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Probing long distance QCD effects at LHC through the total cross-section

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We present a model for exploring the confinement region of QCD by means of the total cross-section. Our results are compared with the present LHC data.

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1. Introduction

Total cross-sections provide a way to access the question of confinement from a phenomenological point of view. This is so for the obvious reason that the bulk of particle scattering takes place at large distance, with only a tiny minority of events occurring at the small distances whose dynamics is described by perturbative QCD. To access very large distances, one needs a formalism linking an observable such as the total cross-section to very low momentum gluons. We have proposed to do it through the mechanism of soft gluon resummation. Since Bloch and Nordsieck wrote their fundamental paper on the radiation field of electrons [1], summation of soft quanta emitted in a collision has remained of central interest both in QED and QCD, for different reasons. In QED, this was due to the importance of extracting information about theoretical quantities from the measurements which are irreducibly affected by soft photon emission. In QCD, the focus is not only on the calculation of hadronic backgrounds for high energy experiments (often identified as minimum bias effects), but , as we propose, on the possiblity to use the resummation tool to study the infra red (IR) region.

Our program includes providing a formalism in which soft gluon resummation is linked to the total cross-section, but also a revisitation of soft gluon resummation to include the infrared region, and an ansatz for the effective soft gluon coupling to the quark field when the gluon momenta go to zero. This ansatz relates the one gluon exchange potential to the infrared coupling for soft gluons. We are then able to link the singularity of the infrared coupling to the asymptotic Froissart bound.

In the following, after describing the model we have developed with our collaborators, we summarize our most recent results for total cross-section phenomenology [2].

2. The Froissart bound for the total cross-section

Although the most popular and successful parametrization for the energy dependence of the total cross-section has been the Regge inspired formula by Donnachie and Landshoff [3], i.e.

$$\sigma_{total} = X s^{-\eta} + Y s^{\varepsilon} \tag{2.1}$$

with $\eta \sim 0.5$ and $\varepsilon \lesssim 0.1$, this formulation is at variance with the expectations from the Froissart bound based unitarity and analyticity, namely that $\sigma_{total} \lesssim \log^2 s$. This bound was established in the 1960s [4, 5, 6], but it is already present in the Heisenberg calculation of the total cross-section [7]. According to Heisenberg, the energy dependence for production of mesons in a high energy reaction can be a *constant* or rise as much as $\log^2 s$, depending on whether the average pion energy rises proportionally to the energy or is limited by a constant as the energy increases. In either case, the derivation is based on a limitation of the spatial extension of the emitted pion cloud, namely on the existence of a cut-off, b_{max}, in impact parameter space. A similar condition is also present in the derivation of the Froissart bound: namely, the existence of a maximum value L_{max} in the partial wave expansion of the elastic scattering amplitude is integral part of the derivation. If one wishes to understand total cross-sections in QCD it is thus necessary to uncover the presence of a cut-off which can produce the observed logarithmic rise of the total cross-section at high energies. This has been the driving idea behind our approach, as would become clearer when we discuss an infra red (IR) singular expression for the strong coupling α_s.

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In our approach, an asymptotic cut-off is provided by a singular behaviour of the effective quark-gluon coupling for gluon momenta $0 \leq k_t \leq \Lambda$, where Λ is O(Λ_{QCD}). The argument we developed in [8] is based on an asymptotic behaviour of the total cross-section such that

$$\sigma_{tot}(s) \approx 2\pi \int db^2[1 - exp[-C(s)e^{-(b\bar{\Lambda})^{2p}})]] \tag{2.2}$$

In this expression, $C(s)$ is obtained from a leading order (LO) calculation of QCD mini-jet cross-sections, which increase as $\sim s^{\varepsilon}$. The cut-off in b-space, obtained for $1/2 \leq p \leq 1$, appears from emission of singular infrared gluons, as we discuss in Sect. 3. Eq. (2.2), whose derivation is based on the model described in the next section, leads to an asymptotic behaviour compatible with the Froissart bound, i.e.

