## The quest for light scalar quarkonia from an $N_{f}=3$ linear sigma model with vectors and axial-vectors

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We address the question whether it is possible to interpret the low-lying scalar mesons $f_{0}(600)$ and $a_{0}(980)$ as $\bar{q} q$ states within a $U(3) \times U(3)$ Linear Sigma Model containing vector and axialvector degrees of freedom.

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## 1. Introduction

Debate regarding the structure of scalar mesons has lasted for several decades but still without a clear-cut result. Experimental data [1] suggest the existence of at least seven $I J^{P}=00^{+}$states in the region up to 1.75 GeV . These states are important both in vacuum phenomenology as well as in the restoration of the Chiral Symmetry of the QCD that is spontaneously broken in vacuum. Additionally, the larger-than-expected number of isoscalars in vacuum implies that not all of them can be $\bar{q} q$ states. In this work, we discuss a possibility to identify the elusive scalar quarkonia.

## 2. The Model and its Implications

We utilise a $U(3)_{L} \times U(3)_{R}$ linear sigma model with vectors and axial-vectors both in the non-strange and strange sectors $[2,3]:$

$$
\begin{align*}
\mathscr{L} & =\operatorname{Tr}\left[\left(D^{\mu} \Phi\right)^{\dagger}\left(D^{\mu} \Phi\right)\right]-m_{0}^{2} \operatorname{Tr}\left(\Phi^{\dagger} \Phi\right)-\lambda_{1}\left[\operatorname{Tr}\left(\Phi^{\dagger} \Phi\right)\right]^{2}-\lambda_{2} \operatorname{Tr}\left(\Phi^{\dagger} \Phi\right)^{2} \\
& -\frac{1}{4} \operatorname{Tr}\left[\left(L^{\mu v}\right)^{2}+\left(R^{\mu v}\right)^{2}\right]+\operatorname{Tr}\left[\left(\frac{m_{1}^{2}}{2}+\Delta\right)\left(L^{\mu}\right)^{2}+\left(R^{\mu}\right)^{2}\right]+\operatorname{Tr}\left[H\left(\Phi+\Phi^{\dagger}\right)\right] \\
& +c_{1}\left(\operatorname{det} \Phi-\operatorname{det} \Phi^{\dagger}\right)^{2}+i \frac{g_{2}}{2}\left(\operatorname{Tr}\left\{L_{\mu v}\left[L^{\mu}, L^{v}\right]\right\}+\operatorname{Tr}\left\{R_{\mu v}\left[R^{\mu}, R^{v}\right]\right\}\right) \\
& +\frac{h_{1}}{2} \operatorname{Tr}\left(\Phi^{\dagger} \Phi\right) \operatorname{Tr}\left[\left(L^{\mu}\right)^{2}+\left(R^{\mu}\right)^{2}\right]+h_{2} \operatorname{Tr}\left[\left(\Phi R^{\mu}\right)^{2}+\left(L^{\mu} \Phi\right)^{2}\right]+2 h_{3} \operatorname{Tr}\left(\Phi R_{\mu} \Phi^{\dagger} L^{\mu}\right) \tag{2.1}
\end{align*}
$$

where

$$
\Phi=\frac{1}{\sqrt{2}}\left(\begin{array}{ccc}
\frac{\left(\sigma_{N}+a_{0}^{0}\right)+i\left(\eta_{N}+\pi^{0}\right)}{\sqrt{2}} & a_{0}^{+}+i \pi^{+} & K_{S}^{+}+i K^{+}  \tag{2.2}\\
a_{0}^{-}+i \pi^{-} & \frac{\left(\sigma_{N}-a_{0}^{0}\right)+i\left(\eta_{N}-\pi^{0}\right)}{\sqrt{2}} & K_{S}^{0}+i K^{0} \\
K_{S}^{-}+i K^{-} & \bar{K}_{S}^{0}+i \bar{K}^{0} & \sigma_{S}+i \eta_{S}
\end{array}\right)
$$

is a matrix containing the scalar and pseudoscalar degrees of freedom, $L^{\mu}=V^{\mu}+A^{\mu}$ and $R^{\mu}=$ $V^{\mu}-A^{\mu}$ are, respectively, the left-handed and the right-handed matrices containing vector and axial-vector degrees of freedom with

