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Crossing symmetry in the $\pi\pi$ D- and F-wave scattering amplitudes and new precise results for the S-wave amplitude*

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Recently presented new-one subtracted dispersion relations with imposed crossing symmetry condition for the $\pi\pi$ S- and P-wave scattering amplitudes and the well known Roy's equations with two subtractions have led to a set of many partial wave amplitudes in very wide energy range [1]. They allow e.g. for a very precise and unambiguous determination of scattering lengths and parameters of the $f_0(600)$ (often called σ) and $f_0(980)$ resonances in the S wave.
Similarly, one subtracted dispersion relations for the D and F waves have also been recently derived and presented [2]. Here, general structure of these equations with imposed crossing symmetry condition and results of their first practical application in the testing of the input amplitudes obtained in [1] is presented. It can be seen that these equations are very demanding i.e. produce D and F wave output amplitudes with very small errors. This significantly increases the accuracy of determined amplitudes and indirectly can further improve the precision of parameters in the other waves, such as S and P.
It is worthy noting that although the presented amplitudes of the D and F waves were not fitted directly to dispersion relations in [1], they fulfill crossing symmetry quite well up to ~ 800 MeV (some work is still needed).
Recently, new and very precise dispersive analysis of the S and P wave amplitudes appeared [3]. Using similar to presented here once subtracted dispersion equations (plus other dispersion relations) and very recent K_{l4} experimental results we have calculated set of $\pi\pi$ partial wave amplitudes fulfilling, inter alia, crossing symmetry condition. Making analytic continuation to the complex plane we have found the $f_0(600)$ pole at $(457^{+14}_{-13} - i279^{+11}_{-7})$ MeV and $f_0(980)$ pole at $(996 \pm 7 - i25^{+10}_{-6})$ MeV.
Those new dispersion relations, for the D and F waves, together with the previous ones (called GKPY - see [1]) for the S and P waves form a complementary set of theoretical constraints that imposed on the experimental amplitude can define them clearly and precisely. The analysis is based only on unitarity, analyticity and crossing symmetry.

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1. Dispersion relations for the $\pi\pi$ D and F wave amplitudes

General structure of equations for amplitudes $f_\ell^I(s)$ with spin $\ell = 0,1,2,3$ and isospin $I = 0,1,2$ (sum $I+\ell$ must be even) reads (for detailed discussion see [1] and [2]):

$$\mathrm{Re}\, f_\ell^I(s) = ST_\ell^I + KT_\ell^I(s) + DT_\ell^I(s) \qquad (s = m_{\pi\pi}^2), \tag{1.1}$$

where for the D and F partial waves: $ST_\ell^I = -\frac{1}{24}(a_0^0 - \frac{5}{2}a_0^2)\delta_{I1}\delta_{l3}$ - "Subtracting term" ,

$KT_\ell^I(s) = \sum\limits_{I'=0}^{2}\sum\limits_{\ell'=0}^{3} \fint_{4m_\pi^2}^{s'_{max}} ds' K_{\ell\ell'}^{II'}(s,s')\mathrm{Im}\, f_{\ell'}^{I'}(s')$ - "Kernel term" and $DT_\ell^I(s)$ - so called "Driving term". Amplitudes $f_\ell^I(s)$ are related with phase shifts $\delta_\ell^I(s)$ and inelasticities $\eta_\ell^I(s)$ by

