical quark mass matrices with a texture zero in the 1-3 element it is straightforward to derive the phase sum rule [3]

$$\alpha = \delta_{12}^d - \delta_{12}^u \,, \tag{1.1}$$

which is correct to leading order in an expansion of the small quark mixing angles and where $\delta_{12}^{d/u}$ are the arguments of the complex 1-2 quark rotation angles. Therefore mass matrices with purely real and purely imaginary elements can predict correctly $\alpha \approx 90^\circ$, see also [4]. In [5] it was shown that TBM and the right-angled CKM unitarity triangle can emerge from the spontaneous breaking of discrete family and discrete shaping symmetries.

2. The Method: Discrete Vacuum Alignment

The considered class of models, we discuss here, is based on the previously discussed method of discrete vacuum alignment [5], which has as its ingredients a discrete family (like A_4 or S_4) and shaping symmetry (like a product of Z_n's), spontaneous CP violation and a SUSY unified gauge group. The unified gauge group is not strictly necessary, but it is very powerful, because it relates the mixing and the CP violation in the quark and the lepton secton to each other.

The method can be described in a simple algorithm. First, use the family symmetry to align the flavon vevs, so that only one complex parameter x is left undetermined, e.g. $\langle\phi\rangle \propto (0,0,x)^T$ or $\langle\phi\rangle \propto (x,x,x)^T$. Then add for each flavon ϕ the following type of terms to the superpotential

$$P\left(\frac{\phi^n}{\Lambda^{n-2}} \mp M^2\right) \,, \tag{2.1}$$

which are allowed by the discrete Z_n shaping symmetries, and where M and Λ are real mass parameters. By solving the F-term condition, $F_P = 0$, the phase of the flavon vev is fixed to be

$$\arg(\langle\phi\rangle) = \arg(x) = \begin{cases} \frac{2\pi}{n} q \,, & q = 1,\dots,n \quad \text{for "$-$" in Eq. (2.1)} \,, \\ \frac{2\pi}{n} q + \frac{\pi}{n} \,, & q = 1,\dots,n \quad \text{for "$+$" in Eq. (2.1)} \,. \end{cases} \tag{2.2}$$

If the shaping symmetries are only Z_2 or Z_4 symmetries the phases can easily be arranged to fulfill the phase sum rule in Eq. (1.1).

As an example we sketch now the A_4 model from [5], where an S_4 model is given as well. The A_4 model has the symmetry $SU(5) \times A_4 \times Z_4^4 \times Z_2^2 \times U(1)_R$ and five flavons with the alignments

$$\langle\phi_1\rangle \propto \begin{pmatrix}1\\0\\0\end{pmatrix}, \; \langle\phi_2\rangle \propto \begin{pmatrix}0\\-\mathrm{i}\\0\end{pmatrix}, \; \langle\phi_3\rangle \propto \begin{pmatrix}0\\0\\1\end{pmatrix}, \; \langle\phi_{23}\rangle \propto \begin{pmatrix}0\\1\\-1\end{pmatrix}, \; \langle\phi_{123}\rangle \propto \begin{pmatrix}1\\1\\1\end{pmatrix} . \tag{2.3}$$

Note that only $\langle\phi_2\rangle$ has a purely imaginary vev, while all other vevs are real. These flavon vevs form the entries of the Yukawa matrices, for which we obtain in the quark sector

$$Y_d = \begin{pmatrix} 0 & \mathrm{i}\,\varepsilon_2 & 0 \\ \varepsilon_{123} & \varepsilon_{23} + \varepsilon_{123} & -\varepsilon_{23} + \varepsilon_{123} \\ 0 & 0 & \varepsilon_3 \end{pmatrix} \quad \text{and} \quad Y_u = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{22} & a_{23} \\ 0 & a_{23} & a_{33} \end{pmatrix} , \tag{2.4}$$

where the ε_i and a_{ij} are real coefficients. First note that $\delta_{12}^d = \arg((Y_d)_{12}/(Y_d)_{22}) = 90°$, due to the purely imaginary 1-2 element of Y_d, and $\delta_{12}^u = 0°$, because Y_u is real. The 1-3 elements in Y_d and Y_u vanish and the sum rule from Eq. (1.1) can be applied successfully.

In the neutrino sector we find exact TBM, which is disturbed by corrections coming from the charged lepton sector inducing, e.g. a non-vanishing $\theta_{13}^{\mathrm{PMNS}} \approx 3°$. It is also interesting to note, that we predict all CP phases in the lepton sector, which turn out to be close to 0° or 180° in the A_4 model.

3. Summary

Discrete symmetries are not only powerful in describing leptonic mixing angles, but they can also be used to predict the right-angled CKM unitarity triangle by means of spontaneous CP violation. In combination with a unified gauge group this gives close relations between the CP violation in the quark and the lepton sector. In fact, in this new class of models all physical phases can be predicted up to a discrete choice. For example in the A_4 and S_4 model from [5] apart from $\alpha \approx 90°$ in the quark sector, the leptonic Dirac and Majorana CP phases are all close to 0°, 90°, 180° or 270°. These predictions, especially for the leptonic Dirac CP phase, can be tested at ongoing and forthcoming neutrino experiments

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