$$
V^{\mu}=\frac{1}{\sqrt{2}}\left(\begin{array}{ccc}
\frac{\omega_{N}+\rho^{0}}{\sqrt{2}} & \rho^{+} & K^{\star+}  \tag{2.3}\\
\rho^{-} & \frac{\omega_{N}-\rho^{0}}{\sqrt{2}} & K^{\star 0} \\
K^{\star-} & \bar{K}^{\star 0} & \omega_{S}
\end{array}\right)^{\mu}, A^{\mu}=\frac{1}{\sqrt{2}}\left(\begin{array}{ccc}
\frac{f_{1 N}+a_{1}^{0}}{\sqrt{2}} & a_{1}^{+} & K_{1}^{+} \\
a_{1}^{-} & \frac{f_{1 N}-a_{1}^{0}}{\sqrt{2}} & K_{1}^{0} \\
K_{1}^{-} & \bar{K}_{1}^{0} & f_{1 S}
\end{array}\right)^{\mu}
$$

and $\Delta=\operatorname{diag}\left(\delta_{N}, \delta_{N}, \delta_{S}\right)$ describes explicit breaking of the chiral symmetry in the (axial-)vector channel. The explicit symmetry breaking in the (pseudo)scalar sector is described by $\operatorname{Tr}[H(\Phi+$ $\left.\left.\Phi^{\dagger}\right)\right]$ with $H=1 / 2 \operatorname{diag}\left(h_{0 N}, h_{0 N}, \sqrt{2} h_{0 S}\right), h_{0 N}=$ const., $h_{0 S}=$ const. Also, $D^{\mu} \Phi=\partial^{\mu} \Phi-i g_{1}\left(L^{\mu} \Phi-\right.$ $\left.\Phi R^{\mu}\right)$ is the covariant derivative; $L^{\mu \nu}=\partial^{\mu} L^{v}-\partial^{v} L^{\mu}, R^{\mu v}=\partial^{\mu} R^{v}-\partial^{v} R^{\mu}$ are, respectively, the left-handed and right-handed field strength tensors and the term $c_{1}\left(\operatorname{det} \Phi-\operatorname{det} \Phi^{\dagger}\right)^{2} \operatorname{describes}$ the $U(1)_{A}$ anomaly [3]. All the states present in Eqs. (2.2) and (2.3) are $\bar{q} q$ states [2].

The assignment of the fields in the model is discussed in Ref. [3]. In this work it suffices to note that Eq. (2.2) contains two scalars: $\sigma_{N}=\bar{n} n$ and $\sigma_{S}=\bar{s} s$ ( $n$ and $s$ denote nonstrange and strange quarks, respectively). Mixing of $\sigma_{1}$ and $\sigma_{2}$ produces states denoted henceforth as $\sigma_{1}$ (predominantly $\bar{n} n$ ) and $\sigma_{2}$ (predominantly $\bar{s} s$ ). As described in Ref. [3], $\sigma_{1}$ and $\sigma_{2}$ can be assigned either to the pair $\left\{f_{0}(600), f_{0}(1370)\right\}$ [conversely: $f_{0}(600)$ is predominantly a $\bar{n} n$ state and $f_{0}(1370)$ is predominantly a $\bar{s} s$ state] or to the pair $\left\{f_{0}(1370), f_{0}(1710)\right\}$ [conversely: $f_{0}(1370)$ is predominantly a $\bar{n} n$ state and $f_{0}(1710)$ is predominantly a $\bar{s} s$ state].