$$\sigma_{tot}(s) \to [\varepsilon \log(s)]^{1/p}. \tag{2.3}$$

3. Model in impact parameter space

We describe here the formalism we use in order to link soft gluon resummation to the total cross-section. It is based on the eikonal representation for the scattering amplitude, i.e., for $t = -q^2$,

$$F(s,t) = \int d^2\mathbf{b} f(b,s) = i \int d^2\mathbf{b} e^{i\mathbf{q}\cdot\mathbf{b}}[1 - e^{i\chi(b,s)}] \tag{3.1}$$

from where one obtains

$$\frac{d^2\sigma_{elastic}}{d^2\mathbf{b}} = |1 - e^{i\chi(b,s)}|^2 \tag{3.2}$$

$$\sigma_{total}(s) = 2\int d^2\mathbf{b}\Re e[1 - e^{i\chi(b,s)}] = 2\int d^2\mathbf{b}[1 - \cos\Re e\chi(b,s)e^{-\Im m\chi(b,s)}] \tag{3.3}$$

$$\sigma_{inel} = \sigma_{total} - \sigma_{elastic} = \int d^2\mathbf{b}[1 - e^{-2\Im m\chi(b,s)}] \tag{3.4}$$

Experimentally, the real part of the scattering amplitude at $t = 0$ is only a fraction of the imaginary part, so that, for what concerns the calculation of $\sigma_{total}(s)$, one can put $\Re e\chi(b,s) \approx 0$ in Eq. (3.3). This is an approximation which will need to be revised when dealing with $t \neq 0$, such as is the case for the differential elastic cross-section.

Mini-jet models attribute the rise of the total cross-section to the rising number of low-x parton collisions. We follow the description advanced quite some time ago by Durand and collaborators [9], who embedded mini-jets in the eikonal reprentation, with $\Im m\chi(b,s) \propto \sigma_{mini-jets}$. As mentioned, and as is well known, if one uses actual parton density functions, PDFs, as parametrized at LO through Deep Inelastic Scattering, mini-jet cross-sections are seen to rise with energy as a power law, s^{ε}, with $\varepsilon \sim 0.3 - 0.4$. If the eikonal is factorized into a cross-section and an impact parameter distribution with no residual s-dependence, it is difficult to describe both the early, almost turbulent, rise and the subsequent gentle behaviour of the total cross-section, without introducing *ad hoc* parametrizations of the mini-jet contributions [10]. Our proposal is a model in which both the rise and the cut-off in b-space producing the levelling off, are obtained from QCD. This model uses library available PDFs at LO and LO parton cross-sections to calculate the mini-jet contribution, i.e., for collisions between two hadrons A and B, we have

$$\sigma^{AB}_{mini-jets}(s,pt)min) = \int_{p_{tmin}}^{\sqrt{s}/2} dp_t \int_{4p_t^2/s}^{1} dx_1 \int_{4p_t^2/(x_1 s)}^{1} dx_2 \sum_{i,j,k,l} f_{i|A}(x_1,p_t^2) f_{j|B}(x_2,p_t^2) \frac{d\hat{\sigma}^{kl}_{ij}(\hat{s})}{dp_t} \tag{3.5}$$

where p_{min} separates the perturbative from the non -perturbative QCD regime. $f_{i|A}(x_1,p_t^2)$ are PDFs for extracting partons of type i from hadron A, and are DGLAP evolved at $Q^2 = p_t^2$. We write

$$2\Im m\chi(b,s) = \bar{n}_{soft}b,s + \bar{n}_{mini-jet}(b,s) \tag{3.6}$$

At low energies, $\sqrt{s} \lesssim 10\ GeV$, the mini-jet contribution is very small, and the average number of collisions $\bar{n}_{soft}(b,s)$ is parametrized as in [11]. For the term $\bar{n}_{mini-jet}(b,s)$ which is proportional to the mini-jet contribution, we propose a QCD explanation for the impact parameter distribution of partons, as follows: the mini-jet expression of Eq. (3.5) assumes the colliding partons to be collinear. Initial state radiative corrections, which are energy dependent, change this acollinearity. The acollinearity reduces the rise of the mini-jet cross-section and is the missing factor in QCD models for mini-jet production. In our approach, we use the Fourier transform of the initial state transverse momentum distribution arising from soft gluon emission to model the b-dependence of the mini-jet term of the eikonal.