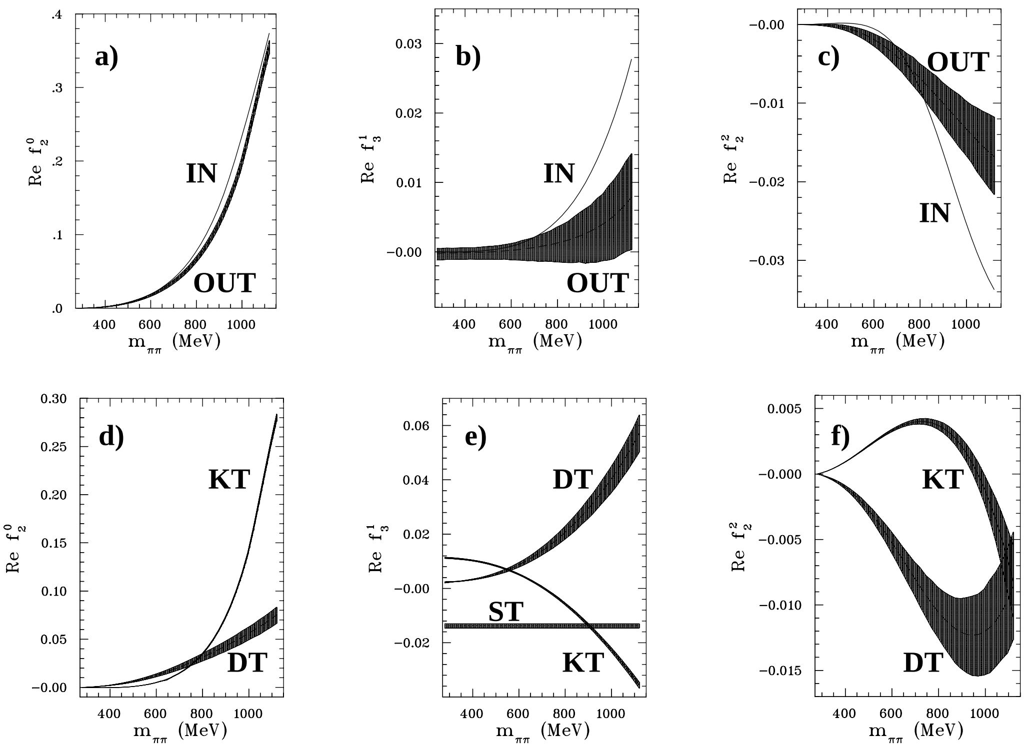
$$f_\ell^I(s) = \frac{\sqrt{s}\eta_\ell^I(s)e^{i\delta_\ell^I(s)} - 1}{2i\sqrt{s-4m_\pi^2}} \tag{1.2}$$

and a_ℓ^I are scattering lengths given by threshold expansion $Re\, f_\ell^I(k) = k^{2\ell}(a_\ell^I + k^2 b_\ell^I + ...)$ at $k = \sqrt{m_{\pi\pi}^2/4 - m_\pi^2} \longrightarrow 0$. Kernels $K_{\ell\ell'}^{II'}(s,s')$ are constructed imposing $s-t$ crossing symmetry condition and have been derived and presented in [2]. Driving terms $DT_\ell^I(s)$ account for contributions of all partial waves ($\ell = 0...3$) at effective two pion mass $m_{\pi\pi}$ higher then $\sqrt{s'_{max}} = 1.42$ GeV. This value is given by maximal two pion mass up to which experimental data are sufficiently precise to determine the amplitudes $f_\ell^I(s)$. Definition of these terms are given in [1] and [2].

On figures a), b) and c) the outputs (left side of the equation (1)) are compared with corresponding real parts of the input amplitudes $Im\, f_{\ell'}^{I'}(s)$ (in the $KT(s)$ and $DT(s)$). All input amplitudes were taken from [1]. Crossing symmetry requires $\Delta = (\text{input} - \text{output}) \longrightarrow 0$. On figures d), e) and f) we compare the subtracting, kernel and driving terms for given partial wave. The nonzero ST is only for the F wave.

As is seen on Figures a)-c), differences between inputs and outputs below $m_{\pi\pi} \approx 800$ MeV are smaller or comparable with output uncertainties. It is worthy to note here that this quite good agreement was achieved despite the fact that none of amplitudes presented here (D and F) was directly fitted to dispersion relations in [1]. Possible fits of these amplitudes to dispersion relations presented in [2] could improve this agreement. Large size difference between the amplitudes f_0^2 and f_3^1, f_2^2 is due to presence of the well known resonance $f_2(1270)$ in the D wave with isospin 0.

Analysis of Figures d)-f) shows that in all three amplitudes kernel and driving terms are of comparable size. A little bigger kernel term for the $f_0^2(s)$ amplitude is again due to presence of the quite precisely known resonance $f_2(1270)$. Its dominant role leads also to very small errors of the kernel term in this amplitude.



Figures a) - c): comparison of output and input amplitudes, figures d) - f): subtracting (ST), kernel (KT) and driving terms (DT). Dashed bands represent uncertainties.

2. Conclusions

Presented here first results for dispersion relations with imposed crossing symmetry condition for for the $\pi\pi$ *D* and *F* wave amplitudes indicate on possible important role of these relations in tests or determinations of the $\pi\pi$ amplitudes. Small output uncertainties generated by the dispersion relations presented in [2] indicate that these relations are very demanding and useful. Together with dispersion relations for the *S* and *P* wave amplitudes, described in [1], those presented here for the *D* and *F* waves create complementary set of very useful and easy to use tools for both experimental and theoretical studies. First and very promising results of such precise dispersive analysis can be found in [3].

References

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