Assumption I: Scalars below 1 GeV are $\bar{q} q$ states. A fit is needed to determine model parameters in Eq. (2.1). It includes all states except for $\sigma_{1,2}$. It enforces that $\vec{a}_{0}=a_{0}(980)$ and $K_{S}=K_{0}^{\star}$ (800) in Eq. (2.2). Then decay widths are calculated. We obtain satisfying results in the decay channels $\sigma_{1,2} \rightarrow \pi \pi$ and $\sigma_{1,2} \rightarrow K K$ if we set $m_{\sigma_{1}}=705 \mathrm{MeV}$ and $m_{\sigma_{2}}=1200 \mathrm{MeV}$ leading to $\Gamma_{\sigma_{1} \rightarrow \pi \pi}=305 \mathrm{MeV}$ and $\Gamma_{\sigma_{2} \rightarrow \pi \pi}=207 \mathrm{MeV}$ in the former and $\Gamma_{\sigma_{1} \rightarrow K K}=0$ and $\Gamma_{\sigma_{2} \rightarrow K K}=240$ MeV in the latter channel. These results correspond well to the data [1]. The results also suggest, however, that $f_{0}(1370)$ should predominantly decay into kaons (as $\Gamma_{\sigma_{2} \rightarrow K K} / \Gamma_{\sigma_{2} \rightarrow \pi \pi}=1.15$ ) - not surprising for a predominantly $\bar{s} s$ state but clearly at odds with data [1].
An even larger problem arises in the (axial-)vector channel. The model exhibits interdependence of $\Gamma_{a_{1}(1260) \rightarrow \rho \pi}$ and $\Gamma_{\rho \rightarrow \pi \pi}$ [3]. We obtain $\Gamma_{a_{1}(1260) \rightarrow \rho \pi}>10 \mathrm{GeV}$ if we set $\Gamma_{\rho \rightarrow \pi \pi}=149.1 \mathrm{MeV}$ (as suggested by Ref. [1]). Alternatively, if one forces $\Gamma_{a_{1}(1260) \rightarrow \rho \pi}<600 \mathrm{MeV}$ to comply with the data, then $\Gamma_{\rho \rightarrow \pi \pi}<38 \mathrm{MeV}$ is obtained - approximately 100 MeV less than the PDG value.
Thus the assumption of scalar $\bar{q} q$ states below 1 GeV is strongly disfavoured.
Assumption II: Scalars above 1 GeV are $\bar{q} q$ states. We calculate our parameters as above; however, we now consider $\vec{a}_{0}=a_{0}(1450)$ and $K_{S}=K_{0}^{\star}(1430)$ in Eq. (2.2). i.e., scalar $I=1$ and $I=1 / 2$ states are now assumed above 1 GeV .
The overall fit is better than in the previous case. We observe $\sigma_{1}$ as a broad resonance: $\Gamma_{\sigma_{1} \rightarrow \pi \pi} \simeq$ $(200-350) \mathrm{MeV}$ for $m_{\sigma_{1}} \in[1200,1500] \mathrm{MeV}$. We thus assign $\sigma_{1}$ to $f_{0}(1370)$. The state $\sigma_{2}$ corresponds to $f_{0}(1710)$ : the PDG suggests $\Gamma_{f_{0}(1710) \rightarrow \pi \pi}^{\text {exp. }}=(29.28 \pm 6.53) \mathrm{MeV}$ [1] yielding $m_{\sigma_{2}}^{(1)}=1613$ MeV and $m_{\sigma_{2}}^{(2)}=1677 \mathrm{MeV} ; m_{\sigma_{2}}$ is very close to $m_{f_{0}(1710)}^{\text {exp. }}=(1720 \pm 6) \mathrm{MeV}$. We thus conclude that $f_{0}(1370)$ is predominantly a $\bar{n} n$ state and that $f_{0}(1710)$ is predominantly a $\bar{s} s$ state.
We also observe that $\Gamma_{a_{1}(1260) \rightarrow \rho \pi}$ is dramatically smaller than in the previous case: we obtain $\Gamma_{a_{1}(1260) \rightarrow \rho \pi} \simeq 860 \mathrm{MeV}$. This is admittedly larger than $250 \mathrm{MeV} \leq \Gamma_{a_{1}(1260)}^{\text {exp. }} \leq 600 \mathrm{MeV}$ [1]. However, only a small change of $\Gamma_{\rho \rightarrow \pi \pi}$ is necessary in this case for $\Gamma_{a_{1}(1260) \rightarrow \rho \pi}$ to be within the data: $\Gamma_{a_{1}(1260) \rightarrow \rho \pi}=590 \mathrm{MeV}$ is obtained for $\Gamma_{\rho \rightarrow \pi \pi}=129 \mathrm{MeV}$ (previously: $\Gamma_{\rho \rightarrow \pi \pi}=38 \mathrm{MeV}$ ).

## 3. Summary and Outlook

We have presented a $U(3)_{L} \times U(3)_{R}$ Linear Sigma Model with (axial-)vector mesons. Structure of scalar states $f_{0}(600), f_{0}(1370)$ and $f_{0}(1710)$ has been discussed within the model. Our results suggest that $f_{0}(1370)$ and $f_{0}(1710)$ are strongly favoured as predominantly $\bar{n} n$ and $\bar{s} s$ states, respectively, whereas the consideration of $f_{0}(600)$ and $f_{0}(1370)$ as predominantly $\bar{n} n$ and $\bar{s} s$ states, respectively, yielded unphysically broad $a_{1}(1260)$ state (decay width $>10 \mathrm{GeV}$ ).
Further calculations may include, e.g, a glueball field mixing with the already-present $\bar{q} q$ states.

## References

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