4. A model for Soft Gluon Resummation in the Infrared region

In order to probe large distances, as mentioned, we need to study very low momenta. In QED, the infrared divergence due to the zero mass of the associated gauge field, does not cancel in the amplitude, only in the cross-sections. Thus what is important is to find the probability for overall emission of soft quanta. Resummation of soft photons to all orders led to the well known expression [12, 13, 14] for the probability of emission of a 4-momentum K_μ in charged particle collisions:

$$d^4P(K) = \int \frac{d^4K}{(2\pi)^4} e^{iK\cdot x} exp[-\int \frac{d^3k}{2k_0} |j_\mu(k)|^2 (1-e^{-ik\cdot x})] \tag{4.1}$$

where

$$j_\mu(k) = \frac{ie}{(2\pi)^{3/2}} \frac{\sum_i \varepsilon_i p_{i\mu}}{p_i \cdot k} \tag{4.2}$$

for emission of a real photon of momentum **k** from an electron or positron of momentum $p_{i\mu}$, with $\varepsilon_i = \pm 1$ depending on whether the electron or positron is entering or leaving. The expression in Eq. (4.1) exhibits the summation of all soft photons into an exponential factor as well as the cancellation of the infrared divergence between real and virtual photons. The real photon contribution is multiplied by the factor $e^{ik\cdot x}$. This factor correlates the individual emitted photons of momentum k_μ to the total energy-momentum K_μ of the emitted radiation. The cancellation between real and virtual photons as the individual photon 4-momentum goes to zero can occurr because the distinction between a real, $k^2 = 0$, and a virtual, $k^2 \neq 0$, photon disappears as the photon 4- momentum goes to zero. Indeed, individual soft photons are not an observable.

When dealing with resummation in QCD, our knowledge of the coupling is limited to high Q^2 values and this usually prevents studying the infrared region. However, one can distnguish our

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ignorance about the value of the coupling in region of confinement, from the question of whether a cancellation between real and virtual gluons takes place when the gluon momenta go to zero. To overcome the difficulty, one can first ignore the details of the interaction, QED or QCD, and try to derive an expression similar to Eq. (4.1) on purely statistical basis, as done in [15] for the case of photons. The argument, semi-classical, runs as follows.

Consider the probability of having a total energy-momentum loss K_μ due to gluon emission in a scattering process, such as the one between quarks. This total emission can be constructed through the many possible ways in which $n_{\mathbf{k}}$ gluons of momentum $\mathbf{k}$ can give rise to a given total energy loss K_μ and then summing on all the values of $\mathbf{k}$. In this formulation, one obtains a total energy-momentum loss K_μ through emission of $n_{\mathbf{k}_1}$ gluons of momentum $\mathbf{k}_1$, $n_{\mathbf{k}_2}$ gluons of momentum $\mathbf{k}_2$ and so on. If one can assume that the gluons are all emitted independently (the effect of their emission on the source particle is neglected), each one of these distributions is a Poisson distribution, and the probabilty of a 4-momentum loss in the interval d^4K is written as

$$d^4P(K) = \sum_{n_{\mathbf{k}}} \Pi_{\mathbf{k}} P(\{n_{\mathbf{k}}, \bar{n}_{\mathbf{k}}\}) \delta^4(K - \sum_k k n_{\mathbf{k}}) d^4K \tag{4.3}$$

where the Bloch and Nordsiek's result of independent emission is introduced through the Poisson distribution $P(\{n_{\mathbf{k}}, \bar{n}_{\mathbf{k}}\})$,

$$P(\{n_{\mathbf{k}}, \bar{n}_{\mathbf{k}}\}) = \frac{\bar{n}_{\mathbf{k}}^{n_{\mathbf{k}}}}{n_{\mathbf{k}}!} exp[-\bar{n}_{\mathbf{k}}] \tag{4.4}$$

and four momentum conservation is ensured through the 4-dimensional δ-function, which selects the distributions $\{n_{\mathbf{k}}, \bar{n}_{\mathbf{k}}\}$ with the right energy momentum loss K_μ. Using the integral representation for the δ-function, one can invert the order between performing the sum with the product in Eq. (4.3). One can then perform the sum over the $n_{\mathbf{k}}$ and obtain

$$d^4P(K) = \frac{d^4K}{(2\pi)^4} \int d^4x \, exp[-h(x) + iK \cdot x] \tag{4.5}$$

