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Matter swapping between two braneworlds from the equivalence between two-brane worlds and noncommutative two-sheeted spacetimes

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It is shown that a two-brane world made of two domain walls can be seen as a noncommutative two-sheeted spacetime under certain assumptions. This equivalence implies a model-independent phenomenology: Matter swapping between the two 3-branes (or sheets) is predicted through fermionic oscillations induced by magnetic vector potentials. This phenomenon, which might be experimentally studied, could reveal the existence of extra dimensions in a new and very affordable way.

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1. Fermion dynamics in a two-brane world and noncommutative geometries

Regarding the dynamics of spin-$1/2$ particles, at low energies any two-brane world related to a domain wall approach [1] is formally equivalent to a noncommutative two-sheeted spacetime $M_4 \times Z_2$ [2]. The demonstration [2] of this result is inspired by quantum chemistry and the construction of molecular orbitals, here extended to branes. Let us consider, for instance, a two-brane world made of two domain walls (two kink-like solitons) on a continuous $M_4 \times R_1$ manifold from:

$$S = \int \left[-\frac{1}{4G^2} \mathscr{F}_{AB} \mathscr{F}^{AB} + \frac{1}{2} g^{AB} \left(\partial_A \Phi\right)\left(\partial_B \Phi\right) - V(\Phi) + \overline{\Psi} \left(i\Gamma^A \left(\partial_A + i\mathscr{A}_A\right) - \lambda \Phi \right) \Psi \right] \sqrt{g} d^5 x \tag{1.1}$$

The continuous real extra dimension R_1 can be replaced by an effective phenomenological discrete two-point space Z_2. At each point along the discrete extra dimension Z_2 there is then a four-dimensional spacetime M_4 endowed with its own metric field. Both branes/sheets are then separated by a phenomenological distance δ which is inversely proportional to the overlap integral of the extra-dimensional fermionic wave functions of each 3-brane over the fifth dimension R_1. Considering the electromagnetic gauge field, it has been also demonstrated that the five-dimensional $U(1)$ bulk gauge field is substituted by an effective $U(1) \otimes U(1)$ gauge field acting in the $M_4 \times Z_2$ spacetime. It is important to stress that the equivalence between the continuous two-domain wall approaches and the noncommutative two-sheeted spacetime model is rather general and does not rely for instance on the domain walls features or on the bulk dimensionality.

The dynamics of a spin-$1/2$ fermion can be then described with a two-brane Dirac equation [2,3]. The derivative operator is: $D_\mu = \mathbf{1}_{8\times 8} \partial_\mu$ ($\mu = 0,1,2,3$) and $D_5 = ig\sigma_2 \otimes \mathbf{1}_{4\times 4}$ with $g = 1/\delta$. One can build the Dirac operator defined as $\not{D} = \Gamma^N D_N = \Gamma^\mu D_\mu + \Gamma^5 D_5$ where: $\Gamma^\mu = \mathbf{1}_{2\times 2} \otimes \gamma^\mu$ and $\Gamma^5 = \sigma_3 \otimes \gamma^5$. γ^μ and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ are the usual Dirac matrices and σ_k ($k = 1,2,3$) the Pauli matrices. By introducing a general mass term M, a two-brane Dirac equation is then derived:

$$(i\not{D}_A - M)\Psi = \begin{pmatrix} i\gamma^\mu(\partial_\mu + iqA_\mu^+) - m & ig\gamma^5 - im_r + i\gamma^5\Upsilon \\ ig\gamma^5 + im_r + i\gamma^5\overline{\Upsilon} & i\gamma^\mu(\partial_\mu + iqA_\mu^-) - m \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = 0 \tag{1.2}$$

with $\psi_\pm$ the wave functions in the branes $(\pm)$ and $\overline{\Upsilon} = \gamma^0\Upsilon^\dagger\gamma^0$. The off-diagonal mass term m_r can be justified from a two-brane(-domain wall) structure of the Universe. The electromagnetic field $\left\{A_\mu^\pm, \Upsilon\right\}$ is introduced through: $\not{D}_A \to \not{D} + \not{A}$, according to the $U(1) \otimes U(1)$ gauge group. Assuming that the electromagnetic field of a brane couples only with the particles belonging to the same brane, we set $\Upsilon \sim 0$.

2. Two-brane Pauli equation and model-independent phenomenology

The non-relativistic limit of the Dirac equation is derived to obtain a two-brane Pauli equation [2,3]: $i\hbar\partial_t \Psi = \{\mathbf{H}_0 + \mathbf{H}_{cm} + \ldots\}\Psi$, with $\mathbf{H}_0 = diag(\mathbf{H}_+, \mathbf{H}_-)$, where $\mathbf{H}_\pm$ are simply the usual four-dimensional Pauli Hamiltonian expressed in each branes. Moreover, a new fundamental coupling term appears [2,3] specific to the two-brane world:

$$\mathbf{H}_{cm} = igg_s\mu \frac{1}{2} \begin{pmatrix} 0 & -\sigma \cdot \{\mathbf{A}_+ - \mathbf{A}_-\} \\ \sigma \cdot \{\mathbf{A}_+ - \mathbf{A}_-\} & 0 \end{pmatrix} \tag{2.1}$$

$\mathbf{A}_{\pm}$ are the magnetic vector potentials in the branes $(\pm)$. $g_s\mu$ is the magnetic moment of the particle. This specific term induces a coupling between the two branes through the magnetic vector potentials of each brane and the fermionic magnetic moment.

The two-brane Pauli equation supports resonant solutions [4]. Let us consider a neutron under the influence of a rotative magnetic vector potential $\mathbf{A}_p$ (with an angular frequency ω) localized in our brane. The probability to find the neutron in the second brane is [4]:

$$P(t) = \frac{4\Omega_p^2}{(\Omega_0-\omega)^2+4\Omega_p^2}\sin^2\left((1/2)\sqrt{(\Omega_0-\omega)^2+4\Omega_p^2}t\right) \tag{2.2}$$

where $\Omega_p = gg_s\mu A_p/(2\hbar)$ and $\Omega_0 = (V_+ - V_-)/\hbar$, which defines the interactions of the particle with its environment ($V_{\pm}$ are the potential energies of the particle in each brane). When $\omega = \Omega_0$, the particle then resonantly oscillates between the branes.

To investigate this matter swapping effect, a possibility would be to study the population of a stored ultracold neutron gas. The disappearance of neutrons by the presently discussed mechanism could be observed by counting the remaining neutrons in a vessel. Two kind of experimental devices could be used to that end. Some papers [5] suggest the existence of an astrophysical ambient magnetic vector potential from astrophysical magnetic fields. Maybe, this ambient field could be responsible for non-resonant swapping, which could be soon be tested by teams working with ultracold neutrons [2,3]. Also, a resonant experiment [4] involving pulsed monochromatic electromagnetic radiation could be considered as an artificial means to produce this matter swapping.

3. Conclusions and outlooks

A universe which contains at least two branes can be modeled by a $M_4 \times Z_2$ two-sheeted spacetime in the formalism of the noncommutative geometry. The dynamics of fermions in a two-brane world can then be studied independently of the domain wall formalism. A new effect, which corresponds to an exchange of fermionic matter between the two braneworlds may be revealed by using convenient magnetic vector potentials. This matter swapping might be investigated by using the current technology. A work is in progress to compare the predictions of the model with already published experimental data. In a forthcoming study, it will be shown that this presently discussed effect could also be included in the superstring formalism. The study of the dynamics induced by the variations of the coupling constant g is scheduled.

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