with

$$h(x) = \sum_{\mathbf{k}} \big(1 - exp[-ik \cdot x]\big) \bar{n}_{\mathbf{k}} \tag{4.6}$$

or, passing from the discrete to the continuum,

$$h(x) = \int d^3\bar{n}_{\mathbf{k}} \big(1 - exp[-ik \cdot x]\big) \tag{4.7}$$

The above derivation does not specify what the soft gluon distribution $d^3\bar{n}_{\mathbf{k}}$ is, only that it is possible to define its integral. Since the expression in the round bracket of Eq. (4.7) goes to zero as $k \to 0$, this procedure shows that the integral can be finite even if the single guon spectrum is singular. Writing $d^3\bar{n}_{\mathbf{k}} = (d^3k/2k) g(k)/k^2$, for this procedure to be finite even when the integral extends down to $k = 0$, one must require the function $g(k)$ to be less singular than $1/k$ as $k \to 0$.

Integrating the expression in Eq. (4.5) over the energy and longitudinal momentum variables, one can obtain a transverse momentum distribution for emitted radiation [16, 17]. To proceed further in QCD, just as in QED, one needs to specify the single particle, gluon or photon, distribution $d^3\bar{n}_{\mathbf{k}}$. In perturbative QCD, using the asymptotic freedom expression for the strong coupling constant, one can only specify this distribution for $k > \Lambda_{QCD} \sim 100 - 200 \; MeV$. With a lower cut-off in the single gluon momentum, as was done in [17] and in [18] for the case of Drell-Yan pairs, the second term in Eq. (4.7) is no longer necessary, since the infrared region is excluded from integration. The integrals can then be done analytically.

Since our aim is to study the infrared region, we propose instead to retain the second term and use an *ad hoc* expression for the coupling in the infrared region, such as allow the integration. There are a number of possibilities, among them a frozen $\alpha_s(k_t < \Lambda) = \bar{\alpha}$, as in [19]. Our proposal is a power law behaviour , i.e. $\alpha_s(k_t < \Lambda) = (\Lambda/k_t)^{2p}$, with $p < 1$ for the integral in Eq. (4.7) to be finite. This proposal has led us to obtain the intrinsic transverse momentum of Drell-Yan pairs as given by

$$< p_t^2(\sqrt{s}) >= constant \int_0^\Lambda k_t dk_t \alpha_s(k_t) \ln[2\sqrt{s}/k_t] \tag{4.8}$$

To summarize, our proposal for the transverse momentum distribution of a pair of initially collinear partons, which acquire acollinearity through soft gluon emission is

$$\begin{aligned}
\Pi(K_t) &\equiv \frac{d^2P()K_t}{d^2K_t} = \int \frac{d^2b}{(2\pi)^2} e^{-i\mathbf{K}_t\cdot\mathbf{b} - h(b)} \\
h(b,E) &= \frac{16}{3\pi}\int_0^E \frac{dk_t}{k_t}\alpha_{eff}(k_t)\ln[\frac{2E}{k_t}](1-J_0(k_tb)) \\
\alpha_{eff}(k_t) &= \frac{12\pi}{33-2N_f}\frac{p}{\log[1+p(k_t/\Lambda)^{2p}]}
\end{aligned} \tag{4.9}$$

The expression we have used for the strong coupling constant is such as to interpolate between our proposed singular but integrable α_s and the usual one loop asymptotic freedom expression. Such an expression is what we propose to use to probe QCD at large distances through total cross-section phenomenology. The Fourier transform of the above distribution is used to describe the distribution of partons in impact parameter space at high energy, in lieu of the Fourier transform of the EM form factors, as is more customary in the impact parameter representation. In the following section, we apply the above equations to estimate total cross-sections at LHC.

5. Comparison between the model and LHC data at $\sqrt{s} = 7\ TeV$

Our model is based on using library available LO PDFs, such as GRV [20] or MRST [21], as input to the following set of equations:

$$\begin{aligned}
\sigma_{total} &= 2\int d^2\mathbf{b}[1-e^{-\bar{n}(b,s)/2}] \qquad \Re e\chi(b,s)\approx 0 \\
\bar{n}(b,s) &= \bar{n}_{low}(b,s) + \bar{n}_{mini-jet}(b,s) \\
\bar{n}_{mini-jet} &= A(b,s)\sigma_{mini-jet}(s,p_{tmin}) \\
A(b,s) &= \frac{e^{-h(b,q_{max})}}{\int d^2\mathbf{b}\, exp[-h(b,q_{max})]}
\end{aligned} \tag{5.1}$$

In these equations, the low energy component $\bar{n}_{low}(b,s)$ is parametrized with no rising term, and the rise with energy is obtained solely through the high energy term $\bar{n}_{mini-jet}(b,s)$. A part from the

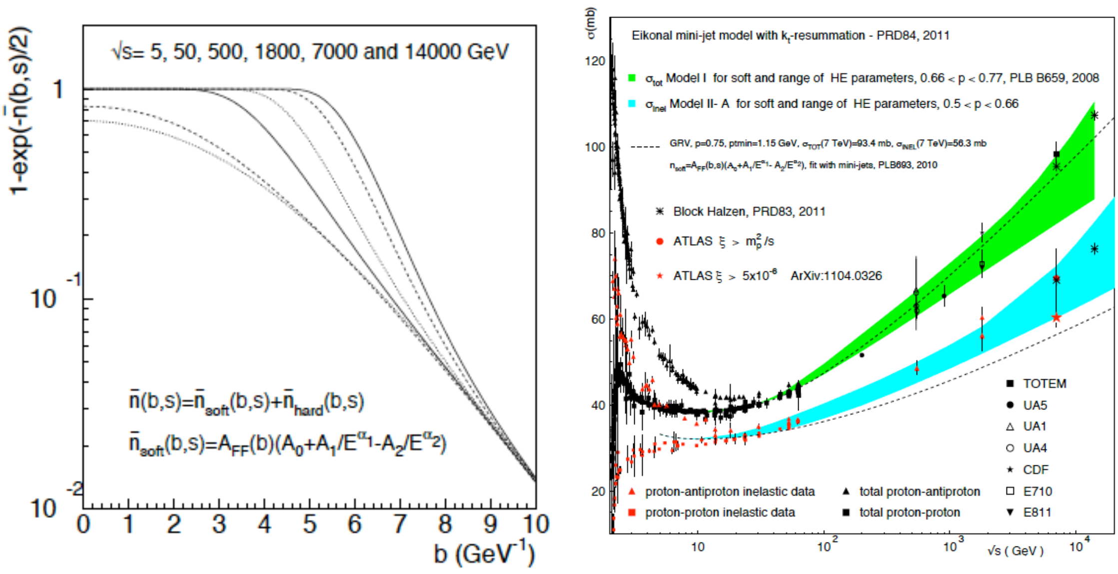

Figure 1: The curves in the panel at left and the two dotted lines in the panel at right correspond to the same set of high energy parameters, $p_{tmin} = 1.15\ GeV, p = 0.75$, GRV [20] densities. The two bands in the panel at right correspond to choosing a range of high energy parameters for $\bar{n}(b,s)$ as discussed in [2]. Our results are compared with data up to the recent ATLAS[22], CMS [23] and TOTEM [24]experiment. We also compare predictions and data with the recent description from Block and Halzen [25].

low energy parameters, the model depends on a set of high energy parameters such as the choice of PDFs and p_{tmin} and on the infrared power p . The following scales define the high energy dependence :

- for a given choice of PDFs, the value of p_{tmin} determines the beginning of the rise: to describe the CERN Intersecting Storage Ring data, it is chosen to be $\sim 1.1\ GeV$;
- the quantity q_{max} depends on p_{tmin} and on the chosen PDFs, and defines the maximum transverse momentum allowed for single gluon emission in a given collision, averaged over the PDFs: it is a slowly varying function of $\sqrt{s}$ and of the order of p_{tmin};
- the parameter p, with the condition $1/2 < p < 1$, respectively for consistency with a rising one-gluon potential and an integrable spectrum, regulates the singularity of the single gluon distribution and controls the softening of the rise: everything else being equal, higher values of p correspond to more saturation and thus to a slower rise,
- $\Lambda \sim 100\ MeV$ is the scale in the effective coupling constant in the resummed spectrum.

We show in Fig. 1 our result for the elastic amplitude in impact parameter space at different c.m. energies, at left, and, at right, the total and inelastic cross-sections as obtained from our model. A detailed description of our results can be found in [2] and references therein.

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